TP B.21
Small-gap-combination throw effects

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu

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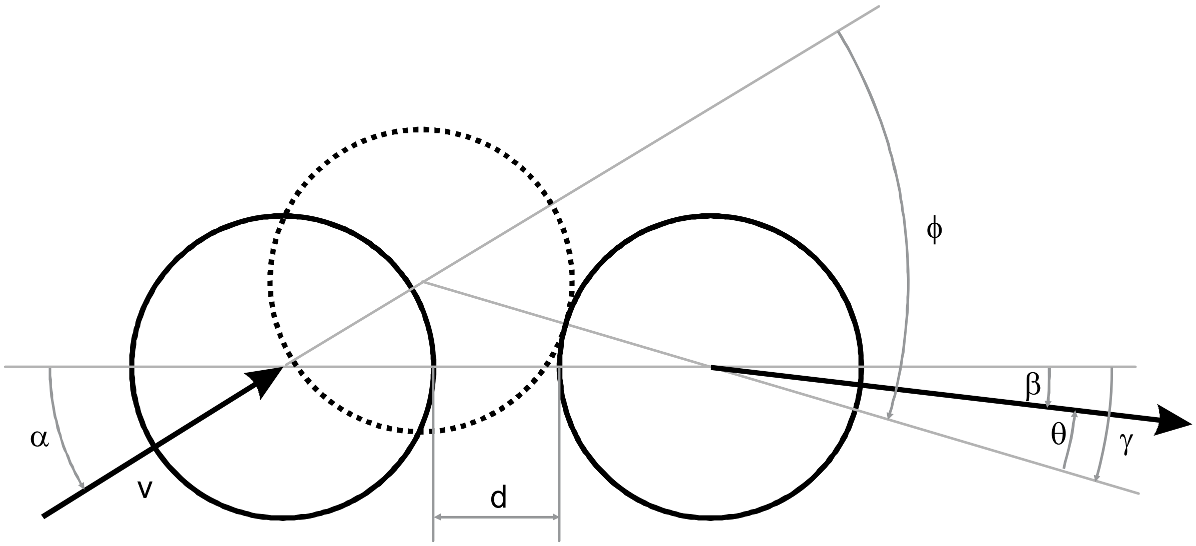


The first OB is hit with speed v at angle α (relative to the original line of centers of the OBs) resulting in cut angle ϕ, and impact line-of-centers angle γ. Cut-induced throw (CIT) θ results in the second OB heading at angle β relative to the original line of centers.

The law of sines applied to the gray triangle in the diagram above, with ball radius R, gives:

$$R := \frac{9}{8} \cdot in = 1.125 \cdot in$$

$$\frac{\sin(\alpha)}{2R} = \frac{\sin(180 \cdot deg - \alpha - \gamma)}{2R + d} = \frac{\sin(\alpha + \gamma)}{2R + d} = \frac{\sin(\phi)}{2R + d}$$

This can be solved for the line-of-centers angle γ giving:

$$\gamma(\alpha, d) := \operatorname{asin}\left(\frac{2R + d}{2R} \cdot \sin(\alpha)\right) - \alpha$$

The cut angle for the shot is:

$$\phi(\alpha, d) := \alpha + \gamma(\alpha, d)$$

From TP A.23, the ball-hit fraction corresponding to this cut angle is:

$$f(\alpha, d) := 1 - \sin(\phi(\alpha, d))$$

The 1st-ball angle α is related to shot cut angle ϕ as:

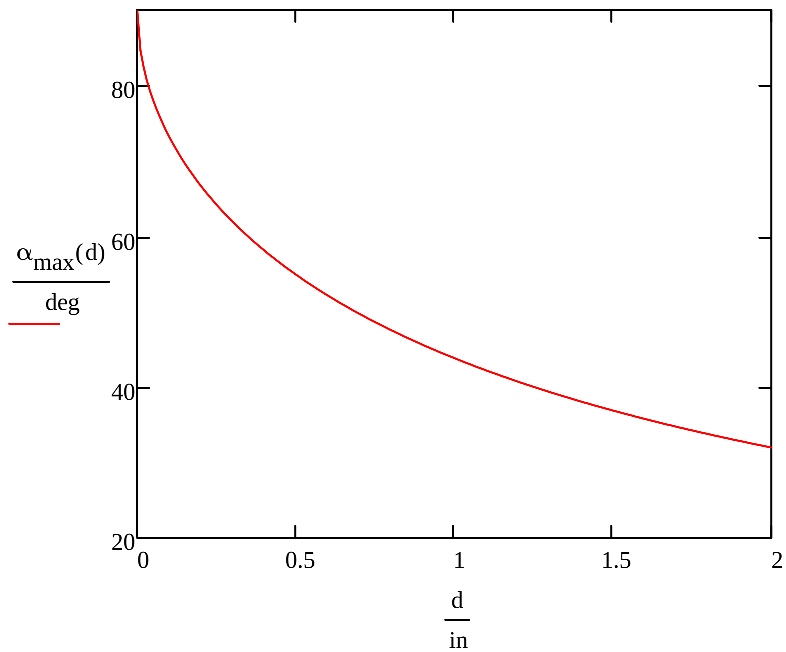
$$\alpha(\phi, d) := \operatorname{asin}\left(\frac{2R\cdot\sin(\phi)}{2R + d}\right)$$

