technical proof

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TP 4.1
Distance required for stun and normal roll to develop

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
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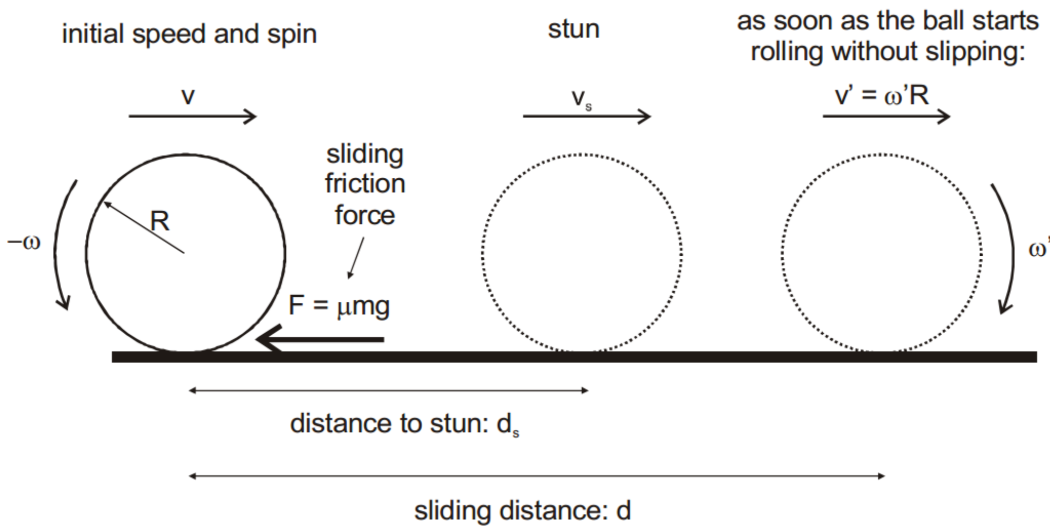

m: ball mass

t_s: time for stun to develop over distance d_s

t_d: time for sliding to stop over distance d

ω: ball angular speed ω < 0: bottom spin ω > 0: topspin ω = 0: stun

The direction of the friction force is as shown when the CB has "overspin" (where ω > v/R). Therefore, the constant linear acceleration (negative implies deceleration) of the ball can be expressed as:

$$a = -sign\left(\frac{v}{R} - \omega\right) \cdot \frac{F}{m} = -sign\left(\frac{v}{R} - \omega\right) \cdot \mu \cdot g = -\pm \cdot \mu \cdot g$$

<u>with no overspin (ω < v/R):</u>

$$sign\left(\frac{v}{R} - \omega\right) = 1$$ ±: +

$$a = -\mu \cdot g$$

<u>with overspin (ω > v/R):</u>

$$sign\left(\frac{v}{R} - \omega\right) = -1$$ ±: -

$$a = \mu \cdot g$$

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Linear speeds at stun (v_s) and when sliding stops (v'):

$$v_s = v + a \cdot t_s = v - \mu \cdot g \cdot t_s$$

$$v' = v - \pm \cdot \mu \cdot g \cdot t_d$$

Constant angular acceleration caused by the moment of the friction force about the ball center:

$$\alpha = \pm \cdot \frac{F \cdot R}{I} = \pm \cdot \frac{\mu \cdot m \cdot g \cdot R}{\frac{2}{5} \cdot m \cdot R^2} = \pm \cdot \frac{5 \cdot \mu \cdot g}{2 \cdot R}$$

Angular speed when sliding stops:

$$\omega' = \omega + \alpha \cdot t_d = \omega + \pm \cdot \frac{5 \cdot \mu \cdot g}{2 \cdot R} \cdot t_d$$

At time t_s, the ball is in stun (i.e., no spin), so:

$$t_s = \frac{-\omega}{\alpha} = \frac{2 \cdot R \cdot (-\omega)}{5 \cdot \mu \cdot g}$$

This equation applies only if ω<0 to begin with, giving t_s>0.

At time t_d, the ball is rolling without slipping, so:

$$v' = \omega' \cdot R$$

$$v - \pm \cdot \mu \cdot g \cdot t_d = R \cdot \omega + \pm \cdot \frac{5 \cdot \mu \cdot g}{2} \cdot t_d$$

$$\pm \cdot t_d \cdot \mu \cdot g \cdot \left(\frac{5}{2} + 1\right) = v - R \cdot \omega$$

$$t_d = \pm \cdot \frac{2}{7 \cdot \mu \cdot g} \cdot (v - R \cdot \omega)$$

for a stun shot (ω=0):

$$t_d = \frac{2 \cdot v}{7 \cdot \mu \cdot g}$$

Distance and time are related with the following constant acceleration relation:

$$x = v \cdot t + \frac{1}{2} \cdot a \cdot t^2 = v \cdot t - \pm \cdot \frac{1}{2} \cdot \mu \cdot g \cdot t^2$$

So the distance for stun to develop (with ω<0) is:

$$d_s = -v \cdot \frac{2 \cdot R \cdot \omega}{5 \cdot \mu \cdot g} - \frac{1}{2} \cdot \mu \cdot g \cdot \left(\frac{2 \cdot R \cdot (-\omega)}{5 \cdot \mu \cdot g}\right)^2 = \frac{2 \cdot R \cdot (-\omega)}{5 \cdot \mu \cdot g} \cdot \left(v - \frac{(-\omega) \cdot R}{5}\right)$$

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And the total distance for sliding to stop and rolling to begin is:

$$d = \pm \cdot \frac{2 \cdot v}{7 \cdot \mu \cdot g} \cdot (v - R \cdot \omega) - \pm \cdot \frac{1}{2} \cdot \mu \cdot g \cdot \left(\frac{2}{7 \cdot \mu \cdot g} \cdot (v - R \cdot \omega) \right)^2$$

$$d = \pm \cdot \frac{2}{7 \cdot \mu \cdot g} \cdot \left(v^2 - R \cdot \omega \cdot v \right) - \pm \cdot \frac{2}{49 \cdot \mu \cdot g} \cdot \left(v^2 - 2 \cdot R \cdot \omega \cdot v + R^2 \cdot \omega^2 \right)$$

$$d = \pm \cdot \frac{2}{49 \cdot \mu \cdot g} \cdot \left(6 \cdot v^2 - 5 \cdot v \cdot R \cdot \omega - (R \cdot \omega)^2 \right)$$

for a non-overspin shot ($\omega < v/R$),

$$d = \frac{2}{49 \cdot \mu \cdot g} \cdot \left(6 \cdot v^2 - 5 \cdot v \cdot R \cdot \omega - (R \cdot \omega)^2 \right)$$

for a stun-drag shot ($\omega=0$):

$$d = \frac{12 \cdot v^2}{49 \cdot \mu \cdot g}$$

The final ball speed, after sliding stops and rolling begins, is given by:

$$v' = v - \pm \cdot \mu \cdot g \cdot t_d = v - \pm \cdot \mu \cdot g \cdot \left(\pm \cdot \frac{2}{7 \cdot \mu \cdot g} \cdot (v - R \cdot \omega) \right) = \frac{5}{7} \cdot v + \frac{2}{7} \cdot R \cdot \omega$$

Note that the final ball speed is independent of the ball and table conditions. Also, 5/7 (71.4%) of the final speed comes from the initial translational speed (v), and 2/7 (28.6%) comes from the spin component (Rω).

For a stun shot ($\omega=0$):

$$v' = \frac{5}{7} \cdot v$$

Therefore, the final ball speed, for an initially sliding ball, is always 5/7 of the initial speed!

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During sliding, the linear and angular speeds change over distance (x) according to:

$$v(x)^2 - v^2 = 2 \cdot a \cdot x$$

$$v(x) = \sqrt{v^2 - \pm \cdot 2 \cdot \mu \cdot g \cdot x}$$

$$x = v \cdot t - \pm \cdot \frac{1}{2} \cdot \mu \cdot g \cdot t^2 \qquad \pm \cdot \frac{1}{2} \cdot \mu \cdot g \cdot t^2 - v \cdot t + x = 0$$

$$t = \frac{v - \sqrt{v^2 - \pm \cdot 2 \cdot \mu \cdot g \cdot x}}{\pm \cdot \mu \cdot g}$$

