technical proof technical proof

TP A.18

Distance required for stun and natural roll to develop for different tip offsets

supporting:

"The Illustrated Principles of Pool and Billiards"

http://billiards.colostate.edu

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originally posted: 8/9/2005 last revision: 2/10/2009

ball size and physical parameters:

$D := 2.25 \cdot in$ $\quad R := \frac{D}{2}$

$g = 32.174 \cdot \frac{ft}{sec^2}$ gravity

$\mu := 0.2$ typical sliding friction between ball and table cloth

$b_{max} := \frac{9}{16} \cdot in$ maximum suggested tip offset (see TP A.12)

typical ball speeds:

$v_{slow} := 3 \cdot mph$ $\quad v_{medium} := 7 \cdot mph$ $\quad v_{fast} := 12 \cdot mph$

From TP A.12, the forward angular spin (ω) is related to tip offset above center (b) and ball speed (v) according to:

$$\omega(b, v) := \frac{5 \cdot b \cdot v}{2 \cdot R^2} \qquad (1)$$

From TP 4.1, the time required for stun to develop (with b<0) is:

$$t_s = \frac{-\omega}{\alpha} = \frac{2 \cdot R \cdot (-\omega)}{5 \cdot \mu \cdot g}$$

Substituting Equation 1 gives:

$$t_s = \frac{2 \cdot R \cdot \left[\frac{5 \cdot (-b) \cdot v}{2 \cdot R^2}\right]}{5 \cdot \mu \cdot g} = \frac{\left(\frac{-b}{R}\right) \cdot v}{\mu \cdot g} \qquad (2)$$

From TP 4.1, the time required for roll to develop is:

$$t_d = \text{sign}(v - \omega \cdot R) \cdot \frac{2}{7 \cdot \mu \cdot g} \cdot (v - R \cdot \omega)$$

Substituting Equation 1 gives:

$$t_d = \text{sign}\left(1 - \frac{5 \cdot b}{2 \cdot R}\right) \cdot \left[\frac{2 \cdot v}{7 \cdot \mu \cdot g} \cdot \left(1 - \frac{5 \cdot b}{2 \cdot R}\right)\right] \qquad (3)$$

Distance traveled is related to time according to:

$$d = v \cdot t - \frac{1}{2} \cdot \mu \cdot g \cdot t^2 \qquad (4)$$

Substituting Equation 2 into Equation 4 gives the distance required for stun to develop:

$$d_s = v \cdot \frac{\left(\frac{-b}{R}\right) \cdot v}{\mu \cdot g} - \frac{1}{2} \cdot \mu \cdot g \cdot \left[\frac{\left(\frac{-b}{R}\right) \cdot v}{\mu \cdot g}\right]^2$$

$$d_s(b, v) := \frac{v^2}{\mu \cdot g} \cdot \left(\frac{-b}{R}\right) \cdot \left[1 - \frac{1}{2} \cdot \left(\frac{-b}{R}\right)\right] \qquad (5)$$

Substituting Equation 3 into Equation 4 gives the total distance for sliding to stop and rolling to begin:

$$d = v \cdot \left[\text{sign}\left(1 - \frac{5 \cdot b}{2 \cdot R}\right) \cdot \left[\frac{2 \cdot v}{7 \cdot \mu \cdot g} \cdot \left(1 - \frac{5 \cdot b}{2 \cdot R}\right)\right]\right] - \frac{1}{2} \cdot \mu \cdot g \cdot \left[\frac{2 \cdot v}{7 \cdot \mu \cdot g} \cdot \left(1 - \frac{5 \cdot b}{2 \cdot R}\right)\right]^2$$

$$d(b, v) := \text{sign}\left(1 - \frac{5 \cdot b}{2 \cdot R}\right) \cdot \frac{2 \cdot v^2}{7 \cdot \mu \cdot g} \cdot \left(1 - \frac{5 \cdot b}{2 \cdot R}\right) - \frac{2 \cdot v^2}{49 \cdot \mu \cdot g} \cdot \left[1 - 5 \cdot \left(\frac{b}{R}\right) + \frac{25}{4} \cdot \left(\frac{b}{R}\right)^2\right] \qquad (6)$$

