technical proof technical proof

TP B.14
Draw shot cue ball angle approximations

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
by David G. Alciatore, PhD, PE ("Dr. Dave")

originally posted: 9/24/2011 last revision: 9/24/2011

Relevant physical constants and parameters

$e := 0.95$ — typical coefficient of restitution between balls

$D := \frac{2.25 \cdot in}{m}$ $\quad R := \frac{D}{2}$ — ball dimensions, converter to meters

$v := \frac{5 \cdot mph}{\frac{m}{s}}$ — typical slow-medium CB speed

$\varphi := 0 \cdot deg, 1 \cdot deg .. 90 \cdot deg$ — cut angle range

$f := 0, 0.01 .. 1$ — ball-hit fraction range

From TP A.23, cut angle (ϕ) and ball-hit fraction (f) are related according to:

$$\varphi_f(f) := asin(1 - f) \qquad f_\varphi(\varphi) := 1 - \sin(\varphi)$$

From TP A.4, the final cue ball deflection angle for any shot is:

$$\theta_c = \tan^{-1}\left(\frac{5v\sin(\phi)\cos(\phi)}{5v\sin^2(\phi) - 2R\omega}\right)$$

From TP A.20, the amount of backspin for a typical good action draw shot is:

$$\omega = 0.625 \cdot \frac{v}{R}$$

giving a CB deflection angle of:

$$\theta_c = \tan^{-1}\left(\frac{\sin(\phi)\cos(\phi)}{\sin^2(\phi) - 1/4}\right)$$

Therefore, the angle between the orignal CB aiming and the draw line is:

$$\theta_{actual}(\varphi) := 180 \cdot deg - atan2\left(\sin(\varphi)^2 - \frac{1}{4}, \sin(\varphi) \cdot \cos(\varphi)\right)$$

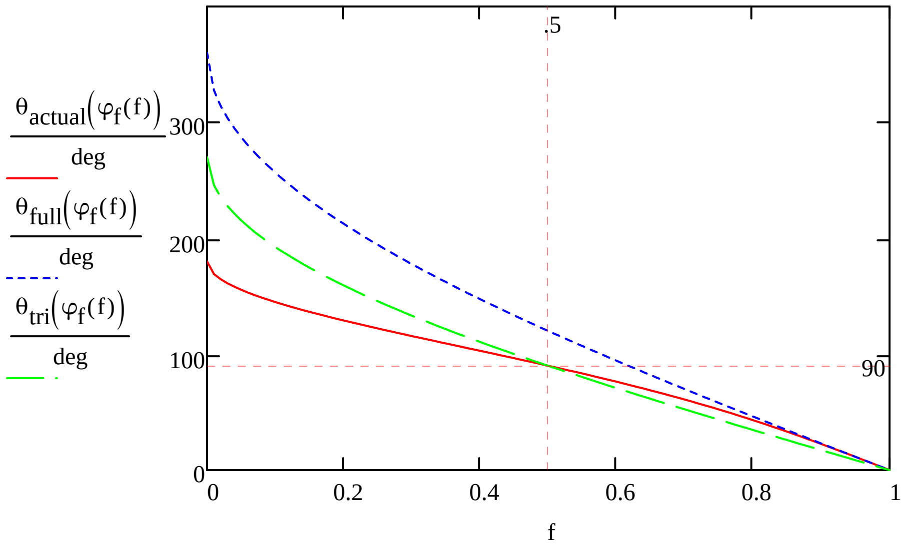
From TP A.20, here are two approximations that apply to certain ranges of draw shots:

