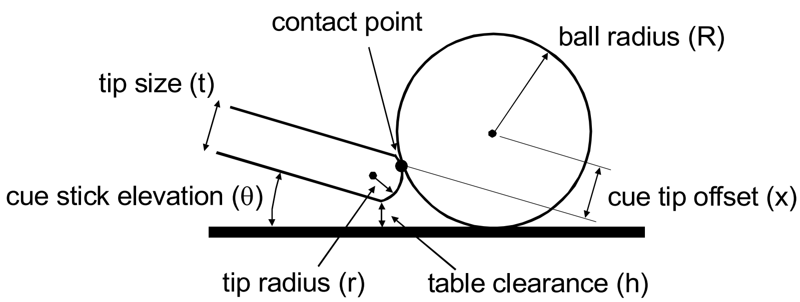
technical proof technical proof

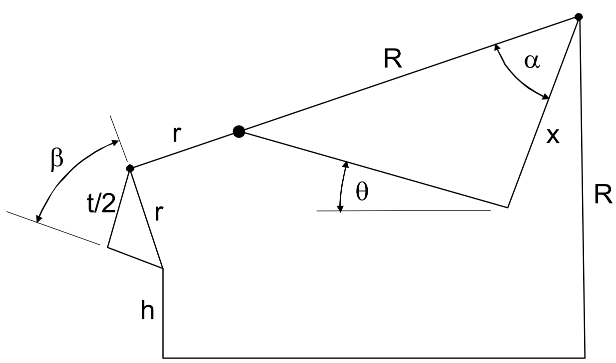
TP A.22
"Tips" of English

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu

by David G. Alciatore, PhD, PE ("Dr. Dave")

originally posted: 5/2/2006 last revision: 5/2/2006





ball radius:

$$R := \frac{2.25 \cdot in}{2}$$

tip sizes:

$$t_{min} := 11 \cdot mm$$

$$t_{max} := 14 \cdot mm$$

$$t_{avg} := 12.5 \cdot mm$$

tip curvatures:

$$r_{dime} := \frac{0.705 \cdot in}{2}$$

$$r_{nickel} := \frac{0.835 \cdot in}{2}$$

$$r_{flat} := 1 \cdot in$$

Geometry (see the drawing above)

$$\alpha(x) := acos\left(\frac{x}{R}\right) \qquad \beta(t,r) := asin\left[\frac{\left(\frac{t}{2}\right)}{r}\right]$$

$$h(x,t,r,\theta) := R - (R + r) \cdot cos(\theta + \alpha(x)) - r \cdot sin(\theta + \beta(t,r))$$

Based on TP A.3, a typical minimum cue stick elevation is about:

$$\theta_{min} := 2 \cdot deg$$

An average cue stick elevation (for most non elevated shots) is probably closer to:

$$\theta_{avg} := 4 \cdot deg$$

An average angle for an elevated cue stick is:

$$\theta_{elev} := 20 \cdot deg$$

The minimum tip radius possible for a given tip size is:

$$r_{min}(t) := \frac{t}{2}$$

Note that even for a large tip, a small tip radius is still possible:

$r_{min}(t_{max}) = 0.276\ in$ which is still smaller than: $r_{dime} = 0.353\ in$

The maximum offset that can be applied corresponds to when the table clearance is zero:

$$R - (R + r) \cdot \cos(\theta + \alpha(x)) - r \cdot \sin(\theta + \beta(t,r)) = 0$$

$$\cos(\theta + \alpha(x)) = \frac{R - r \cdot \sin(\theta + \beta(t,r))}{(R + r)}$$

$$\alpha(x) = acos\left(\frac{x}{R}\right) = acos\left[\frac{R - r \cdot \sin(\theta + \beta(t,r))}{(R + r)}\right] - \theta$$

$$\frac{x}{R} = \cos\left[acos\left[\frac{R - r \cdot \sin(\theta + \beta(t,r))}{(R + r)}\right] - \theta\right]$$

$$x_{max}(t, r, \theta) := R \cdot \cos\left[acos\left[\frac{R - r \cdot \sin(\theta + \beta(t,r))}{(R + r)}\right] - \theta\right]$$

TP A.12 defines the "offset factor" as x/R. The largest recommended offset factor (to prevent miscues) is 1/2 (0.5).