With a non-overspin shot, Equation 6 becomes:

$$d = \frac{v^2}{98 \cdot \mu \cdot g} \cdot \left[\left[28 - 70 \cdot \left(\frac{b}{R}\right)\right] - \left[4 - 20 \cdot \left(\frac{b}{R}\right) + 25 \cdot \left(\frac{b}{R}\right)^2\right]\right]$$

$$d = \frac{v^2}{98 \cdot \mu \cdot g} \cdot \left[24 - 50 \cdot \left(\frac{b}{R}\right) - 25 \cdot \left(\frac{b}{R}\right)^2\right]$$

This equation can be verified at special cases:

$\frac{b}{R} = \frac{2}{5}$	gives	$d = 0$	which agrees with TP 4.2
$b = 0$	gives	$d = \frac{12v^2}{49 \cdot \mu \cdot g}$	which agrees with TP 4.1

Here are typical values for different types of shots:

stun-drag shot: $b := 0 \cdot in$

$d(b, v_{slow}) = 0.737 \cdot ft$ $d(b, v_{medium}) = 4.012 \cdot ft$ $d(b, v_{fast}) = 11.789 \cdot ft$

draw-drag shots:

half of the maximum recommended tip offset: $b := \frac{-b_{max}}{2}$ $\frac{b}{R} = -0.25$

$d_s(b, v_{slow}) = 0.658 \cdot ft$ $d_s(b, v_{medium}) = 3.583 \cdot ft$ $d_s(b, v_{fast}) = 10.53 \cdot ft$

$d(b, v_{slow}) = 1.073 \cdot ft$ $d(b, v_{medium}) = 5.84 \cdot ft$ $d(b, v_{fast}) = 17.162 \cdot ft$

maximum recommended tip offset: $b := -b_{max}$ $\frac{b}{R} = -0.5$

$d_s(b, v_{slow}) = 1.128 \cdot ft$ $d_s(b, v_{medium}) = 6.143 \cdot ft$ $d_s(b, v_{fast}) = 18.052 \cdot ft$

$d(b, v_{slow}) = 1.312 \cdot ft$ $d(b, v_{medium}) = 7.146 \cdot ft$ $d(b, v_{fast}) = 20.999 \cdot ft$

follow shots:

half of the maximum recommended tip offset: $b := \frac{b_{max}}{2}$ $\frac{b}{R} = 0.25$

$sign(v_{slow} - \omega(b, v_{slow}) \cdot R) = 1$ at this tip offset, the vertical spin is less than roll to begin with

$d(b, v_{slow}) = 0.305 \cdot ft$ $d(b, v_{medium}) = 1.661 \cdot ft$ $d(b, v_{fast}) = 4.881 \cdot ft$

maximum recommended tip offset: $b := b_{max}$ $\frac{b}{R} = 0.5$

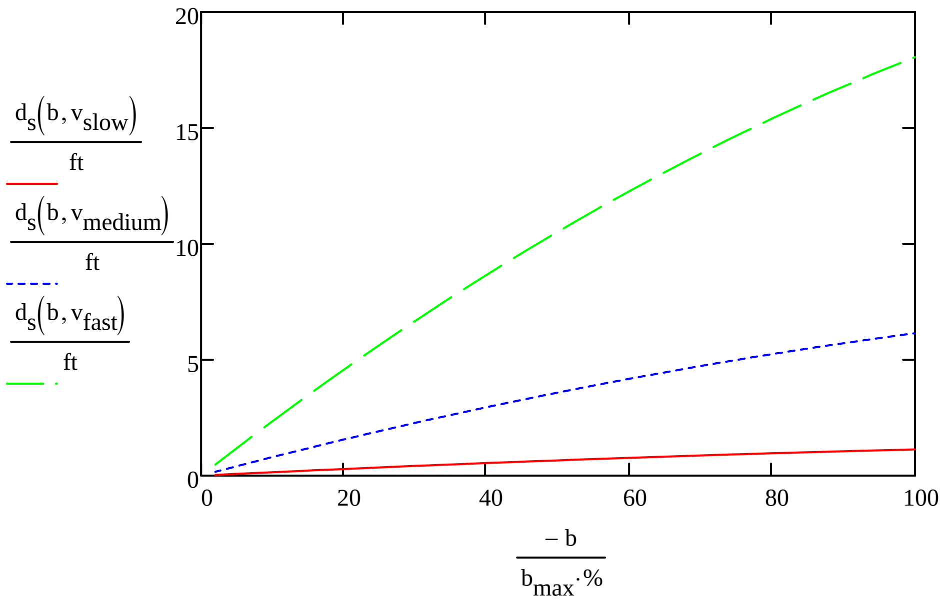
$sign(v_{slow} - \omega(b, v_{slow}) \cdot R) = -1$ at this tip offset, the vertical spin is more than roll to begin with