For a given gap distance d, the largest the 1st-ball angle (α_{max}) can be, where the cut angle ϕ is 90 degrees, is:

$$\alpha_{max}(d) := \operatorname{asin}\left(\frac{2R}{2R + d}\right)$$

Here's a plot showing how the maximum 1st-ball angle (α_{max}) varies with gap distance d (it goes from 90 degrees at d=0 to 0 degrees at d=∞):

$$d := 0\cdot in, 0.01\cdot in .. 2\cdot in$$



From TP A.14, for stun shot with no sidespin (with $\omega_x = \omega_z = 0$), the amount of throw θ (with speed in units of m/s) is given by:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} := \begin{pmatrix} 9.951 \times 10^{-3} \\ 0.108 \\ 1.088 \end{pmatrix} \qquad \mu(v) := a + b\cdot e^{-c\cdot v}$$

$$\theta(v, \phi) := \operatorname{atan}\left[\frac{\min\left(\frac{\mu(v\cdot\sin(\phi))\cdot v\cdot\cos(\phi)}{v\cdot\sin(\phi)}, \frac{1}{7}\right)\cdot(v\cdot\sin(\phi))}{v\cdot\cos(\phi)}\right]$$

The resulting OB angle is the line-of-centers angle adjusted by throw:

$$\beta(v, \alpha, d) := \gamma(\alpha, d) - \theta(v, \phi(\alpha, d))$$

To determine the optimal gap size to have throw exactly cancel cut with relatively small 1st-ball angles, we can use a small-angle approximation for everything, where for small angle x:

$$\sin(x) = x \qquad \cos(x) = 1 \qquad \tan(x) = x \qquad \operatorname{atan}(x) = x$$

The small-angle approximation for the line-of-centers angle γ is:

$$\gamma = \left(\frac{2R + d}{2R} - 1\right)\alpha = \frac{d}{2R}\cdot\alpha$$

At small angles, throw θ is in the gearing region and simplifies to:

$$\theta = \frac{1}{7}\cdot\phi = \frac{1}{7}\cdot\left(\frac{2R + d}{2R}\right)\cdot\alpha$$

To have the OB head perfectly straight:

$$\beta = \gamma - \theta = 0$$

Giving:

$$\frac{d}{2R}\cdot\alpha - \frac{1}{7}\cdot\left(\frac{2R + d}{2R}\right)\cdot\alpha = 0$$

$$\alpha\left[d - \frac{1}{7}\cdot(2R + d)\right] = 0$$

$$\frac{6}{7}\cdot d = \frac{2}{7}\cdot R$$

$$d = \frac{1}{3}\cdot R = \frac{1}{3}\cdot\left(\frac{9}{8}\cdot in\right) = \frac{3}{8}\cdot in$$

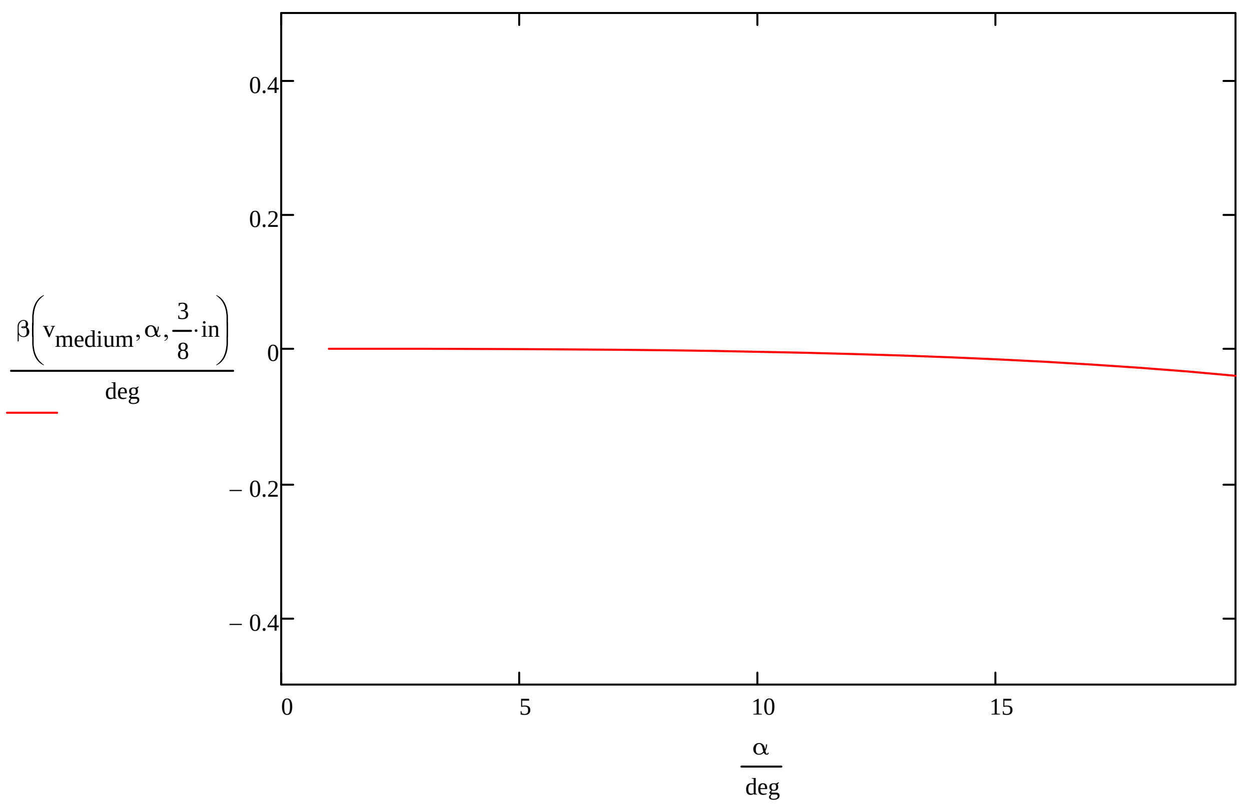
So to have the OB head straight over a fairly wide range of 1st-ball angles, the optimal gap size is 3/8" (9.5mm). Fortunately, 9.5mm is just slightly smaller than the diameter of typical low-CB-deflections shafts, so it is easy to judge the optimal gap size with your tip. Alternatively, you can visualize the gap size as 1/3 the radius of a ball.

