technical proof

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TP 3.4
Margin of error based on distance and cut angle

from:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
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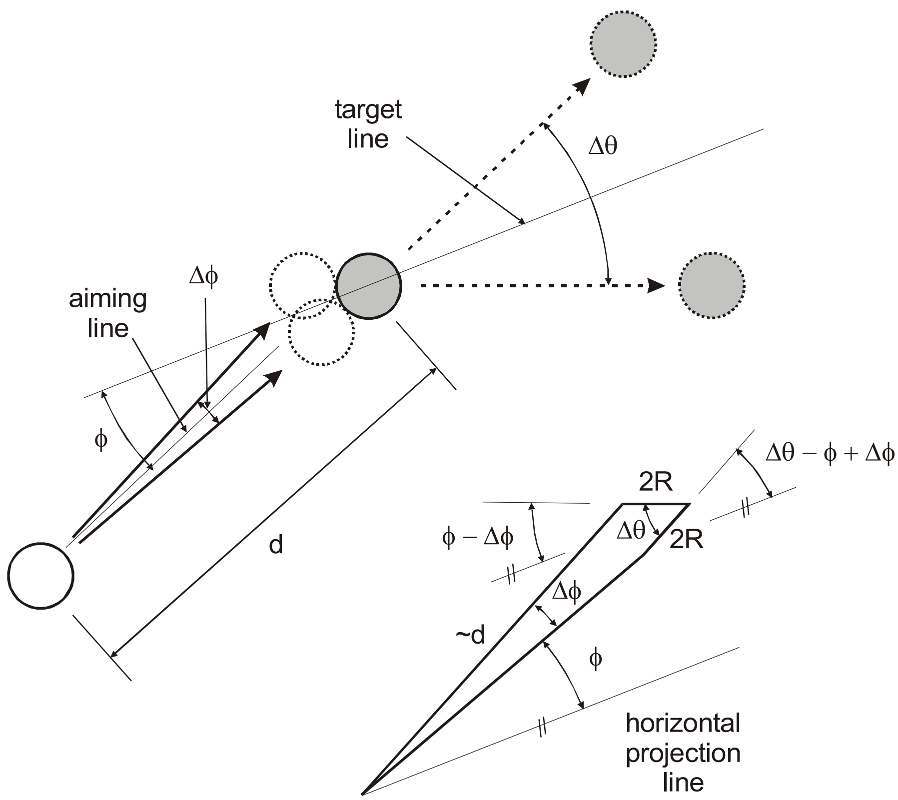


ball radius: $R := 1.125$ ϕ: cut angle θ: object ball leaving angle

d: distance between cue ball (CB) and object ball (OB)

Assume: d >> R

From the bottom right portion of the figure above, equating the vertical components of the loop (perpendicular to the horizontal project line) gives:

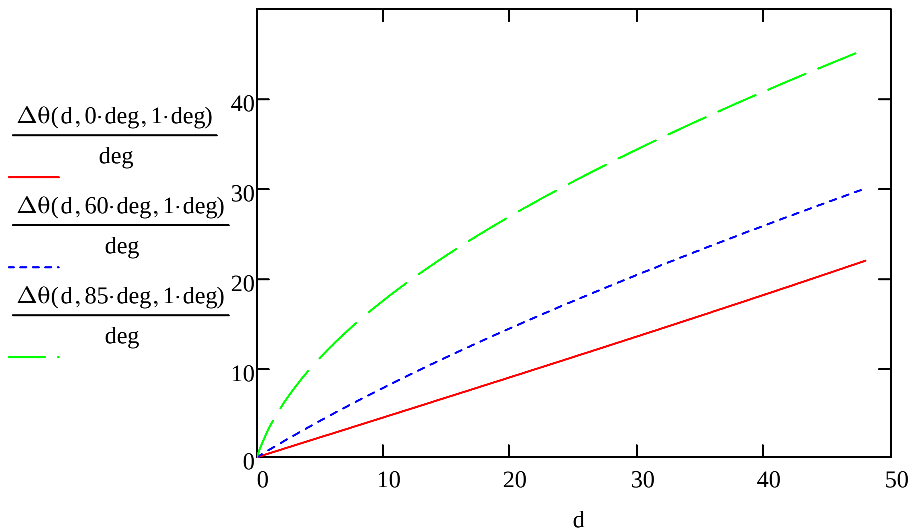
$$d \cdot \sin(\Delta\phi) = 2 \cdot R \cdot \sin(\Delta\theta - \phi + \Delta\phi) + 2 \cdot R \cdot \sin(\phi - \Delta\phi)$$