$d(b, v_{slow}) = 0.207 \cdot ft$ $d(b, v_{medium}) = 1.128 \cdot ft$ $d(b, v_{fast}) = 3.316 \cdot ft$

Below, we will look at how the distance for stun to develop (d_s) compares to the entire distance for sliding to stop and rolling to begin (d). We will also look at how these distances vary for drag shots of different cue ball speeds (v) and tip offsets (b<0 implies a below-center hit). (Note - see TP A.30 to see how cue ball speed varies with cue speed and tip offset.)

$$b_{small} := -25\%\, b_{max} \qquad b_{medium} := -50\%\, b_{max} \qquad b_{large} := -75\%\, b_{max}$$

$$b := -\frac{b_{max}}{50}, -2\frac{b_{max}}{50} .. -b_{max} \qquad v := v_{slow}, (v_{slow} + 0.1\cdot mph) .. v_{fast}$$



The distance to stun increases fairly linearly with tip offset, and is more sensitive to tip offset changes at higher speeds:

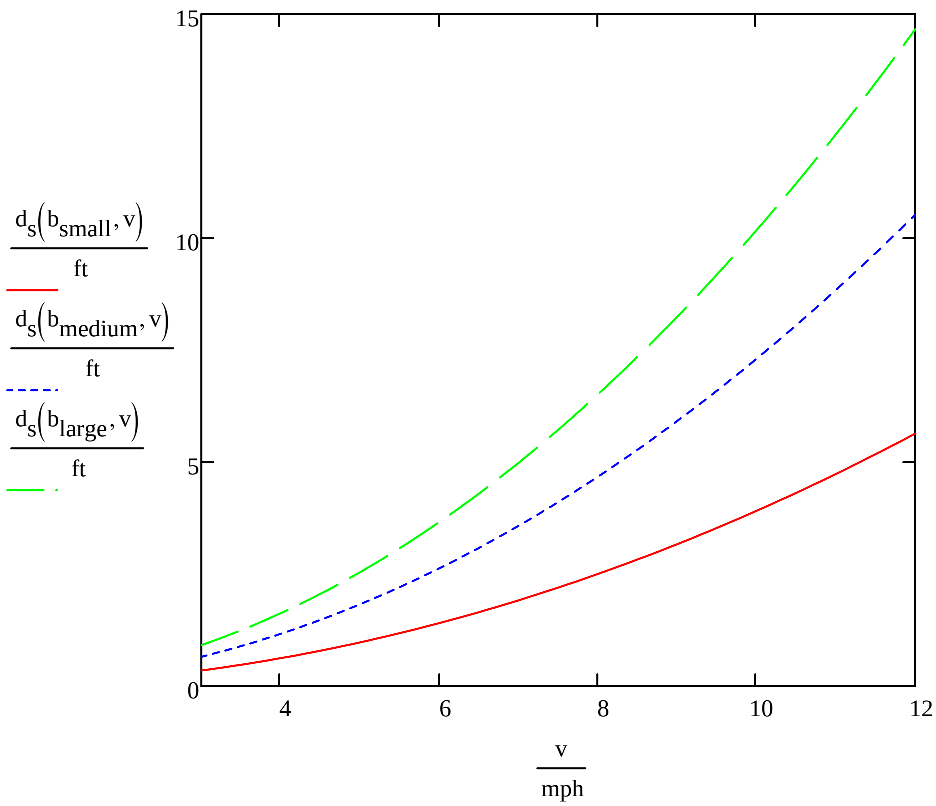
$$\frac{d_s(-2b_{medium}, v_{fast}) - d_s(-b_{medium}, v_{fast})}{d_s(-2b_{medium}, v_{medium}) - d_s(-b_{medium}, v_{medium})} = 2.939$$

Doubling the offset approximately doubles the stun distance, at all speeds:

$$\frac{d_s(-2b_{medium}, v_{medium})}{d_s(-b_{medium}, v_{medium})} = 2.222 \qquad \frac{d_s(-2b_{medium}, v_{fast})}{d_s(-b_{medium}, v_{fast})} = 2.222$$

Doubling the speed increases the stun distance by a factor of 4, at all speeds:

$$\frac{d_s(-b_{max}, 2\cdot v_{medium})}{d_s(-b_{max}, v_{medium})} = 4 \qquad \frac{d_s(-b_{max}, 2\cdot v_{fast})}{d_s(-b_{max}, v_{fast})} = 4$$