From **http://billiards.colostate.edu/threads/speed.html#typical**, a typical range of shot speeds, converted to m/s is:

$$v_{slow} := \frac{1\cdot mph}{\frac{m}{s}} = 0.447 \qquad v_{medium} := \frac{3\cdot mph}{\frac{m}{s}} = 1.341 \qquad v_{fast} := \frac{7\cdot mph}{\frac{m}{s}} = 3.129$$

Here's a close up of how the OB angle β differs from 0 degrees (using a vertical scale of +/- 0.5 degrees) with the optimal 3/8" gap size as the 1st-ball angle α gets larger than a "small angle" (for a typical shot speed):

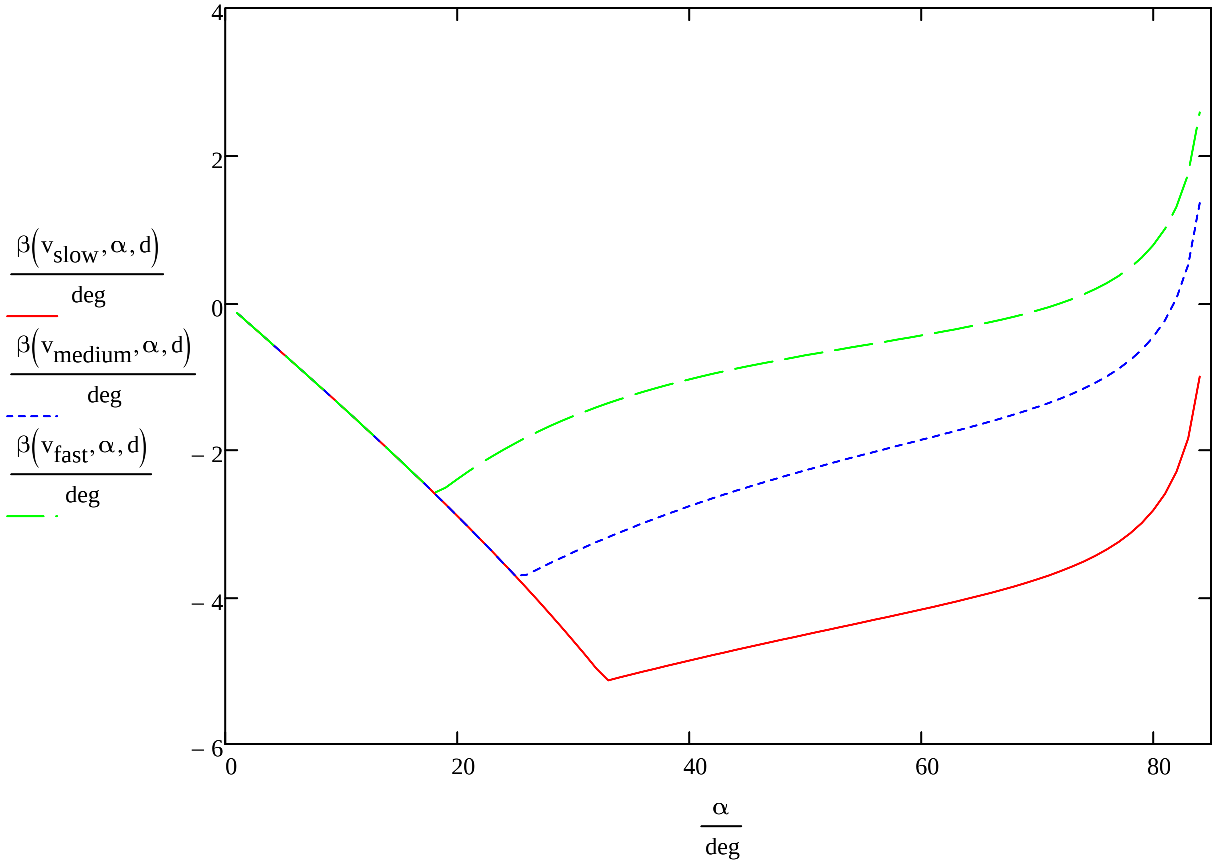
$$\alpha := 0 \cdot \mathrm{deg}, 1 \cdot \mathrm{deg} .. 20 \cdot \mathrm{deg}$$



So the 3/8" gap should yield good OB direction precision over a fairly wide range of 1st-ball angles (+/- 20 degrees)!

