technical proof

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TP 3.6
Effective target sizes for slow shots into a corner pocket at different angles

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu

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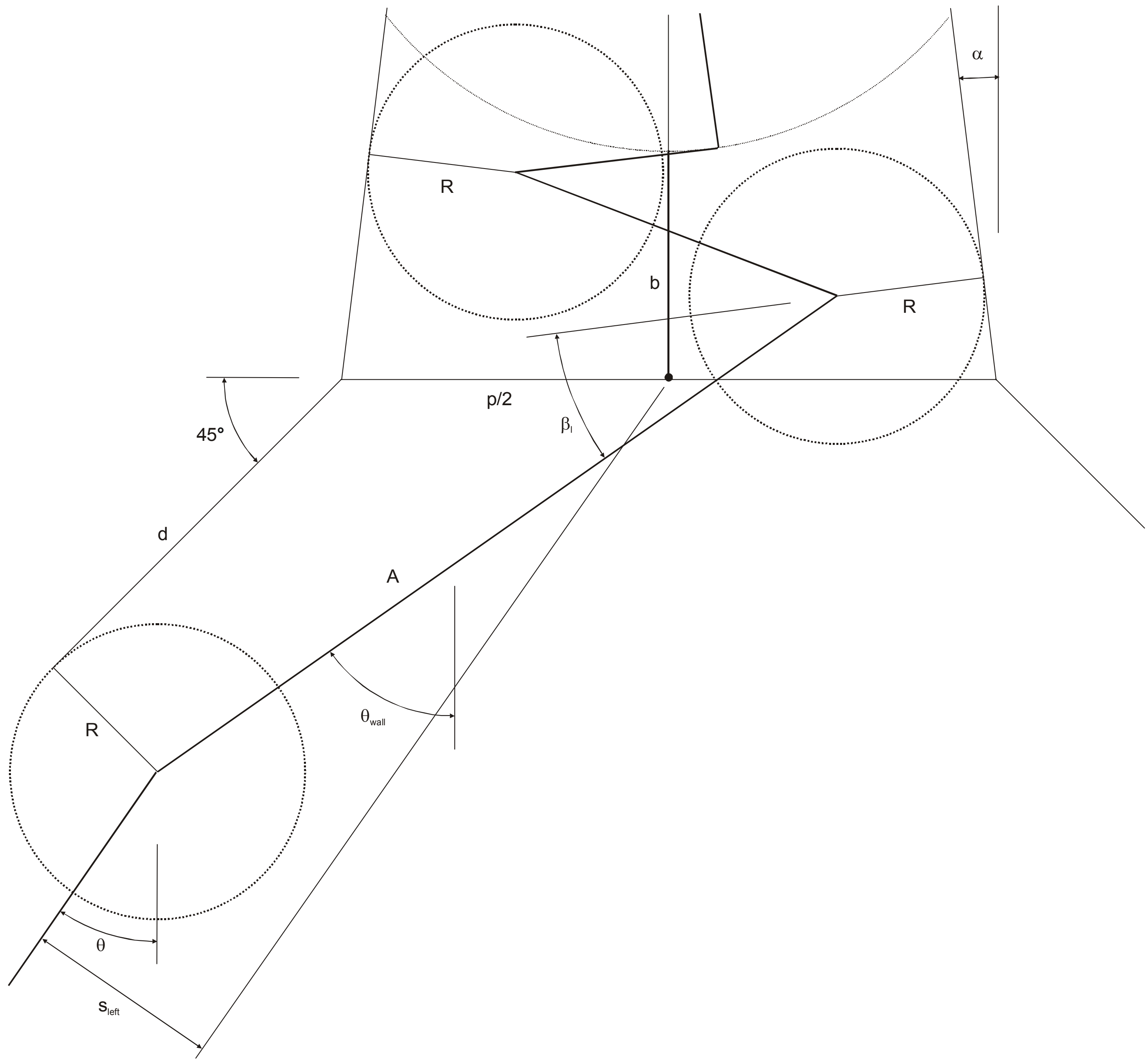


See the general notation presented in TP 3.5

Ball radius:

$R := 1.125$

Pocket-specific parameters:

p: mouth width α: wall angle R_{hole}: hole radius   b:shelf depth to hole

$p := 4.5875 \qquad \alpha := 7\cdot deg \qquad R_{hole} := 2.75 \qquad b := 1.125$

General equations for the size of left portion of target area assuming point deflection, then one wall deflection or straight-in

$$A(\alpha, p, b, R_{hole}, \beta_1, \theta) := \frac{1}{\sin(2\cdot\beta_1 - \theta - \alpha)}\cdot\left[\frac{p}{2}\cdot\cos(\alpha) - R - R_{hole}\cdot\cos(2\cdot\beta_1 - \theta - \alpha) - (R_{hole} + b)\cdot\sin(\alpha)\right]$$

$$poly_{\beta}(\alpha, p, b, R_{hole}, \beta_1, \theta) := R\cdot\sin(\beta_1) + A(\alpha, p, b, R_{hole}, \beta_1, \theta)\cdot\sin(4\cdot\beta_1 - 2\cdot\theta - 2\cdot\alpha) + R_{hole}\cdot\cos(4\cdot\beta_1 - 2\cdot\theta - 2\cdot\alpha) \ldots + \frac{p}{2}\cdot\cos(2\cdot\beta_1 - \theta) + (R_{hole} + b)\cdot\sin(2\cdot\beta_1 - \theta)$$

$$poly_{\beta\_in}(p, b, R_{hole}, \beta_1, \theta) := -R\cdot\sin(\beta_1) + R_{hole} - \frac{p}{2}\cdot\cos(2\cdot\beta_1 - \theta) - (R_{hole} + b)\cdot\sin(2\cdot\beta_1 - \theta)$$

$$\beta_{guess}(\theta) := \left[20\cdot deg + \frac{70}{130}\cdot(\theta + 60\cdot deg)\right] \qquad \beta := \beta_{guess}(0\cdot deg) \qquad \beta = 52.308\cdot deg$$

$$\beta_1(\alpha, p, b, R_{hole}, \theta) := \min\left(\begin{pmatrix} root(poly_{\beta}(\alpha, p, b, R_{hole}, \beta, \theta), \beta) \\ root(poly_{\beta\_in}(p, b, R_{hole}, \beta, \theta), \beta) \end{pmatrix}\right)$$

$$s_{left\_point\_wall}(\alpha, p, b, R_{hole}, \theta) := \frac{p}{2}\cdot\cos(\theta) - R\cdot\sin(\beta_1(\alpha, p, b, R_{hole}, \theta))$$

Size of the left portion of the target area assuming one wall deflection

$$r_{wall}(\alpha, p, b, R_{hole}, \theta) := A(\alpha, p, b, R_{hole}, 90\cdot deg, \theta)\cdot\sin(2\cdot\theta + 2\cdot\alpha) - R_{hole}\cdot\cos(2\cdot\theta + 2\cdot\alpha) \ldots + \frac{p}{2}\cdot\cos(\theta) - (R_{hole} + b)\cdot\sin(\theta)$$

$$s_{left\_wall}(\alpha, p, b, R_{hole}, \theta) := -\left(\frac{p}{2}\cdot\cos(-\theta) - r_{wall}(\alpha, p, b, R_{hole}, -\theta)\right)$$

Angle after rail contact

$$\theta_{rail}(\theta) := 90\cdot deg - \theta$$

Angle after wall contact

$$\theta_{wall}(\theta) := \theta - 2\cdot\alpha$$

Angle after rail, then wall contact

$$\theta_{rail\_wall}(\theta) := \theta_{wall}(-\theta_{rail}(\theta))$$

Size of the left portion of the target area assuming rattle across two pocket walls

$$D_{wall\_wall}(\theta) := \frac{1}{\sin(\theta - \alpha)} \cdot \left(\frac{p}{2} \cdot \cos(\theta - 2 \cdot \alpha) - R \cdot \cos(\theta - \alpha) + s_{left\_wall}(\alpha, p, b, R_{hole}, \theta_{wall}(\theta)) \right)$$

$$s_{left\_wall\_wall}(\theta) := \frac{p}{2} \cdot \cos(\theta) + D_{wall\_wall}(\theta) \cdot \sin(\theta - \alpha) - R \cdot \cos(\theta - \alpha)$$