For a large, flatter tip, large offsets (i.e., close to 0.5R) are not possible (because the table gets in the way [i.e., h is negative]):

$$\frac{x_{max}(t_{max}, r_{flat}, \theta_{min})}{R} = 0.416 \qquad h(0.5 \cdot R, t_{max}, r_{flat}, \theta_{min}) = -0.182\ in$$

With a smaller tip with a smaller radius, larger offsets are possible (i.e., there is ample table clearance for large offset shots):

$$\frac{x_{max}(t_{min}, r_{dime}, \theta_{avg})}{R} = 0.656$$

At the maximum recommended offset (0.5R), the table clearance for a small, round tip is:

$$h\left(\frac{R}{2}, t_{min}, r_{dime}, \theta_{avg}\right) = 0.242 \text{ in}$$

$$h\left(\frac{R}{2}, t_{min}, r_{dime}, \theta_{avg}\right) = 6.144 \text{ mm}$$

(about a 1/4 inch clearance)

With an elevated cue stick, larger offsets are possible, but they are still limited by the tip size and radius:

$$\frac{x_{max}(t_{min}, r_{dime}, \theta_{elev})}{R} = 0.809 \qquad h(0.5 \cdot R, t_{min}, r_{dime}, \theta_{elev}) = 0.57 \text{ in}$$

$$\frac{x_{max}(t_{min}, r_{nickel}, \theta_{elev})}{R} = 0.78 \qquad h(0.5 \cdot R, t_{min}, r_{nickel}, \theta_{elev}) = 0.532 \text{ in}$$

$$\frac{x_{max}(t_{min}, r_{flat}, \theta_{elev})}{R} = 0.589 \qquad h(0.5 \cdot R, t_{min}, r_{flat}, \theta_{elev}) = 0.219 \text{ in}$$

$$\frac{x_{max}(t_{max}, r_{dime}, \theta_{elev})}{R} = 0.792 \qquad h(0.5 \cdot R, t_{max}, r_{dime}, \theta_{elev}) = 0.534 \text{ in}$$

$$\frac{x_{max}(t_{max}, r_{nickel}, \theta_{elev})}{R} = 0.76 \qquad h(0.5 \cdot R, t_{max}, r_{nickel}, \theta_{elev}) = 0.491 \text{ in}$$

$$\frac{x_{max}(t_{max}, r_{flat}, \theta_{elev})}{R} = 0.568 \qquad h(0.5 \cdot R, t_{max}, r_{flat}, \theta_{elev}) = 0.168 \text{ in}$$

Here are some other values for various cases:

$$\frac{x_{max}(t_{min}, r_{dime}, \theta_{avg})}{R} = 0.656 \qquad h(0.5 \cdot R, t_{min}, r_{dime}, \theta_{avg}) = 0.242 \text{ in}$$

$$\frac{x_{max}(t_{min}, r_{nickel}, \theta_{avg})}{R} = 0.629 \qquad h(0.5 \cdot R, t_{min}, r_{nickel}, \theta_{avg}) = 0.208 \text{ in}$$

$$\frac{x_{max}(t_{min}, r_{flat}, \theta_{avg})}{R} = 0.459 \qquad h(0.5 \cdot R, t_{min}, r_{flat}, \theta_{avg}) = -0.091 \text{ in}$$

$$\frac{x_{max}(t_{avg}, r_{dime}, \theta_{avg})}{R} = 0.639 \qquad h(0.5 \cdot R, t_{avg}, r_{dime}, \theta_{avg}) = 0.214 \text{ in}$$

$$\frac{x_{max}(t_{avg}, r_{nickel}, \theta_{avg})}{R} = 0.612 \qquad h(0.5 \cdot R, t_{avg}, r_{nickel}, \theta_{avg}) = 0.18 \text{ in}$$

$$\frac{x_{max}(t_{avg}, r_{flat}, \theta_{avg})}{R} = 0.446 \qquad h(0.5 \cdot R, t_{avg}, r_{flat}, \theta_{avg}) = -0.12 \text{ in}$$

$$\frac{x_{max}(t_{max}, r_{dime}, \theta_{avg})}{R} = 0.621 \qquad h(0.5 \cdot R, t_{max}, r_{dime}, \theta_{avg}) = 0.187 \text{ in}$$

$$\frac{x_{max}(t_{max}, r_{nickel}, \theta_{avg})}{R} = 0.594 \qquad h(0.5 \cdot R, t_{max}, r_{nickel}, \theta_{avg}) = 0.152 \text{ in}$$

$$\frac{x_{max}(t_{max}, r_{flat}, \theta_{avg})}{R} = 0.432 \qquad h(0.5 \cdot R, t_{max}, r_{flat}, \theta_{avg}) = -0.149 \text{ in}$$