The following plots show how the resulting OB angle β varies with 1st-ball angle α at various shot speeds v for different gap sizes d:

$d := \frac{1}{100} \cdot in \qquad \alpha_{max}(d) = 84.608 \cdot deg \qquad \alpha := 0 \cdot deg, 1 \cdot deg .. \alpha_{max}(d)$

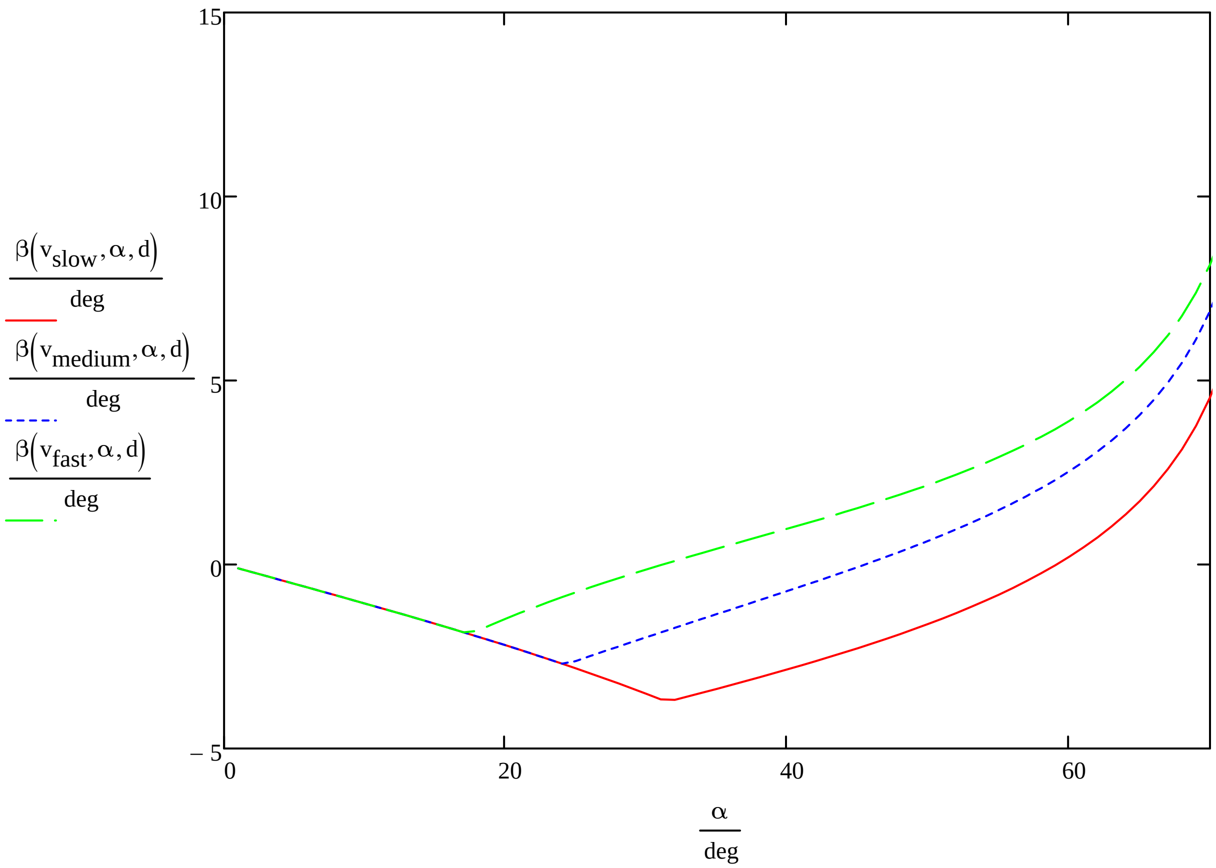


max throw effect at slow speed with:

$\alpha := 33 \cdot deg \qquad \phi(\alpha, d) = 33.166 \cdot deg \qquad f(\alpha, d) = 0.453$

So when the gap between the OBs is extremely small (about 1/100 of an inch), the 2nd OB heads in the throw direction (except at extremely large 1st-ball angles and fast speed), especially at slower speeds (except at really small angles where speed has no effect). The largest throw effect occurs close to a 1/2-ball hit (slightly thinner).

$d := \frac{1}{10} \cdot in$ $\qquad \alpha_{max}(d) = 73.225 \cdot deg$ $\qquad \alpha := 0 \cdot deg, 1 \cdot deg .. \alpha_{max}(d)$

