technical proof

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TP B.8
Draw shot physics

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu

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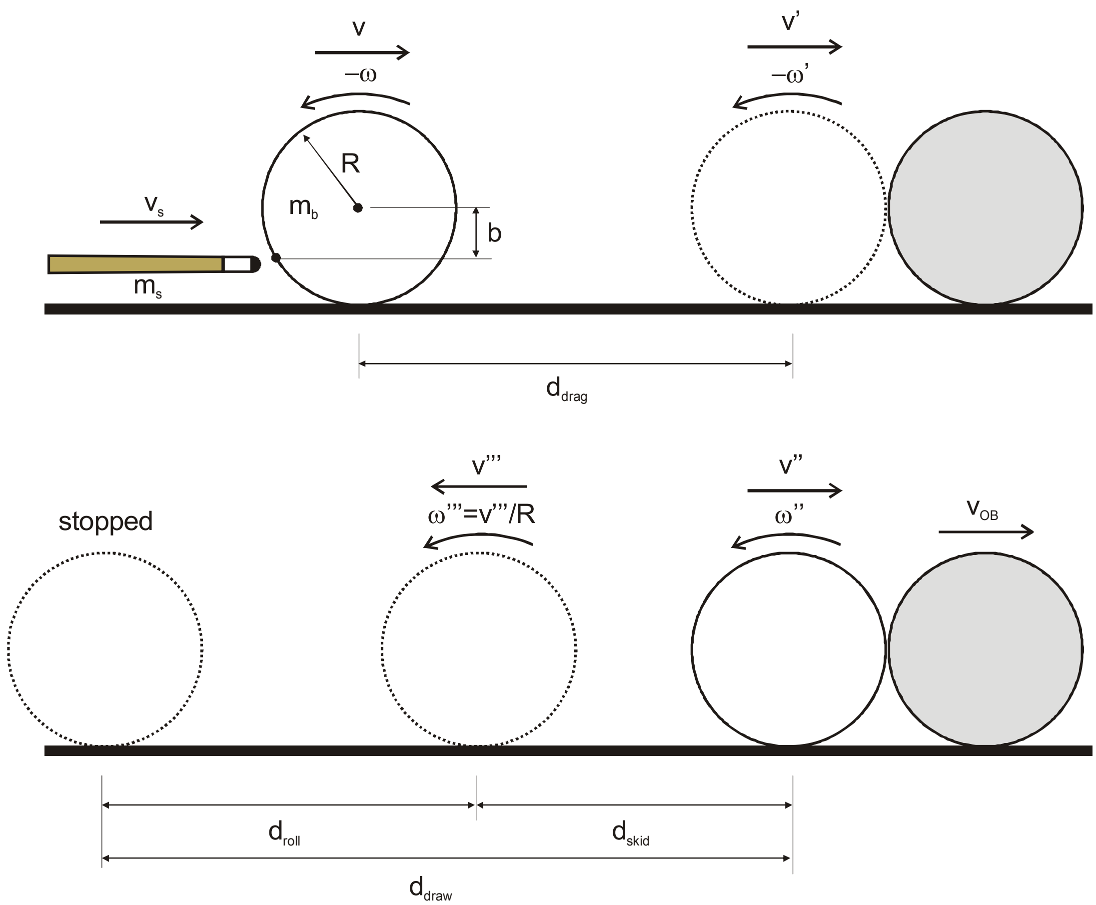


In the analysis below, I will begin by assuming the balls and tip are perfect (e_b=1.0, μ_b=0, η_{tip}=100%), but I will account for these effects later. With perfect balls, v''=0, ω''=-ω', and v_{OB}=v'.

Relevant physical parameters (from "physics" FAQ):

$\mu_s := 0.2$	typical ball-cloth coefficient of sliding friction
$\mu_r := 0.01$	typical ball-cloth coefficient of rolling resistance
$R := 1.125 \cdot in$	ball radius:
$m_r := \frac{6}{19}$	typical ball-mass-to-cue-mass ratio (m_b/m_s):
$e_b := 1$	ball-to-ball coefficient of restitution (assumed perfectly elastic)
$\mu_b := 0.06$	ball-to-ball coefficient of friction (varies with ball surface speed)