Size of the left portion of the target area assuming rattle across three pocket walls

$$D_{wall\_wall\_wall}(\theta) := \frac{1}{\sin(\theta - \alpha)} \cdot \left(\frac{p}{2} \cdot \cos(\theta - 2 \cdot \alpha) - R \cdot \cos(\theta - \alpha) + s_{left\_wall\_wall}(\theta_{wall}(\theta)) \right)$$

$$s_{left\_wall\_wall\_wall}(\theta) := \frac{p}{2} \cdot \cos(\theta) + D_{wall\_wall\_wall}(\theta) \cdot \sin(\theta - \alpha) - R \cdot \cos(\theta - \alpha)$$

Size of the left portion of the target area assuming rail contact and rattle off far point and one wall

$$d_{point\_wall}(\theta) := \frac{1}{\sin(45 \cdot deg - \theta)} \cdot \left(s_{left\_point\_wall}(\alpha, p, b, R_{hole}, -\theta_{rail}(\theta)) + \frac{p}{2} \cdot \sin(\theta) - R \cdot \cos(45 \cdot deg - \theta) \right)$$

$$s_{left\_rail\_point\_wall}(\theta) := \frac{p}{2} \cdot \cos(\theta) + d_{point\_wall}(\theta) \cdot \sin(45 \cdot deg - \theta) - R \cdot \cos(45 \cdot deg - \theta)$$

Size of the left portion of the target area assuming rail contact and rattle off far point and two walls

$$d_{wall\_wall}(\theta) := \frac{1}{\sin(45 \cdot deg - \theta)} \cdot \left(s_{left\_wall\_wall}(-\theta_{rail}(\theta)) + \frac{p}{2} \cdot \sin(\theta) - R \cdot \cos(45 \cdot deg - \theta) \right)$$

$$s_{left\_rail\_wall\_wall}(\theta) := \frac{p}{2} \cdot \cos(\theta) + d_{wall\_wall}(\theta) \cdot \sin(45 \cdot deg - \theta) - R \cdot \cos(45 \cdot deg - \theta)$$

Size of the left portion of the target area assuming deflection off the rail and then one pocket wall

$$d_{wall}(\theta) := \frac{1}{\sin(45 \cdot deg - \theta)} \cdot \left(s_{left\_wall}(\alpha, p, b, R_{hole}, -\theta_{rail}(\theta)) + \frac{p}{2} \cdot \sin(\theta) - R \cdot \cos(45 \cdot deg - \theta) \right)$$

$$s_{left\_rail\_wall}(\theta) := \frac{p}{2} \cdot \cos(\theta) + d_{wall}(\theta) \cdot \sin(45 \cdot deg - \theta) - R \cdot \cos(45 \cdot deg - \theta)$$

Size of the left portion of the target area assuming point contact and rattle off two walls

$$\phi_1(\beta, \theta) := 2 \cdot \beta - 90 \cdot deg - \theta - 2 \cdot \alpha \qquad \phi_2(\beta, \theta) := \phi_1(\beta, \theta) - 2 \cdot \alpha$$

$$A_2(\beta,\theta) := \frac{1}{\cos\left(\phi_2(\beta,\theta)+\alpha\right)} \cdot \left[\frac{p}{2}\cdot\cos(\alpha) - \left(R_{hole}+b\right)\cdot\sin(\alpha) + R_{hole}\cdot\sin\left(\phi_2(\beta,\theta)+\alpha\right) - R\right]$$

$$A_1(\beta,\theta) := \frac{1}{\cos\left(\phi_1(\beta,\theta)+\alpha\right)} \cdot \left[\frac{p}{2}\cdot\cos(\alpha) - R + A_2(\beta,\theta)\cdot\cos\left(\phi_2(\beta,\theta)-\alpha\right) - R_{hole}\cdot\sin\left(\phi_2(\beta,\theta)-\alpha\right) - \left(R_{hole}+b\right)\cdot\sin(\alpha)\right]$$