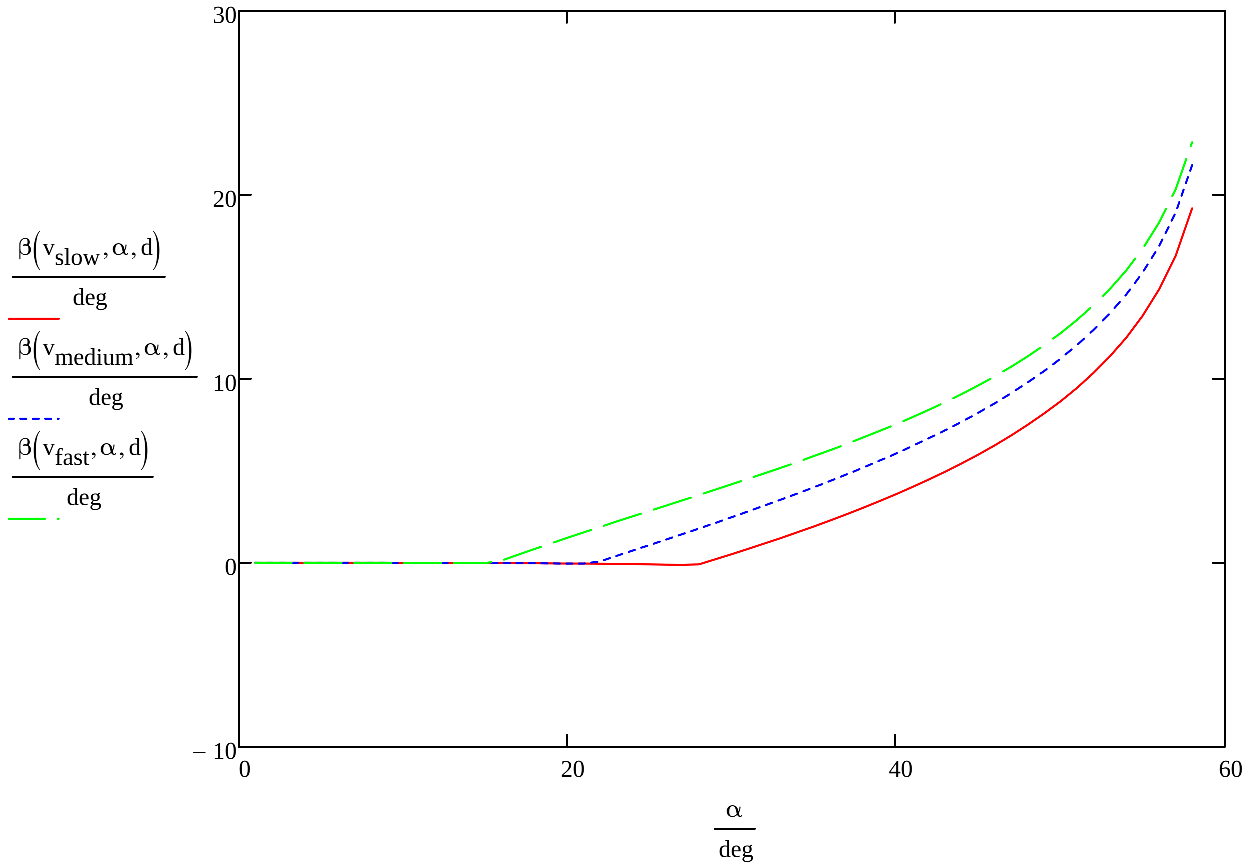


max throw effect at slow speed with:

$\alpha := 31.5 \cdot deg \qquad \phi(\alpha, d) = 33.074 \cdot deg \qquad f(\alpha, d) = 0.454$

So when the gap between the OBs is fairly small (about 1/10 of an inch), the 2nd OB heads in the throw direction at smaller 1st-ball angles and slower speeds (except at really small angles where speed has no effect), but the 2nd OB heads in the cut direction at larger angles and faster speed. The largest throw effect occurs close to a 1/2-ball hit (slightly thinner).

$d := \frac{3}{8} \cdot in$ $\alpha_{max}(d) = 58.997 \cdot deg$ $\alpha := 0 \cdot deg, 1 \cdot deg .. \alpha_{max}(d)$