(only the "-" solution of the quadratic equation is meaningful" because the equations only apply during sliding while the friction force is acting)

$$\omega(x) = \omega + \alpha \cdot t = \omega + \pm \cdot \frac{5 \cdot \mu \cdot g}{2 \cdot R} \cdot t = \omega + \frac{5}{2 \cdot R} \cdot \left(v - \sqrt{v^2 - \pm \cdot 2 \cdot \mu \cdot g \cdot x}\right)$$

Changes in speed and spin over distance with drag shots:

$\mu := 0.2$ typical ball/cloth sliding COF

$D := 2.25 \cdot in \qquad R := \frac{D}{2}$ ball diameter and radius

$v_{slow} := 3 \cdot mph \qquad v_{medium} := 7 \cdot mph \qquad v_{fast} := 12 \cdot mph$

$$d_{skid}(v, \omega) := \frac{2}{49 \cdot \mu \cdot g} \cdot \left(6 \cdot v^2 - 5 \cdot v \cdot R \cdot \omega - (R \cdot \omega)^2\right)$$

$$v_{skid}(v, x, d) := \begin{cases} \text{if } x < d \\ \quad \sqrt{v^2 - 2 \cdot \mu \cdot g \cdot x} \\ \text{else} \\ \quad \sqrt{v^2 - 2 \cdot \mu \cdot g \cdot d} \end{cases}$$

$$\omega_{skid}(v, \omega, x, d) := \begin{cases} \text{if } x < d \\ \quad \left(\omega + \frac{5}{2 \cdot R} \cdot \left(v - \sqrt{v^2 - 2 \cdot \mu \cdot g \cdot x}\right)\right) \\ \text{else} \\ \quad \left(\omega + \frac{5}{2 \cdot R} \cdot \left(v - \sqrt{v^2 - 2 \cdot \mu \cdot g \cdot d}\right)\right) \end{cases}$$

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draw-drag shots:

$$\omega_{slow} := -\frac{v_{slow}}{R} \qquad \omega_{medium} := -\frac{v_{medium}}{R} \qquad \omega_{fast} := -\frac{v_{fast}}{R}$$

$$v := v_{medium} \qquad \omega_{draw} := \omega_{medium}$$

$$d_{skid}\left(v_{slow}, \omega_{slow}\right) = 1.228\ ft \qquad d_{draw} := d_{skid}\left(v_{medium}, \omega_{medium}\right) = 6.686\ ft$$

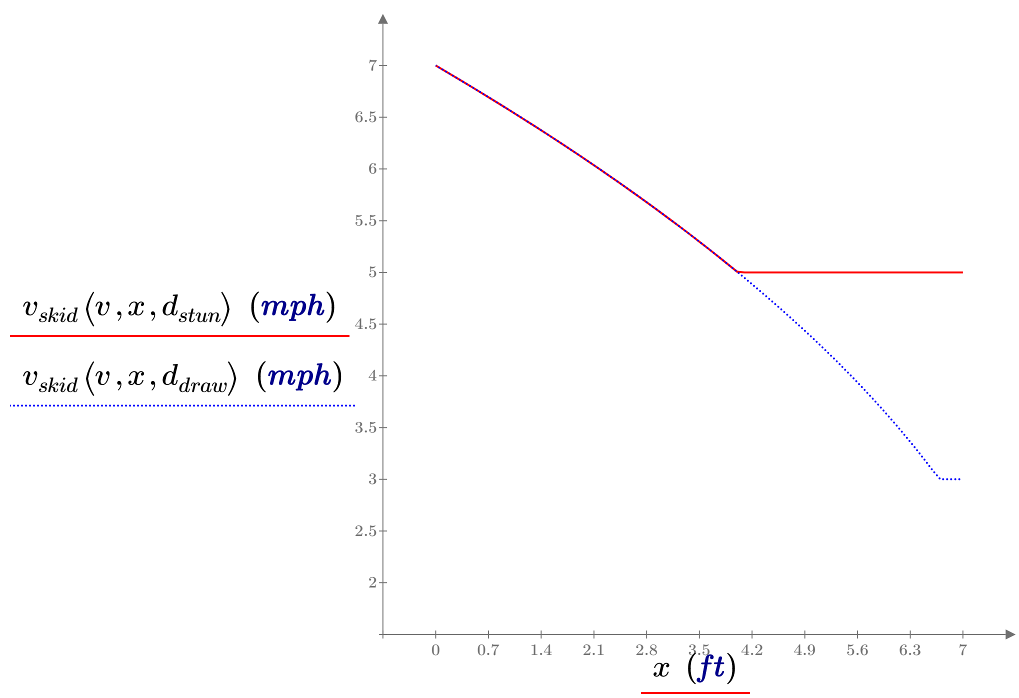
$$d_{skid}\left(v_{fast}, \omega_{fast}\right) = 19.648\ ft$$