$$poly_{\beta ww}(\beta,\theta) := -R\cdot\sin(\beta) - A_1(\beta,\theta)\cdot\cos\left(2\cdot\beta-\theta+\phi_1(\beta,\theta)\right) + A_2(\beta,\theta)\cdot\cos\left(2\cdot\beta-\theta-\phi_2(\beta,\theta)\right) \ldots$$
$$+ R_{hole}\cdot\sin\left(2\cdot\beta-\theta-\phi_2(\beta,\theta)\right) - \frac{p}{2}\cdot\cos(2\cdot\beta-\theta) - \left(R_{hole}+b\right)\cdot\sin(2\cdot\beta-\theta)$$

$\beta_{ww} := 75\cdot deg$ initial guess for beta root function below

$$\beta_{lww}(\theta) := root\left(poly_{\beta ww}\left(\beta_{ww},\theta\right),\beta_{ww}\right)$$

$$s_{left\_point\_wall\_wall}(\theta) := \frac{p}{2}\cdot\cos(\theta) - R\cdot\sin\left(\beta_{lww}(\theta)\right)$$

Size of the left portion of the target area assuming deflection off the rail and then the far point with a two-wall rattle

$$d_{point\_wall\_wall}(\theta) := \frac{1}{\sin(45\cdot deg-\theta)}\cdot\left(s_{left\_point\_wall\_wall}\left(-\theta_{rail}(\theta)\right) + \frac{p}{2}\cdot\sin(\theta) - R\cdot\cos(45\cdot deg-\theta)\right)$$

$$s_{left\_rail\_point\_wall\_wall}(\theta) := \frac{p}{2}\cdot\cos(\theta) + d_{point\_wall\_wall}(\theta)\cdot\sin(45\cdot deg-\theta) - R\cdot\cos(45\cdot deg-\theta)$$

Critical angle between left-rail-point-wall and left-rail-wall-wall

$\theta := 38\cdot deg$

Given

$$s_{left\_rail\_point\_wall}(\theta) = s_{left\_rail\_wall\_wall}(\theta)$$

$\theta_{critical\_C} := Find(\theta)$ $\theta_{critical\_C} = 38.558\,deg$

$s_{left\_rail\_point\_wall}\left(\theta_{critical\_C}\right) = 1.829$ $d_{point\_wall}\left(\theta_{critical\_C}\right) = 10.275$

$s_{left\_rail\_wall\_wall}\left(\theta_{critical\_C}\right) = 1.829$ $d_{wall\_wall}\left(\theta_{critical\_C}\right) = 10.275$

Critical angle between left-point-wall and left-wall-wall

$\theta_{critical} := \theta_{critical\_C} - 90\cdot deg$ $\theta_{critical} = -51.442\,deg$

$\theta := \theta_{critical}$ $\beta := \beta_{guess}(\theta)$

Given

$$poly_{\beta}\left(\alpha,p,b,R_{hole},\beta,\theta\right) = 0$$

$\beta := Find(\beta)$ $\beta = 31.558\cdot deg$ $\beta := \beta_l\left(\alpha,p,b,R_{hole},\theta\right)$ $\beta = 31.558\cdot deg$

$\theta - \alpha + 90\cdot deg - \beta = 1.908\times10^{-14}\cdot deg$

$\theta_{critical} := \beta + \alpha - 90\cdot deg$ $\theta_{critical} = -51.442\cdot deg$