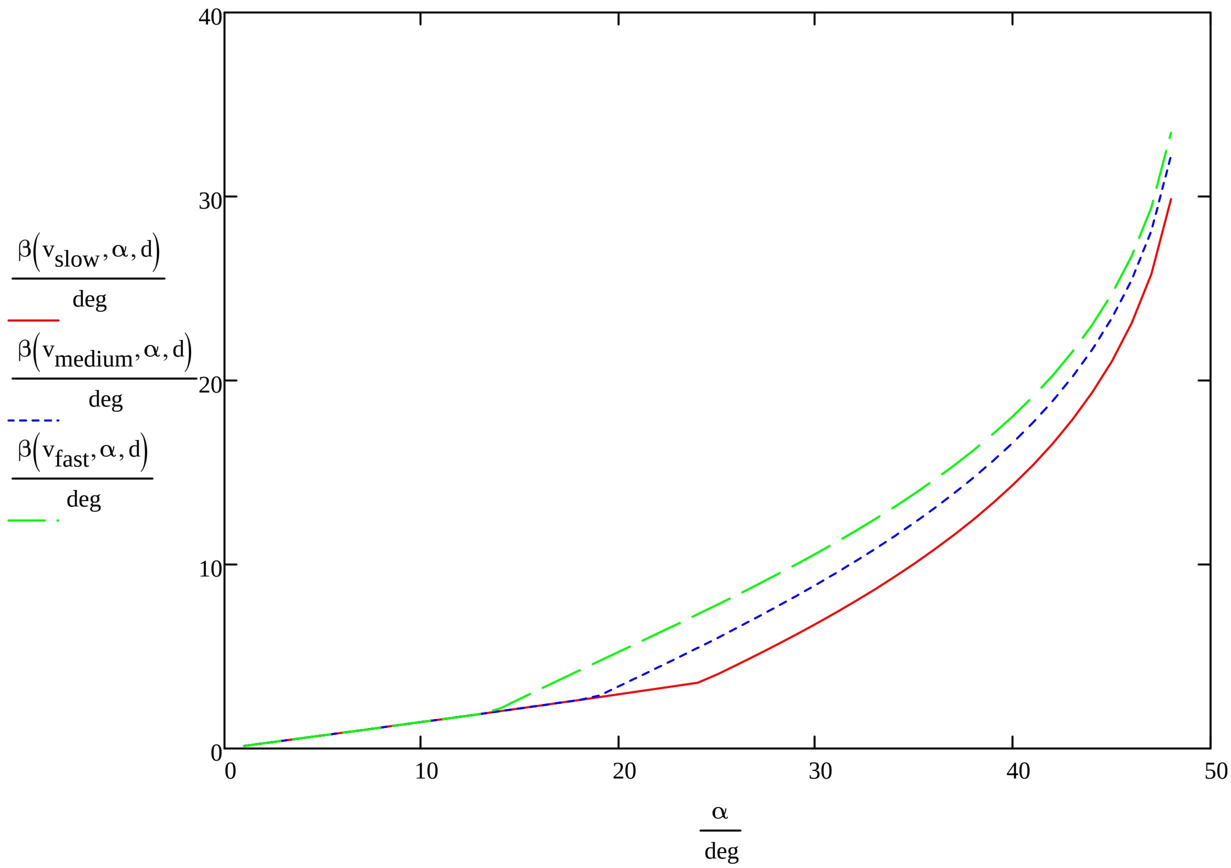


max angle where throw cancels cut at slow speed:

$\alpha := 28 \cdot deg$ $\phi(\alpha, d) = 33.211 \cdot deg$ $f(\alpha, d) = 0.452$

So when the gap between the OBs is the optimal 3/8 of an inch, the 2nd OB heads very straight (i.e., the throw effect cancels the cut effect) over a fairly wide range of 1st-ball angles. At larger angles, the 2nd OB heads in the cut direction (because the cut effect is larger than the throw effect).

$d := \frac{3}{4} \cdot in$ $\alpha_{max}(d) = 48.59 \cdot deg$ $\alpha := 0 \cdot deg, 1 \cdot deg .. \alpha_{max}(d)$

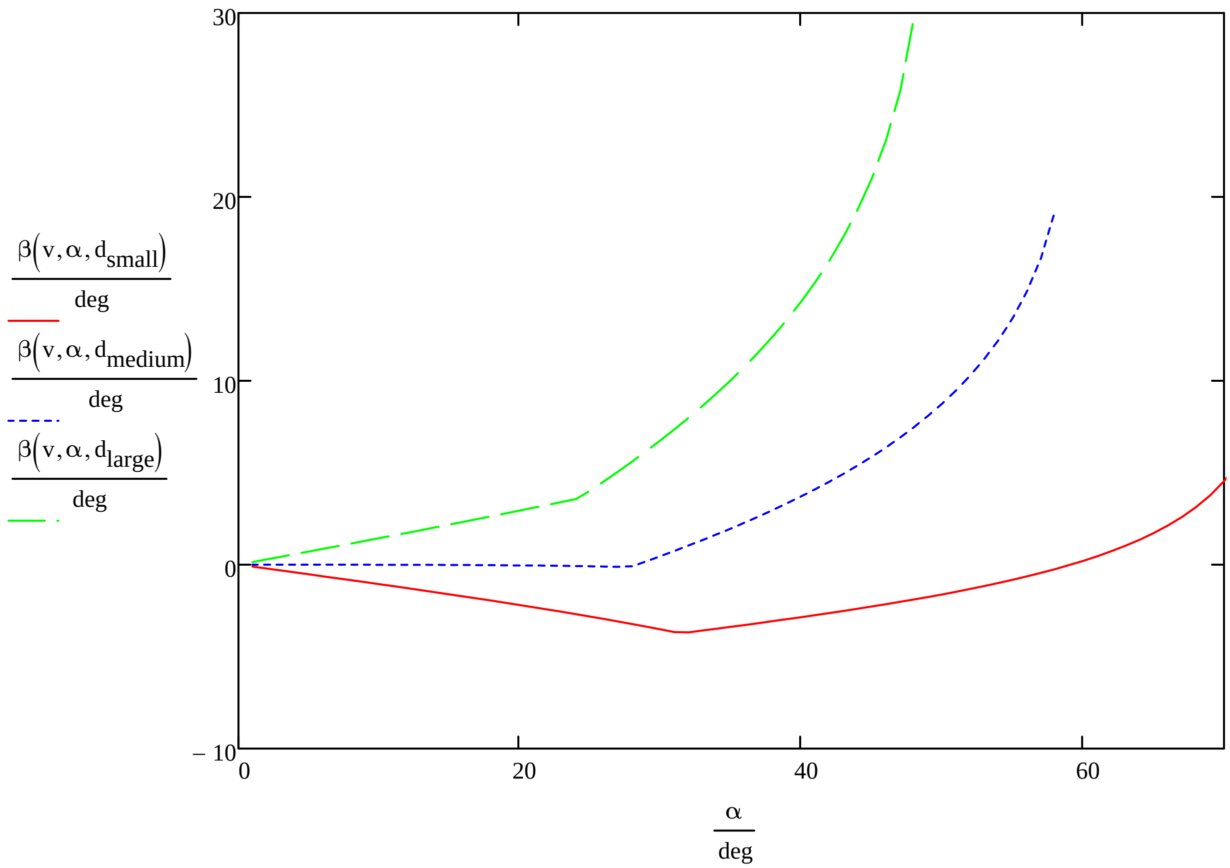


So when the gap between the OBs is larger than 3/8 of an inch (here, 3/4 of an inch), the 2nd OB always heads in the cut direction, regardless of shot speed; although, throw reduces the cut more at slower speed (except at small angles where the speed has no effect).