The distance to stun is more sensitive to speed changes at larger offsets:

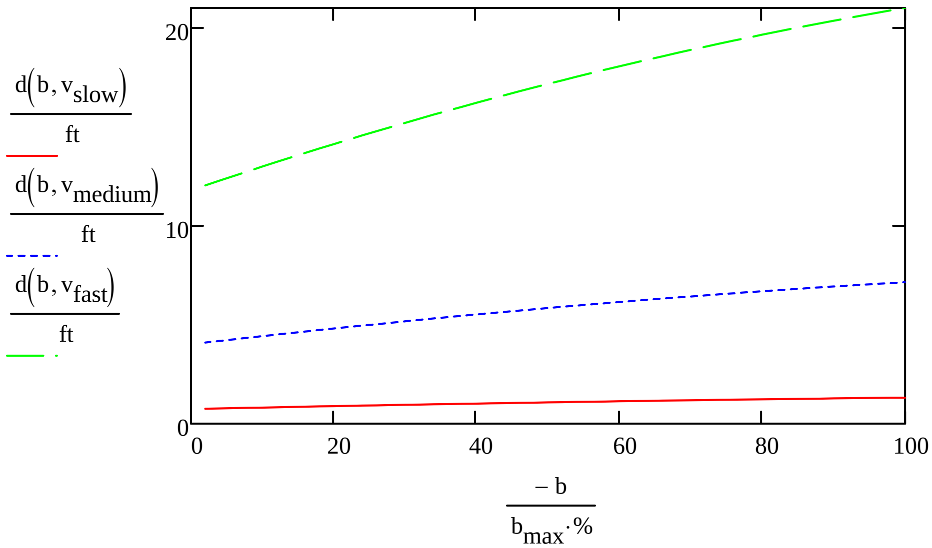
$$\frac{d_s(-b_{large}, 2v_{medium}) - d_s(-b_{large}, v_{medium})}{d_s(-b_{medium}, 2v_{medium}) - d_s(-b_{medium}, v_{medium})} = 1.583$$

Doubling the speed increases the stun distance by a factor of 4, at all offsets:

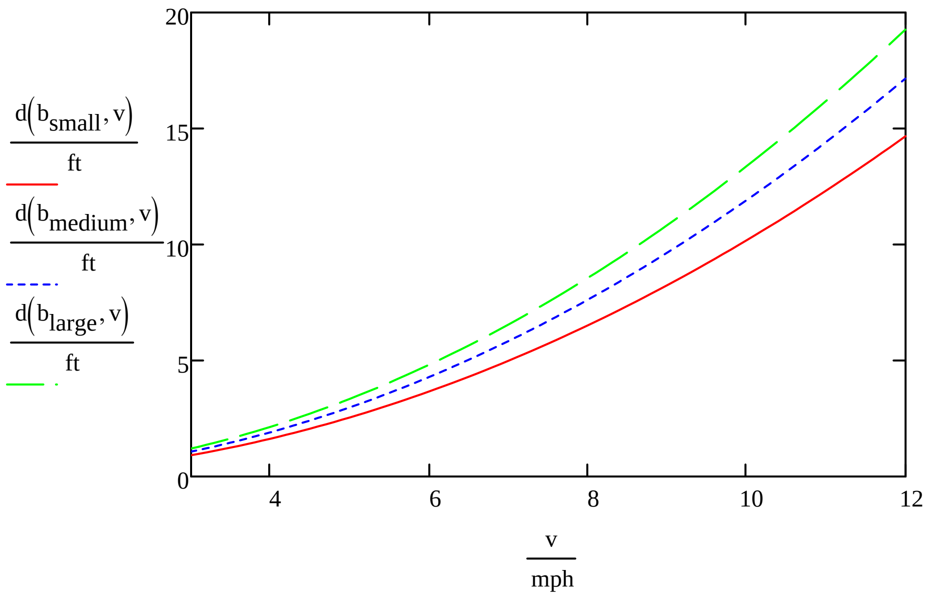
$$\frac{d_s(-b_{medium}, 2 \cdot v_{medium})}{d_s(-b_{medium}, v_{medium})} = 4 \qquad \frac{d_s(-b_{large}, 2 \cdot v_{medium})}{d_s(-b_{large}, v_{medium})} = 4$$