Therefore,

$$\Delta\theta(d,\phi,\Delta\phi) := \phi - \Delta\phi + \operatorname{asin}\left(\frac{d}{2\cdot R}\cdot\sin(\Delta\phi) - \sin(\phi - \Delta\phi)\right) \tag{1}$$

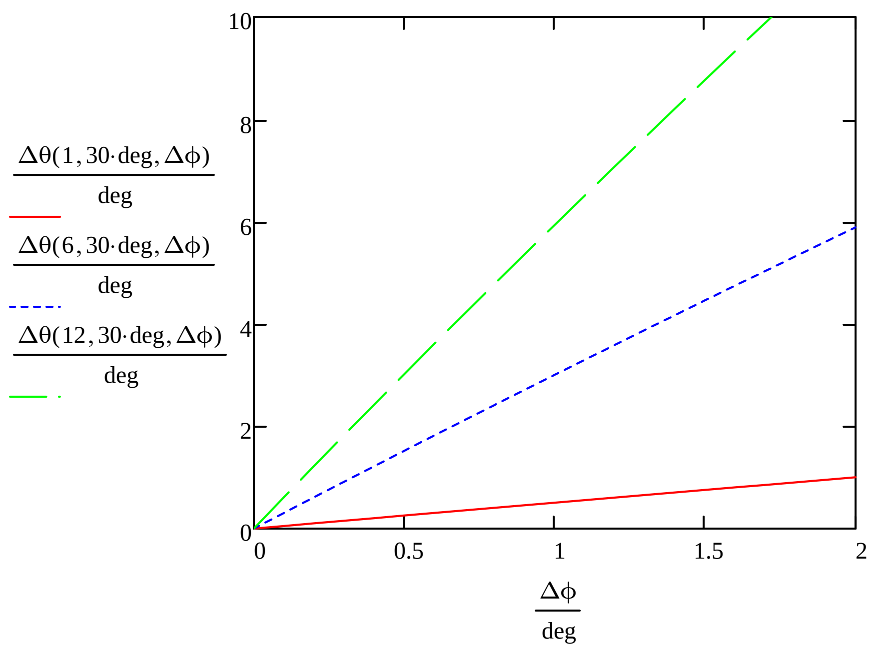
The following plot shows how the angular error in OB motion (Δθ) varies with distance between the CB and OB (d) at different cut angles (ϕ) for a given angular margin for error (Δϕ) in CB motion (1 deg):

$$d := 0, 0.1 .. 48$$



The following plot shows how the angular error in OB motion (Δθ) varies with CB angle margin for error (Δϕ) at different distances between the CB and OB (d) for a 1/2-ball hit (30 deg cut angle):

$$\Delta\phi := 0\cdot deg, 0.1\cdot deg .. 2\cdot deg$$



Example: *What is the allowable object ball (OB) angle error and required cue ball (CB) angle accuracy for a slow, 30-degree cut angle (half-ball hit) shot straight into a corner pocket (i.e., the angle to the pocket is 0 degrees) if the distances between the CB and OB and the OB and pocket are both about 4 diamonds (about 4.5' = 54" on a 9' table)?*

From Figure 3.41 in the book, the approximate allowable object ball angle error is:

$$\Delta\theta_{allowable} := 2 \cdot deg$$

The distance between the CB and OB is:

$$d := 54$$

and the cut angle for a half-ball hit is:

$$\phi := 30 \cdot deg$$

Therefore, the required cue ball angle accuracy is:

initial guess: $\Delta\phi_{required} := 0.1 \cdot deg$

Given

$$\Delta\theta\left(d, \phi, \Delta\phi_{required}\right) = \Delta\theta_{allowable}$$