The following plot shows how the resulting OB angle β varies with 1st-ball angle α at various gap sizes d for a slow-speed shot:

$$d_{small} := \frac{1}{10} \cdot in \qquad d_{medium} := \frac{3}{8} \cdot in \qquad d_{large} := \frac{3}{4} \cdot in$$

$$v := v_{slow} \qquad \alpha_{max}\left(d_{small}\right) = 73.225 \cdot deg \qquad \alpha := 0 \cdot deg, 1 \cdot deg .. \alpha_{max}\left(d_{small}\right)$$

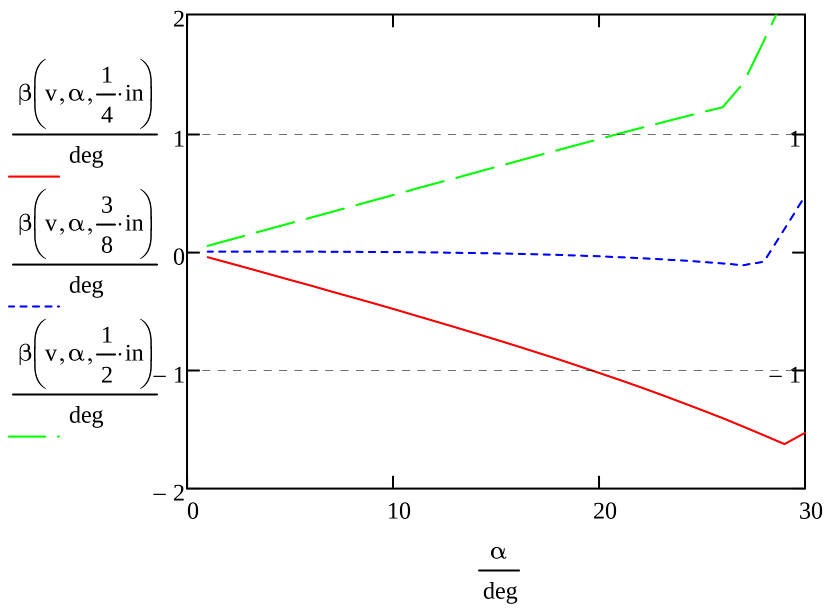


Again, only when the gap between the OBs is 3/8" (9.5 mm) does the 2nd ball head very straight (i.e., the throw and cut effect cancel) over a fairly wide range of 1st-ball angles. With a smaller gap, the throw effect is stronger at small angles and the cut effect is stronger at large angles; and with a larger gap, the cut effect is stronger at all angles.

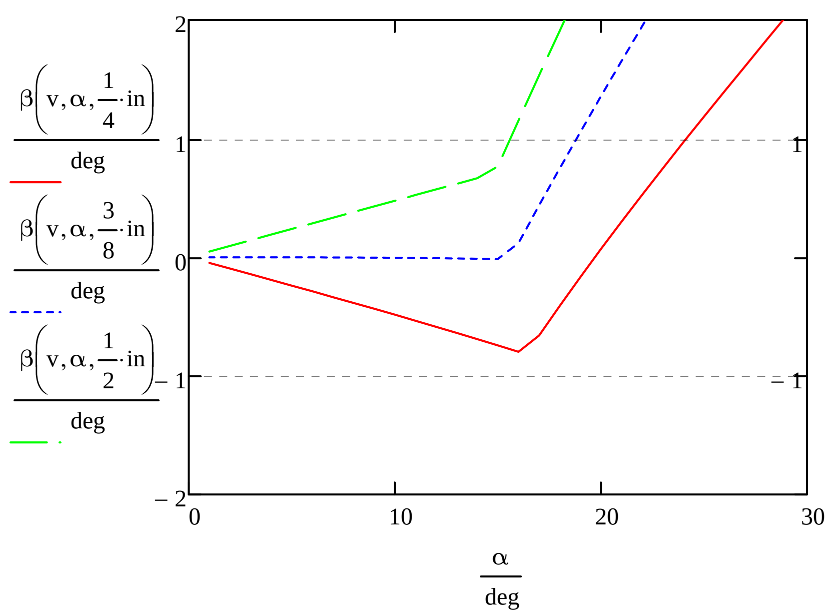
The following plots compares gap sizes close to the optimal 3/8" (9.5 mm). From the last page of TP 3.4, a typical required OB angle error for a medium distance shot is +/- 1 degree, so this is the range OB angle β needs to be if the combo is lined up to the heart of a pocket.