Doubling the speed increases the stun distance by a factor of 4, at all offsets:

$$\frac{d_s(-b_{large}, 2 \cdot v_{medium})}{d_s(-b_{large}, v_{medium})} = 4 \qquad \frac{d_s(-b_{medium}, 2 \cdot v_{medium})}{d_s(-b_{medium}, v_{medium})} = 4$$



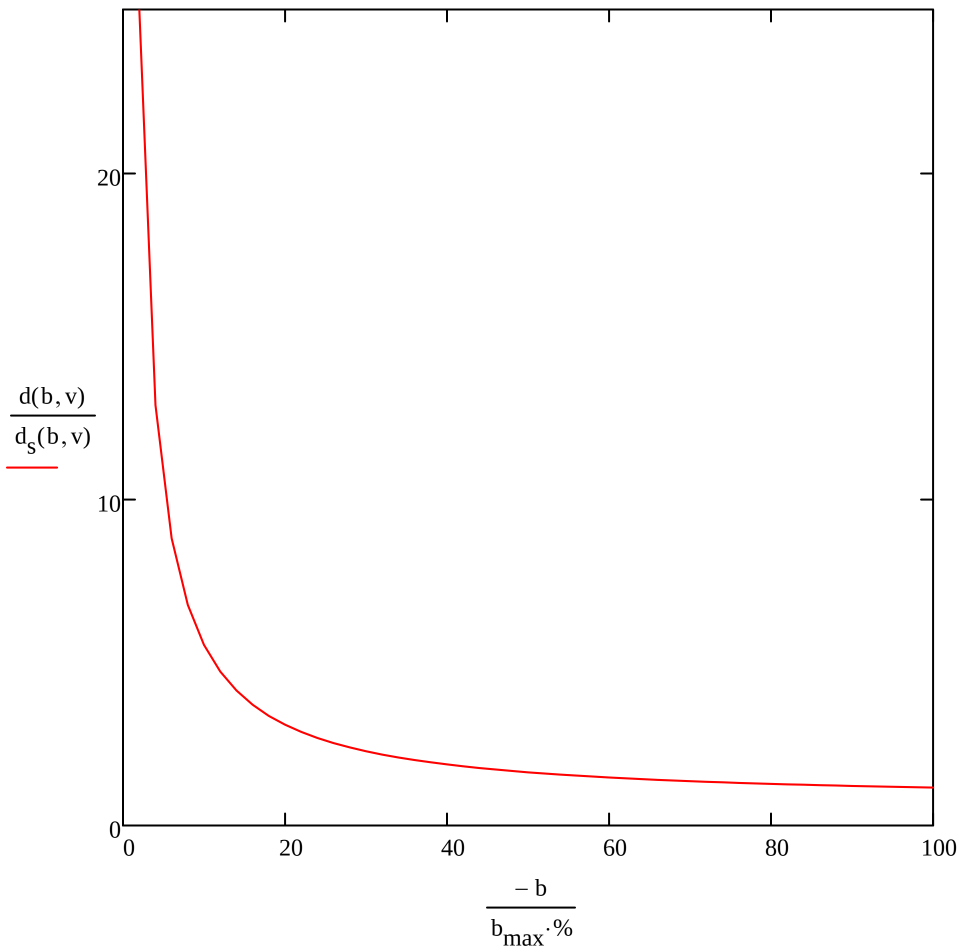
The distance to roll increases fairly linearly with tip offset, and is more sensitive to tip offset changes at higher speeds.



The sensitivity of roll distance to speed change is only a little greater for larger offset hits.

Here's how the ratio of stun distance to roll distance varies with tip offset (it doesn't vary with speed):

$$\frac{d(b_{medium}, v_{fast})}{d_s(b_{medium}, v_{fast})} = 1.63 \qquad \frac{d(b_{medium}, v_{medium})}{d_s(b_{medium}, v_{medium})} = 1.63 \qquad v := v_{medium}$$



$$b := -25\% \cdot b_{max} \qquad \frac{d(b,v)}{d_s(b,v)} = 2.6 \qquad b := -50\% \cdot b_{max} \qquad \frac{d(b,v)}{d_s(b,v)} = 1.63$$

$$b := -75\% \cdot b_{max} \qquad \frac{d(b,v)}{d_s(b,v)} = 1.314$$

$$b := -100\% \cdot b_{max} \qquad \frac{d(b,v)}{d_s(b,v)} = 1.163 \qquad \frac{d(b,v) - d_s(b,v)}{d_s(b,v)} = 16.327\%$$

For large offset drag shots, the distance for roll to develop after stun is a small fraction (about 15%) of the distance required for stun to develop.