From TP A.30, the CB speed (v) and spin (ω) resulting from cue speed v_s and tip offset b, assuming a perfect tip, are:

$$v(v_s, b) := \frac{2 \cdot v_s}{\left[1 + m_r + \frac{5}{2} \cdot \left(\frac{b}{R}\right)^2\right]} \qquad \omega(v_s, b) := -\frac{5}{2} \cdot v(v_s, b) \cdot \frac{b}{R^2}$$

Note - ω<0 implies backspin; ω>0 implies topspin or forward roll

From TP 2.1, the safe miscue limit is generally assumed to be:

$$b_{max} := \frac{R}{2} = 0.563 \cdot in \qquad b := 0, 0.05 \cdot b_{max} .. b_{max}$$

Based on TP B.6, typical draw shot CB speeds (which are higher than other shots) are probably in the following range:

$$v_{slow} := 8 \cdot mph \qquad v_{medium} := 10 \cdot mph \qquad v_{fast} := 14 \cdot mph \qquad v_{power} := 19 \cdot mph$$

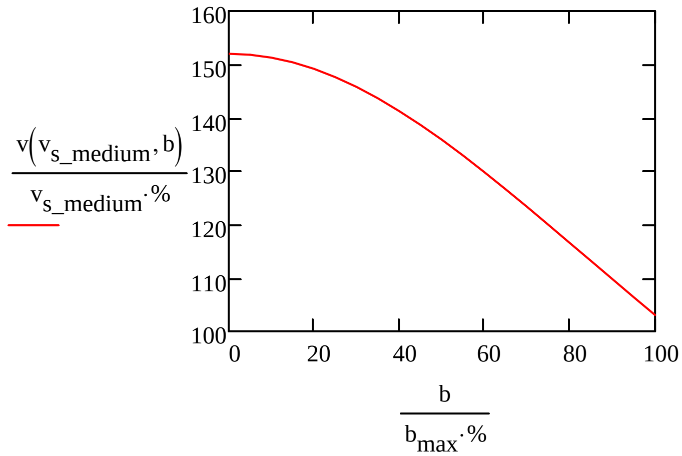
From the equation above for ball speed, these correspond to center-ball-hit cue speeds of:

$$v_s(v) := \frac{(1 + m_r)}{2} \cdot v$$

$$v_{s\_slow} := v_s(v_{slow}) = 5.263 \cdot mph \qquad v_{s\_medium} := v_s(v_{medium}) = 6.579 \cdot mph$$

$$v_{s\_fast} := v_s(v_{fast}) = 9.211 \cdot mph \qquad v_{s\_power} := v_s(v_{power}) = 12.5 \cdot mph$$

Here is how the cue ball speed varies with tip offset for a given cue speed, assuming a perfect tip:

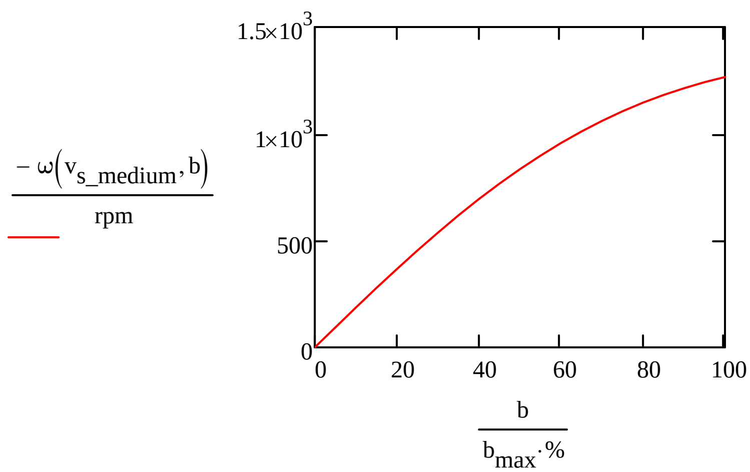


$$\frac{v(v_{s\_medium}, 0)}{v_{s\_medium}} = 152 \cdot \%$$

$$\frac{v(v_{s\_medium}, b_{max})}{v_{s\_medium}} = 103.051 \cdot \%$$

So with a center-ball hit, assuming a perfect tip, the CB speed is about 50% more than (3/2-times) the cue speed; and at a maximum offset shot, the CB and original cue speeds are nearly equal.