From the drawings above, the distance from the centerline of the shaft to the center of the ball is:

$$c(x,r) := (r + R)\cdot\frac{x}{R}$$

Therefore, for a given center line distance, the offset can be found with:

$$x(c,r) := \frac{R\cdot c}{(r + R)}$$

The centerline offset (c) is often expressed as "tips of English" (i.e., as multiples of tip widths):

$$c_{tips}(n,t) := n\cdot t$$

Using the equation above, the offset can be related to "tips of English" as:

$$x_{tips}(n,t,r) := \frac{R\cdot n\cdot t}{(r + R)}$$

An alternative measure of the amount of English is the percentage of the maximum recommended value (0.5R):

$$pE(n,t,r) := \frac{x_{tips}(n,t,r)}{0.5\cdot R}$$

Here is a comparison of "tips of English" and actual offsets for various cases:

one tip of English: $n := 1$

$x := x_{tips}(n,t_{min},r_{dime})$ $\quad \frac{x}{R} = 0.293$ $\quad pE(n,t_{min},r_{dime}) = 58.622\,\%$ $\quad h(x,t_{min},r_{dime},\theta_{avg}) = 0.556\text{ in}$

$x := x_{tips}(n,t_{min},r_{nickel})$ $\quad \frac{x}{R} = 0.281$ $\quad pE(n,t_{min},r_{nickel}) = 56.152\,\%$ $\quad h(x,t_{min},r_{nickel},\theta_{avg}) = 0.555\text{ in}$

$x := x_{tips}(n,t_{avg},r_{dime})$ $\quad \frac{x}{R} = 0.333$ $\quad pE(n,t_{avg},r_{dime}) = 66.616\,\%$ $\quad h(x,t_{avg},r_{dime},\theta_{avg}) = 0.468\text{ in}$

$x := x_{tips}(n,t_{avg},r_{nickel})$ $\quad \frac{x}{R} = 0.319$ $\quad pE(n,t_{avg},r_{nickel}) = 63.809\,\%$ $\quad h(x,t_{avg},r_{nickel},\theta_{avg}) = 0.467\text{ in}$

$x := x_{tips}(n,t_{max},r_{dime})$ $\quad \frac{x}{R} = 0.373$ $\quad pE(n,t_{max},r_{dime}) = 74.61\,\%$ $\quad h(x,t_{max},r_{dime},\theta_{avg}) = 0.381\text{ in}$

$x := x_{tips}(n,t_{max},r_{nickel})$ $\quad \frac{x}{R} = 0.357$ $\quad pE(n,t_{max},r_{nickel}) = 71.466\,\%$ $\quad h(x,t_{max},r_{nickel},\theta_{avg}) = 0.379\text{ in}$

one and a half tips of English: $n := 1.5$

$x := x_{tips}(n, t_{min}, r_{dime})$ $\frac{x}{R} = 0.44$ $pE(n, t_{min}, r_{dime}) = 87.933\,\%$ $h(x, t_{min}, r_{dime}, \theta_{avg}) = 0.334\ in$

$x := x_{tips}(n, t_{min}, r_{nickel})$ $\frac{x}{R} = 0.421$ $pE(n, t_{min}, r_{nickel}) = 84.228\,\%$ $h(x, t_{min}, r_{nickel}, \theta_{avg}) = 0.334\ in$

$x := x_{tips}(n, t_{avg}, r_{dime})$ $\frac{x}{R} = 0.5$ $pE(n, t_{avg}, r_{dime}) = 99.924\,\%$ $h(x, t_{avg}, r_{dime}, \theta_{avg}) = 0.215\ in$

$x := x_{tips}(n, t_{avg}, r_{nickel})$ $\frac{x}{R} = 0.479$ $pE(n, t_{avg}, r_{nickel}) = 95.713\,\%$ $h(x, t_{avg}, r_{nickel}, \theta_{avg}) = 0.214\ in$

$x := x_{tips}(n, t_{max}, r_{dime})$ $\frac{x}{R} = 0.56$ $pE(n, t_{max}, r_{dime}) = 111.915\,\%$ $h(x, t_{max}, r_{dime}, \theta_{avg}) = 0.095\ in$

$x := x_{tips}(n, t_{max}, r_{nickel})$ $\frac{x}{R} = 0.536$ $pE(n, t_{max}, r_{nickel}) = 107.199\,\%$ $h(x, t_{max}, r_{nickel}, \theta_{avg}) = 0.094\ in$

two tips of English: $n := 2$

$x := x_{tips}(n, t_{min}, r_{dime})$ $\frac{x}{R} = 0.586$ $pE(n, t_{min}, r_{dime}) = 117.244\,\%$ $h(x, t_{min}, r_{dime}, \theta_{avg}) = 0.109\ in$