$s_{left\_wall\_wall}\left(\theta_{critical}\right) = 0.841$

$$s_{left\_point\_wall}(\alpha, p, b, R_{hole}, \theta_{critical}) = 0.841$$

Critical angle between left-rail-wall-wall and left-point-wall-wall

$$\theta := 30 \cdot deg$$

given

$$s_{left\_rail\_wall\_wall}(\theta) = s_{left\_point\_wall\_wall}(\theta)$$

$$\theta_{critical\_D} := find(\theta) \qquad \theta_{critical\_D} = 28.167\,deg \qquad \theta_{rail}(\theta_{critical\_D}) = 61.833 \cdot deg$$

$$s_{left\_rail\_wall\_wall}(\theta_{critical\_D}) = 0.945 \qquad d_{wall\_wall}(\theta_{critical\_D}) = 0$$

$$s_{left\_point\_wall\_wall}(\theta_{critical\_D}) = 0.945 \qquad \beta_{lww}(\theta_{critical\_D}) = 73.167 \cdot deg$$

Maximum rail distances and critical angles based on table geometry

$$d_{max}(L) := L - \frac{p}{\cos(45 \cdot deg)} - 2 \cdot R$$

$$d_{max\_long} := d_{max}(8 \cdot 12) \qquad d_{max\_long} = 87.262$$

$$\theta := 43 \cdot deg$$

Given

$$d_{point\_wall\_wall}(\theta) = d_{max\_long}$$

$$\theta_{max\_long} := Find(\theta) \qquad \theta_{max\_long} = 44.058 \cdot deg$$

$$d_{point\_wall\_wall}(\theta_{max\_long}) = 87.262$$

$$s_{left\_rail\_max\_angle}(d_{max}, \theta) := \frac{p}{2} \cdot \cos(\theta) + d_{max} \cdot \sin(45 \cdot deg - \theta) - R \cdot \cos(45 \cdot deg - \theta)$$