$\frac{1}{4}\cdot in = 6.35\cdot mm$ $\quad \frac{3}{8}\cdot in = 9.525\cdot mm$ $\quad \frac{1}{2}\cdot in = 12.7\cdot mm$ $\quad \alpha := 0\cdot deg, 1\cdot deg\,..\,30\cdot deg$

$v := v_{slow}$



$v := v_{fast}$



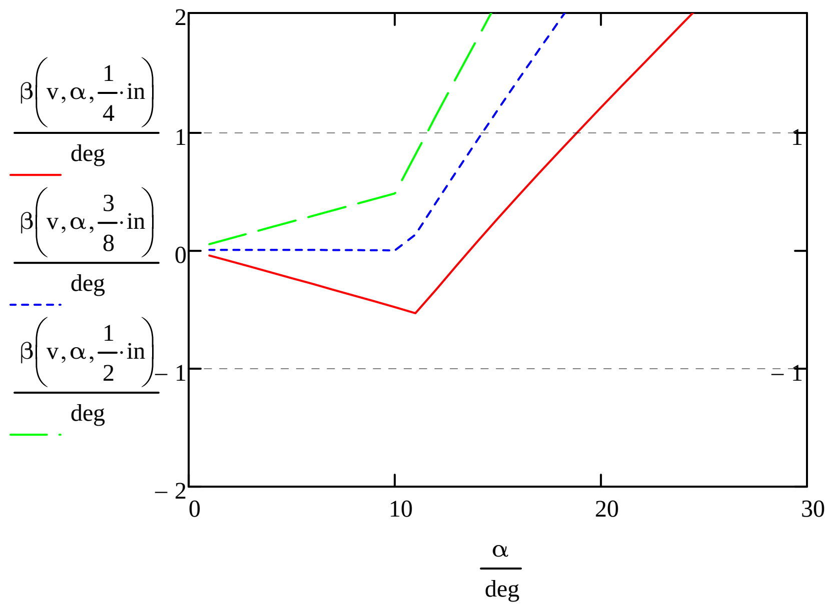
For a slow shot (top plot), a 3/8" gap results in excellent accuracy over a wide range of angles, as proven above and as demonstrated in the following video:
http://billiards.colostate.edu/normal_videos/new/NVJ-1.htm
For a fast speed shot (bottom plot), a 1/4" gap results in less accuracy for small angles (less than about 15 degrees), but the accuracy is within +/- 1 degree over a larger range of 1st-ball angles α. For 1st-ball angles below about 20 degrees, the error is in the throw direction, and for angle above about 20 degrees, the error is in the cut direction.

Here's a good overall simplified "rule of thumb:" If a small-gap combo is wired to the center of a pocket, with a gap size between 1/4" (6 mm, about 1/2 the width of a typical shaft) and 1/2" (13 mm, about the width of a typical shaft), anything fuller than a 3/4-ball hit (14.5 degrees) between the 1st and 2nd ball will result in pocketing the 2nd ball (at any speed).

It is important to point out that the plots in this document will vary slightly with ball conditions. I used what are considered typical values for friction in my analysis, but the amount of throw will vary from one ball set to the next, per the ball cleaning and surface condition effects resource page here:
http://billiards.colostate.edu/threads/balls.html#surface

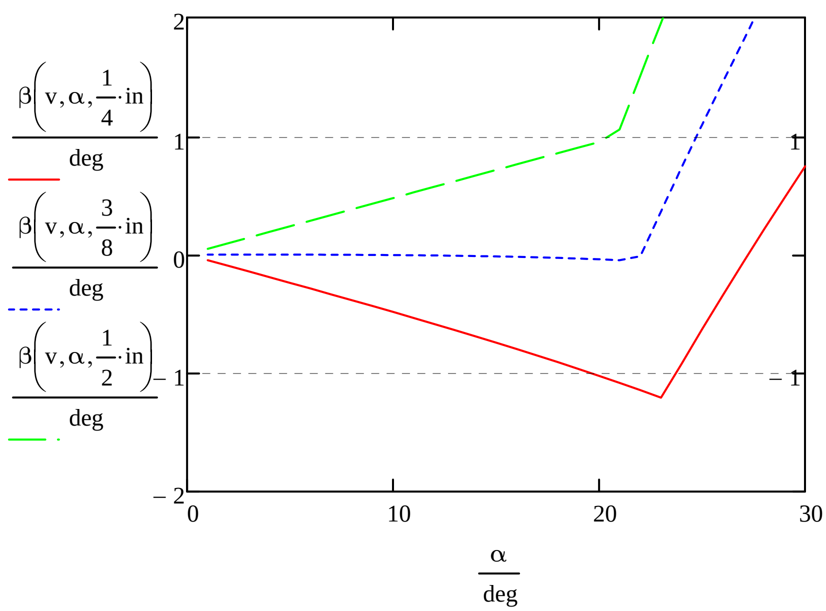
For example, if ball friction were 1/2 of typical values, the last plot on the previous page would change to:

$$\theta(v,\phi) := \mathrm{atan}\left[\frac{\min\left(\frac{\frac{1}{2}\mu(v\cdot\sin(\phi))\cdot v\cdot\cos(\phi)}{v\cdot\sin(\phi)},\frac{1}{7}\right)\cdot(v\cdot\sin(\phi))}{v\cdot\cos(\phi)}\right] \qquad \beta(v,\alpha,d) := \gamma(\alpha,d) - \theta(v,\phi(\alpha,d))$$



And if ball friction were twice typical values, the last plot on the previous page would change to:

$$\theta(v,\phi) := \mathrm{atan}\left[\frac{\min\left[\frac{(2\,\mu(v\cdot\sin(\phi)))\cdot v\cdot\cos(\phi)}{v\cdot\sin(\phi)},\frac{1}{7}\right]\cdot(v\cdot\sin(\phi))}{v\cdot\cos(\phi)}\right] \qquad \beta(v,\alpha,d) := \gamma(\alpha,d) - \theta(v,\phi(\alpha,d))$$



Notice that the 3/8" (9.5mm) optimal value is the same, regardless of the amount of friction.