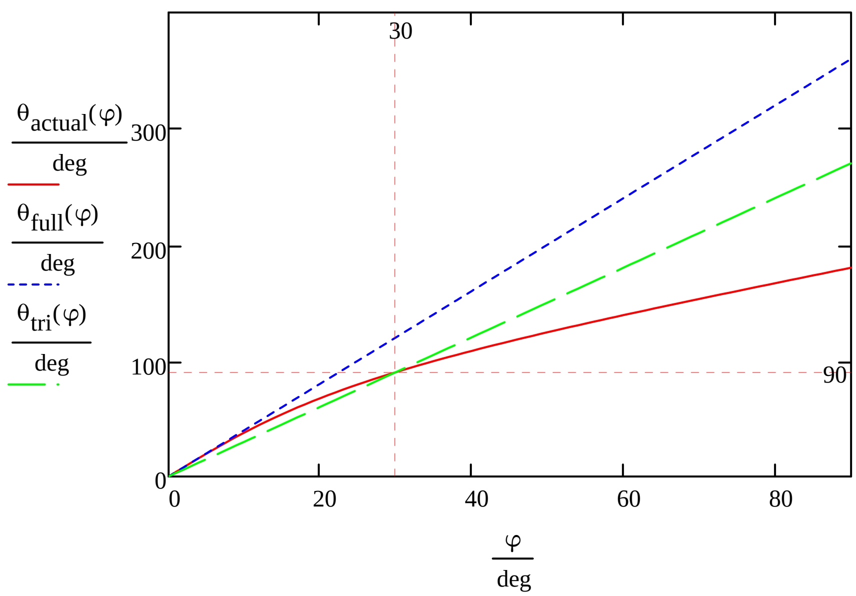
Double-bisect system (double the cut angle and double the total angle again)

$$\theta_{full}(\varphi) := 4\cdot\varphi$$

Trisect system (triple the cut angle):

$$\theta_{tri}(\varphi) := 3\cdot\varphi$$





The double-bisect system approximates the draw angle very well up to about a 20 degree cut (~5/8-ball hit):

$$\varphi_{full} := 0\cdot deg, 1\cdot deg .. 20\cdot deg \qquad f_{full} := f_{\varphi}(20\cdot deg), f_{\varphi}(20\cdot deg) + .01 .. 1$$

$$f_{\varphi}(20\cdot deg) = 0.658 \qquad \frac{5}{8} = 0.625$$

The trisect system approximates the draw angle fairly well for all shots in the 0-40 degree cut range (i.e, greater than ~3/8-ball hit):

$$\varphi_{tri} := 0\cdot deg, 1\cdot deg .. 40\cdot deg \qquad f_{tri} := f_{\varphi}(40\cdot deg), f_{\varphi}(40\cdot deg) + .01 .. 1$$

$$f_{\varphi}(40\cdot deg) = 0.357 \qquad \frac{3}{8} = 0.375$$

The slope of the draw-angle curve can be used to develop additional approximations:

$$slope(\varphi\varphi) := \frac{d}{d\varphi\varphi}\left(180\cdot deg - atan\left(\frac{\sin(\varphi\varphi)\cdot\cos(\varphi\varphi)}{\sin(\varphi\varphi)^2 - \frac{1}{4}}\right)\right) \text{ simplify } \rightarrow \frac{8\cdot\sin(\varphi\varphi)^2 + 4}{8\cdot\sin(\varphi\varphi)^2 + 1}$$

At the important 1/2-ball hit (30 degree cut) benchmark, the slope is:

$$slope(30\cdot deg) = 2$$

so a good approximation for an average cut angle (close to 30 degrees) is:

$$\theta_{avg}(\varphi) := 2\cdot\varphi + 30\cdot deg$$

This approximation applies fairly well for cut angles in the 10-60 degree (~1/8-7/8 ball-hit fraction) range:

$$\varphi_{avg} := 10\cdot deg, 11\cdot deg .. 60\cdot deg \qquad f_{avg} := f_{\varphi}(60\cdot deg), f_{\varphi}(60\cdot deg) + .01 .. f_{\varphi}(10\cdot deg)$$

$$f_{\varphi}(10\cdot deg) = 0.826 \qquad \frac{7}{8} = 0.875 \qquad f_{\varphi}(60\cdot deg) = 0.134 \qquad \frac{1}{8} = 0.125$$

For a 90-degree cut, the slope of the draw-angle curve is:

$$slope(90\cdot deg) = 1.333$$

so a good approximation for a thin cut (close to 90 degrees) is:

$$\theta_{thin}(\varphi) := \frac{4}{3}\cdot\varphi + 60\cdot deg$$

This approximation applies fairly well for cut angles in the 40-90 degree range (i.e., thinner than ~3/8-ball hit):

$$\varphi_{thin} := 40\cdot deg, 41\cdot deg .. 90\cdot deg \qquad f_{thin} := 0, .01 .. f_{\varphi}(40\cdot deg)$$

$$f_{\varphi}(40\cdot deg) = 0.357 \qquad \frac{3}{8} = 0.375$$

Here's a composite all all of the approximations over their good ranges: