TP 3.2
Ball speeds and distances after stun-shot impact

from:
"The Illustrated Principles of Pool and Billiards"
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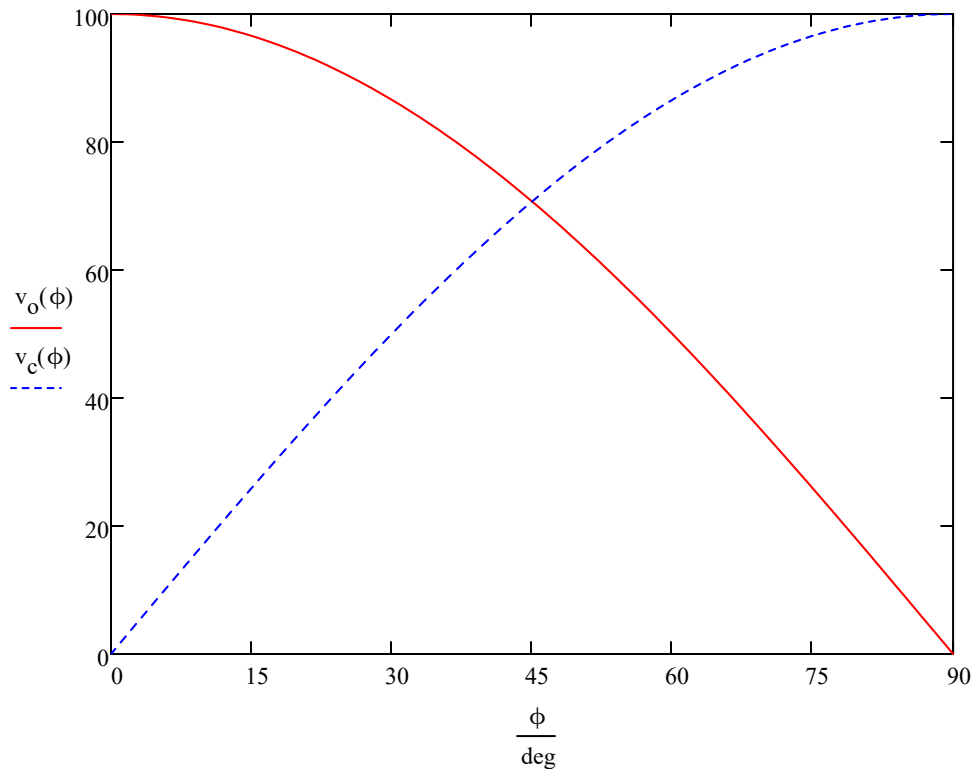**For a rolling cue ball shot, see TP A.16.**
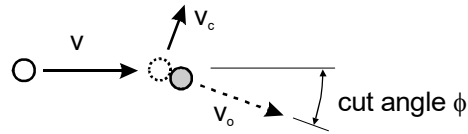
From conservation of momentum (see TP 3.1),

$v := 100$ incoming speed, scaled to 100

$v_o(\phi) := v \cdot \cos(\phi)$ post-impact object ball speed

$v_c(\phi) := v \cdot \sin(\phi)$ post-impact cue ball speed

$\phi := 0 \cdot \deg, 1 \cdot \deg \, .. \, 90 \cdot \deg$

From TP A.23, cut angle and ball-hit fraction are related according to:

$$\phi(f) := \operatorname{asin}(1 - f) \qquad\qquad f(\phi) := 1 - \sin(\phi)$$

Here are the ball speeds and CB-to-OB speed ratios for various ball-hit fractions:

$$f_\phi := 1 \qquad \phi_f := \phi(f_\phi) = 0 \cdot \deg \qquad v_c(\phi_f) = 0 \qquad v_o(\phi_f) = 100 \qquad \frac{v_c(\phi_f)}{v_o(\phi_f)} = 0$$

$$f_\phi := \frac{3}{4} \qquad \phi_f := \phi(f_\phi) = 14.5 \cdot \deg \qquad v_c(\phi_f) = 25 \qquad v_o(\phi_f) = 96.8 \qquad \frac{v_c(\phi_f)}{v_o(\phi_f)} = 0.26$$

$$f_\phi := \frac{1}{2} \qquad \phi_f := \phi(f_\phi) = 30 \cdot \deg \qquad v_c(\phi_f) = 50 \qquad v_o(\phi_f) = 86.6 \qquad \frac{v_c(\phi_f)}{v_o(\phi_f)} = 0.58$$

$$\phi_f := 45 \cdot \deg \qquad f_\phi := f(\phi_f) = 0.29 \qquad v_c(\phi_f) = 70.7 \qquad v_o(\phi_f) = 70.7 \qquad \frac{v_c(\phi_f)}{v_o(\phi_f)} = 1$$

$$f_\phi := \frac{1}{4} \qquad \phi_f := \phi(f_\phi) = 48.6 \cdot \deg \qquad v_c(\phi_f) = 75 \qquad v_o(\phi_f) = 66.1 \qquad \frac{v_c(\phi_f)}{v_o(\phi_f)} = 1.13$$

$$f_\phi := 0 \qquad \phi_f := \phi(f_\phi) = 90 \cdot \deg \qquad v_c(\phi_f) = 100 \qquad v_o(\phi_f) = 0$$