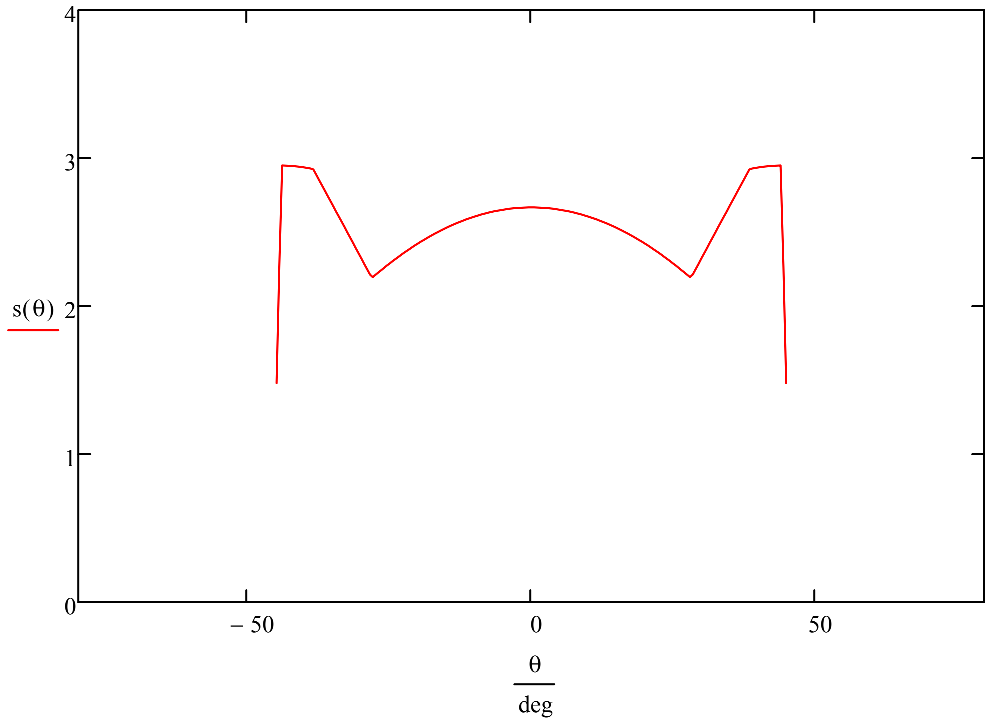
Size of the target area

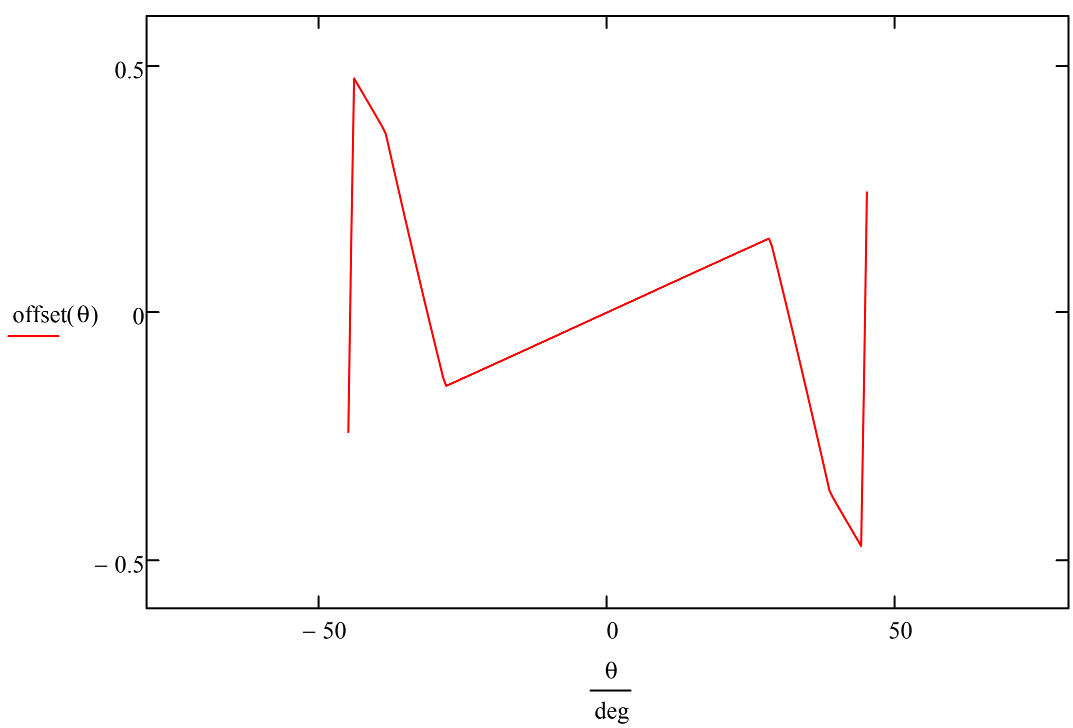
$$s_{left}(\theta) := \begin{cases} s_{left\_rail\_max\_angle}(d_{max\_long}, \theta) & \text{if } (\theta \ge \theta_{max\_long}) \\ s_{left\_rail\_point\_wall}(\theta) & \text{if } (\theta_{critical\_C} \le \theta < \theta_{max\_long}) \\ s_{left\_rail\_wall\_wall}(\theta) & \text{if } (\theta_{critical\_D} \le \theta < \theta_{critical\_C}) \\ s_{left\_point\_wall\_wall}(\theta) & \text{otherwise} \end{cases}$$

$$s_{right}(\theta) := \begin{cases} s_{left\_rail\_max\_angle}(d_{max\_long}, -\theta) & \text{if } (-\theta \ge \theta_{max\_long}) \\ s_{left\_rail\_point\_wall}(-\theta) & \text{if } (\theta_{critical\_C} \le -\theta < \theta_{max\_long}) \\ s_{left\_rail\_wall\_wall}(-\theta) & \text{if } (\theta_{critical\_D} \le -\theta < \theta_{critical\_C}) \\ s_{left\_point\_wall\_wall}(-\theta) & \text{otherwise} \end{cases}$$

$s(\theta) := s_{left}(\theta) + s_{right}(\theta)$ $offset(\theta) := \frac{(s_{right}(\theta) - s_{left}(\theta))}{2}$

$\theta := -45 \cdot deg, -44.5 \cdot deg \,..\, 45 \cdot deg$





Constant margin of error contour lines

$$r(\theta, \Delta\theta) := \frac{1}{2 \cdot \tan(\Delta\theta)} \cdot s(\theta)$$