$x := x_{tips}(n, t_{min}, r_{nickel})$ $\frac{x}{R} = 0.562$ $pE(n, t_{min}, r_{nickel}) = 112.304\,\%$ $h(x, t_{min}, r_{nickel}, \theta_{avg}) = 0.109\ in$

$x := x_{tips}(n, t_{min}, r_{flat})$ $\frac{x}{R} = 0.408$ $pE(n, t_{min}, r_{flat}) = 81.519\,\%$ $h(x, t_{min}, r_{flat}, \theta_{avg}) = 0.112\ in$

$x := x_{tips}(n, t_{avg}, r_{dime})$ $\frac{x}{R} = 0.666$ $pE(n, t_{avg}, r_{dime}) = 133.232\,\%$ $h(x, t_{avg}, r_{dime}, \theta_{avg}) = -0.043\ in$

$x := x_{tips}(n, t_{avg}, r_{nickel})$ $\frac{x}{R} = 0.638$ $pE(n, t_{avg}, r_{nickel}) = 127.618\,\%$ $h(x, t_{avg}, r_{nickel}, \theta_{avg}) = -0.043\ in$

$x := x_{tips}(n, t_{avg}, r_{flat})$ $\frac{x}{R} = 0.463$ $pE(n, t_{avg}, r_{flat}) = 92.635\,\%$ $h(x, t_{avg}, r_{flat}, \theta_{avg}) = -0.039\ in$

$x := x_{tips}(n, t_{max}, r_{dime})$ $\frac{x}{R} = 0.746$ $pE(n, t_{max}, r_{dime}) = 149.22\,\%$ $h(x, t_{max}, r_{dime}, \theta_{avg}) = -0.196\ in$

$x := x_{tips}(n, t_{max}, r_{nickel})$ $\frac{x}{R} = 0.715$ $pE(n, t_{max}, r_{nickel}) = 142.932\,\%$ $h(x, t_{max}, r_{nickel}, \theta_{avg}) = -0.196\ in$

$x := x_{tips}(n, t_{max}, r_{flat})$ $\frac{x}{R} = 0.519$ $pE(n, t_{max}, r_{flat}) = 103.752\,\%$ $h(x, t_{max}, r_{flat}, \theta_{avg}) = -0.19\ in$

number of "tips" of English for maximum offset (x = 0.5R), using the equations above:

$$n_{max}(t, r) := \frac{(r + R)}{2 \cdot t}$$

$n := n_{max}(t_{min}, r_{dime})$ $\quad n = 1.706 \quad$ $h(x_{tips}(n, t_{min}, r_{dime}), t_{avg}, r_{dime}, \theta_{avg}) = 0.214 \text{ in}$

$n := n_{max}(t_{min}, r_{nickel})$ $\quad n = 1.781 \quad$ $h(x_{tips}(n, t_{min}, r_{nickel}), t_{min}, r_{nickel}, \theta_{avg}) = 0.208 \text{ in}$

$n := n_{max}(t_{min}, r_{flat})$ $\quad n = 2.453 \quad$ $h(x_{tips}(n, t_{min}, r_{flat}), t_{min}, r_{flat}, \theta_{avg}) = -0.091 \text{ in}$

$n := n_{max}(t_{avg}, r_{dime})$ $\quad n = 1.501 \quad$ $h(x_{tips}(n, t_{avg}, r_{dime}), t_{avg}, r_{dime}, \theta_{avg}) = 0.214 \text{ in}$

$n := n_{max}(t_{avg}, r_{nickel})$ $\quad n = 1.567 \quad$ $h(x_{tips}(n, t_{avg}, r_{nickel}), t_{avg}, r_{nickel}, \theta_{avg}) = 0.18 \text{ in}$

$n := n_{max}(t_{avg}, r_{flat})$ $\quad n = 2.159 \quad$ $h(x_{tips}(n, t_{avg}, r_{flat}), t_{avg}, r_{flat}, \theta_{avg}) = -0.12 \text{ in}$

$n := n_{max}(t_{max}, r_{dime})$ $\quad n = 1.34 \quad$ $h(x_{tips}(n, t_{max}, r_{dime}), t_{max}, r_{dime}, \theta_{avg}) = 0.187 \text{ in}$

$n := n_{max}(t_{max}, r_{nickel})$ $\quad n = 1.399 \quad$ $h(x_{tips}(n, t_{max}, r_{nickel}), t_{max}, r_{nickel}, \theta_{avg}) = 0.152 \text{ in}$

$n := n_{max}(t_{max}, r_{flat})$ $\quad n = 1.928 \quad$ $h(x_{tips}(n, t_{max}, r_{flat}), t_{max}, r_{flat}, \theta_{avg}) = -0.149 \text{ in}$