stun-drag shots: $\omega_{stun} := 0\ \frac{rad}{sec}$

$$d_{skid}\left(v_{slow}, \omega_{stun}\right) = 0.737\ ft \qquad d_{stun} := d_{skid}\left(v_{medium}, \omega_{stun}\right) \qquad d_{skid}\left(v_{fast}, \omega_{stun}\right) = 11.789\ ft$$

$$d_{stun} = 4.012\ ft$$

balls speed vs. distance: $x := 0 \cdot ft, 0.1 \cdot ft .. 7 \cdot ft$



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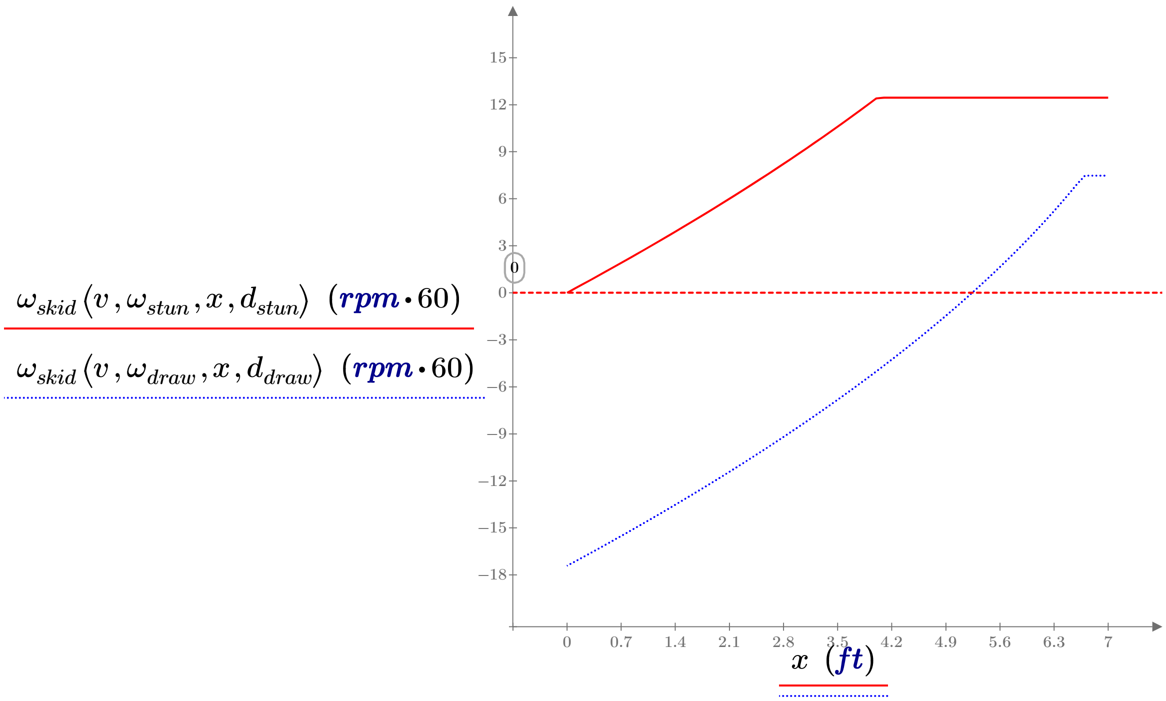
ball spin vs. distance:



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