$$\text{Find}\left(\Delta\phi_{required}\right) = 0.073 \cdot deg$$

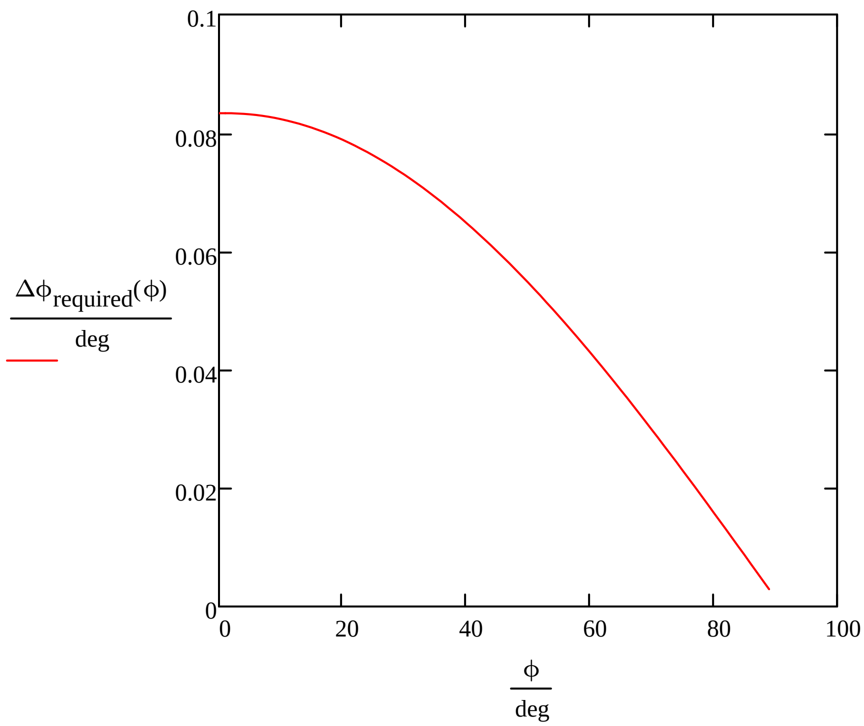
So the margin for error for the CB in this shot is very small
(less than 0.1 degree!)

Allowable cue ball margin for error for various cut angles for the example shot above:

$\Delta\phi := 0.01\cdot\text{deg}$ initial guess

$\Delta\phi_{\text{required}}(\phi) := \text{root}\left(\Delta\theta(d,\phi,\Delta\phi) - \Delta\theta_{\text{allowable}}, \Delta\phi\right)$

$\phi := 0\cdot\text{deg}, 1\cdot\text{deg} .. 90\cdot\text{deg}$



$\Delta\phi_{\text{required}}(0\cdot\text{deg}) = 0.083\cdot\text{deg}$

$\Delta\phi_{\text{required}}(30\cdot\text{deg}) = 0.073\cdot\text{deg}$

$\Delta\phi_{\text{required}}(60\cdot\text{deg}) = 0.043\cdot\text{deg}$

$\left(\frac{\Delta\phi_{\text{required}}(0\cdot\text{deg})}{\Delta\phi_{\text{required}}(30\cdot\text{deg})}\right) = 1.142$

a straight-in shot is 1.15X (15%) easier than a 30 degree cut angle shot.

$\left(\frac{\Delta\phi_{\text{required}}(0\cdot\text{deg})}{\Delta\phi_{\text{required}}(60\cdot\text{deg})}\right) = 1.939$

a straight-in shot is 1.97X (97%) easier than a 60 degree cut angle shot.

NOTE- If the effective size of the pocket (s) is known (e.g., from TP 3.5-3.8), the allowable angular error in the OB direction ($\Delta\theta$), for a given distance to the pocket (d_p), can be calculated with:

$$\Delta\theta = 2 \cdot \tan^{-1}\left(\frac{\frac{s}{2}}{d_p}\right) \qquad (2)$$

A simple way to estimate the effective size of a pocket is to subtract 2 inches (a little less than a ball diameter) from the point-to-point pocket-size measurement (p). For example, for the shot above, with a 4.5" pocket, with the OB 4.5 feet from the pocket:

$$p := 4.5 \cdot in \qquad d_p := 4.5 \cdot ft$$

$$s_{approx} := p - 2 \cdot in = 2.5 \cdot in$$

$$\Delta\theta_{approx} := 2 \cdot atan\left(\frac{\frac{s_{approx}}{2}}{d_p}\right) = 2.7 \cdot deg$$

Now let's answer a common question concerning a **straight shot** with a total distance D between the CB and pocket. Which OB position (distance d from CB, and distance d_p from the pocket) results in the most difficult shot?

For a straight shot, the cut angle ϕ is 0, and as long as the OB is not too close to the CB or the pocket, both the CB angle error $\Delta\phi$ and the allowable OB angle error $\Delta\theta$ are small. Using small-angle approximations (sin(x)=x, tan(x)=x) for Equations 1 and 2 gives:

$$\Delta\theta = \frac{d}{2\cdot R}\cdot\Delta\phi = \frac{s}{d_p}$$

So the allowable CB angle error is:

$$\Delta\phi = \frac{2\cdot R\cdot s}{d\cdot d_p}$$

So the shot is most difficult when the product of the distances between the CB & OB (d) and the OB & pocket (d_p) is greatest. Since the distance between the OB and pocket is:

$$d_p = D - d$$

The shot is most difficult when:

$$\frac{d}{dd}(d(D-d)) = 0$$

$$D - 2d = 0$$

$$d = \frac{D}{2}$$

So a straight shot is the most difficult when the OB is exactly halfway between the CB and the pocket.