From TP 4.1, the final ball speeds (after rolling develops) are 5/7 of the initial sliding speeds. Also, with constant rolling resistance, the travel distance is proportional to the square of the speed (as in TP 4.1), so the approximate relative distances traveled by each ball, compared to the distance the stunned cue ball would travel without collision (dmax), is:

$$d_o(\varphi) := \left(\frac{5}{7}\cdot v_o(\varphi)\right)^2 \qquad d_c(\varphi) := \left(\frac{5}{7}\cdot v_c(\varphi)\right)^2 \qquad d_{max} := d_c(90\cdot deg)$$

$$\phi := 0\cdot deg, 1\cdot deg .. 90\cdot deg$$
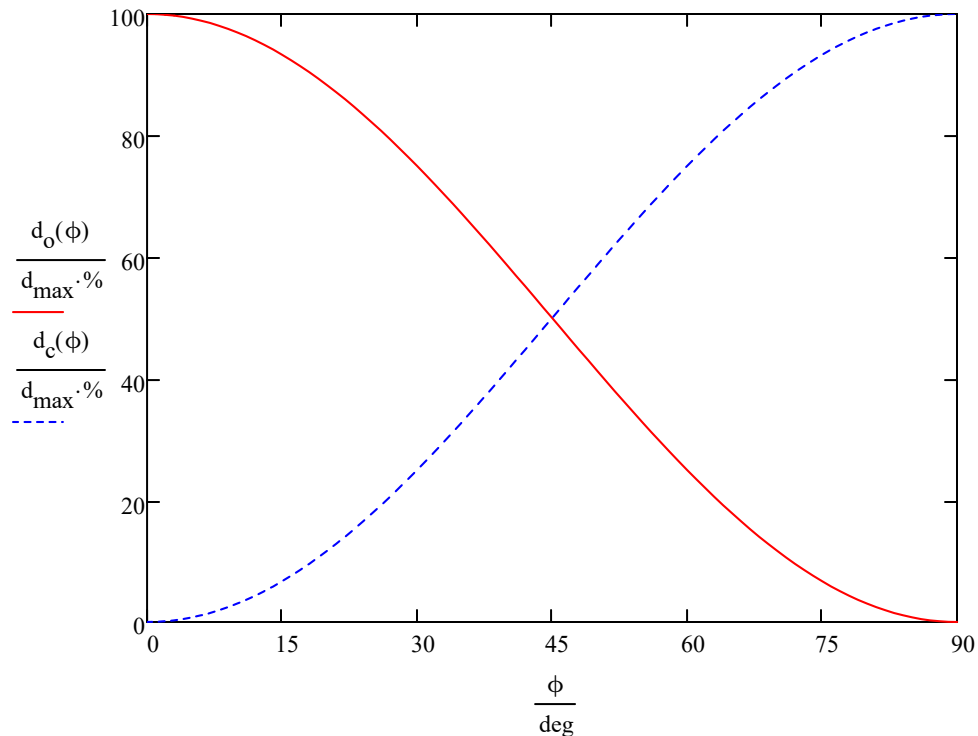
At a 45-degree cut angle, the post-impact speeds are equal and about 70% of the initial cue ball speed (see the graph on the previous page). For the same cut angle, the post-impact ball-travel distances are also equal and 50% of the distance the stunned cue ball would travel if there were no collision (see the graph above).

The graph above can also be expressed in terms of ball-hit fraction: $f(\phi) := 1 - \sin(\phi)$
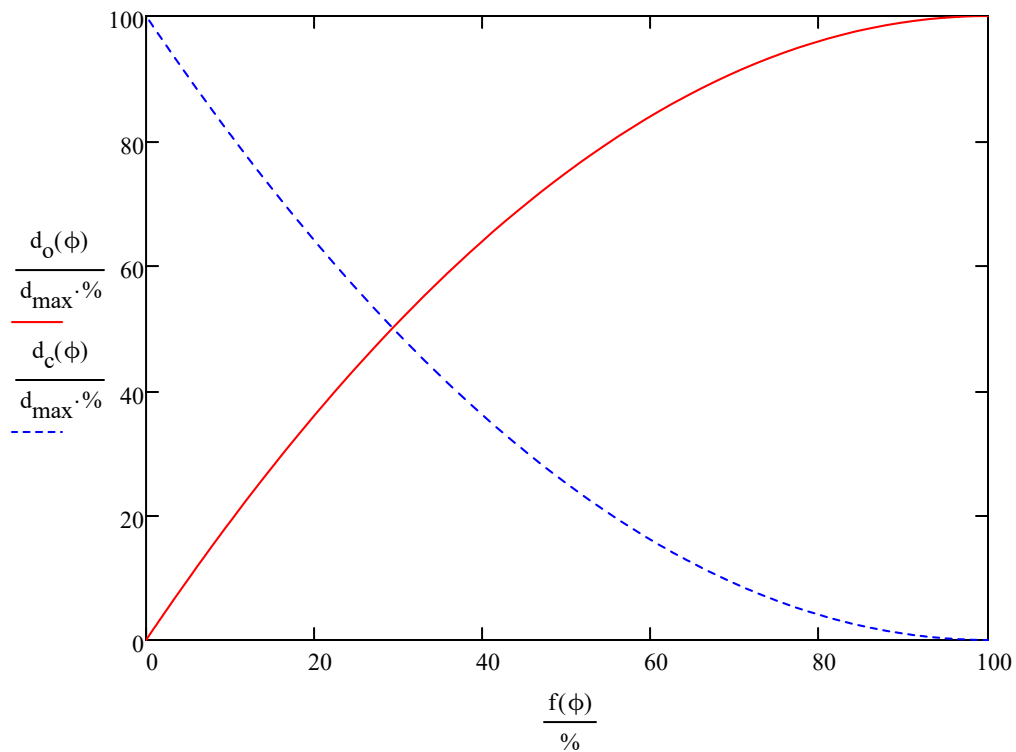
A 45-degree cut angle is between a 1/4-ball and 1/3-ball hit:

$$\frac{1}{4} = 25 \cdot \% \qquad f(45 \cdot \deg) = 29.289 \cdot \% \qquad \frac{1}{3} = 33.333 \cdot \%$$

Interestingly, with a stun shot, **the speed fraction the CB loses when hitting the OB is the same as the ball-hit fraction** (BHF). For example, with a 1/2-ball hit, the CB loses 1/2 of its speed; and with a full hit (BHF=1), it loses all of its speed:

$$\frac{v_c(\phi)}{v} = \sin(\phi) = 1 - f(\phi)$$

Here are travel distance ratios for various ball-hit fractions and cut angles:

$$f(\phi) := 1 - \sin(\phi) \qquad \phi(f) := \mathrm{asin}(1 - f)$$

<table>
<tr><td></td><td></td><td></td><td align="center">**approximate
OB:CB distance
ratio:**</td></tr>
</table>

full-ball hit:

$$f_x := 1 \qquad\qquad \varphi_x := \phi(f_x) = 0 \cdot \deg \qquad\qquad \text{CB stops in place,}$$

CB stops in place,
OB travels same distance CB would have

3/4-ball hit:

$$f_x := \frac{3}{4} \qquad\qquad \varphi_x := \phi(f_x) = 14.5 \cdot \deg \qquad \frac{d_o(\varphi_x)}{d_c(\varphi_x)} = 15 \qquad\qquad 15 : 1$$

1/2-ball hit (30-degree cut):

$$f_x := \frac{1}{2} \qquad\qquad \varphi_x := \phi(f_x) = 30 \cdot \deg \qquad \frac{d_o(\varphi_x)}{d_c(\varphi_x)} = 3 \qquad\qquad 3 : 1$$

45-degree cut:

$$f_x := f(45 \cdot \deg) = 29.289 \cdot \% \qquad \varphi_x := 45 \cdot \deg \qquad \frac{d_o(\varphi_x)}{d_c(\varphi_x)} = 1 \qquad\qquad 1 : 1$$

1/4-ball hit:

$$f_x := \frac{1}{4} \qquad\qquad \varphi_x := \phi(f_x) = 48.6 \cdot \deg \qquad \frac{d_o(\varphi_x)}{d_c(\varphi_x)} = 0.78 \qquad\qquad 1.5 : 2$$

3/16-ball hit (between 1/4 and 1/8):

$$f_x := \frac{3}{16} \qquad\qquad \varphi_x := \phi(f_x) = 54.3 \cdot \deg \qquad \frac{d_o(\varphi_x)}{d_c(\varphi_x)} = 0.51 \qquad\qquad 1 : 2$$

1/8-ball hit:

$$f_x := \frac{1}{8} \qquad\qquad \varphi_x := \phi(f_x) = 61 \cdot \deg \qquad \frac{d_o(\varphi_x)}{d_c(\varphi_x)} = 0.31 \qquad\qquad 1 : 3$$