Here is how the cue ball spin varies with tip offset for a given cue speed:



$$\frac{\omega(v_{s\_medium}, b_{max})}{\omega(v_{s\_medium}, 0.5b_{max})} = 151.695 \cdot \%$$

$$\frac{\omega(v_{s\_medium}, b_{max})}{\omega(v_{s\_medium}, 0.9b_{max})} = 104.313 \cdot \%$$

So, assuming a perfect tip, more tip offset always gives you more spin, but the increase in spin isn't as great with increasing offset. The spin at maximum offset is only about 50% faster than the spin at half the maximum offset, and only about 5% than the spin at 90% maximum offset.

From TP 4.1, the CB speed and spin change during drag over distance x on the way to the OB according to:

$$v_{drag}(v, x, \mu_s) := \sqrt{v^2 - 2\cdot\mu_s\cdot g\cdot x}$$

$$\omega_{drag}(v, \omega, x, \mu_s) := \omega + \frac{5}{2\cdot R}\cdot\left(v - \sqrt{v^2 - 2\cdot\mu_s\cdot g\cdot x}\right)$$

From TP 4.1, the distance required for stun to develop (for ω < 0) is:

$$d_{stun}(v, \omega, \mu_s) := \frac{2\cdot R\cdot(-\omega)}{5\cdot\mu_s\cdot g}\left[v - \frac{(-\omega)\cdot R}{5}\right]$$

From TP B.5, the distance required for a skidding (sliding) CB to develop natural roll, and the speed when roll first develops are:

$$d_{skid}(v, \omega, \mu_s) := \mathrm{sign}\left(\frac{v}{R} - \omega\right)\cdot\frac{2}{49\cdot\mu_s\cdot g}\cdot\left[6\cdot v^2 - 5v\cdot R\cdot\omega - (R\cdot\omega)^2\right]$$

$$v_{skid}(v, \omega) := \frac{5}{7}\cdot v + \frac{2}{7}\cdot R\cdot\omega$$

From TP B.6, the distance required for a rolling ball to come to a stop is:

$$d_{roll}(v, \mu_r) := \mathrm{sign}(v)\cdot\frac{v^2}{2\cdot\mu_r\cdot g}$$

and the speed slows during rolling over distance x according to:

$$v_{roll}(v, x, \mu_r) := \sqrt{v^2 - 2\cdot\mu_r\cdot g\cdot x}$$

For a draw shot, where the CB still has backspin at OB impact,

$$d_{draw} = d_{skid}(-v'', \omega'', \mu_s) + d_{roll}(v''', \mu_r)$$

Assuming perfect balls (v''=0, ω''=-ω'),

$$d_{draw} = d_{skid}(0, -\omega', \mu_s) + d_{roll}(v''', \mu_r)$$

$$\omega'' = -\omega' = -\omega_{drag}(v, \omega, d_{drag}, \mu_s)$$

$$v = v(v_s, b)$$

$$\omega = \omega(v_s, b)$$

$$v''' = v_{skid}(0, -\omega', \mu_s)$$

Combining everything gives:

$$d_{draw} = d_{skid}(0, -\omega', \mu_s) + d_{roll}(v''', \mu_r)$$

$$d_{draw} = \frac{2\cdot(R\cdot\omega')^2}{49\cdot\mu_s\cdot g} + \frac{v'''^2}{2\cdot\mu_r\cdot g}$$

$$d_{draw} = \frac{2\cdot\left(R\cdot\omega_{drag}(v, \omega, d_{drag})\right)^2}{49\cdot\mu_s\cdot g} + \frac{v_{skid}(0, -\omega')^2}{2\cdot\mu_r\cdot g}$$

$$d_{draw} = \frac{2}{49\cdot\mu_s\cdot g}\cdot\left(R\cdot\omega_{drag}(v, \omega, d_{drag})\right)^2 + \frac{1}{2\cdot\mu_r\cdot g}\cdot\left(\frac{2}{7}R\cdot\omega_{drag}(v, \omega, d_{drag})\right)^2$$

$$d_{draw} = \frac{2R^2}{49g}\left(\frac{1}{\mu_s} + \frac{1}{\mu_r}\right)\cdot\omega_{drag}(v, \omega, d_{drag})^2$$

$$d_{draw} = \frac{2R^2}{49g}\left(\frac{1}{\mu_s} + \frac{1}{\mu_r}\right)\cdot\left[\omega + \frac{5}{2\cdot R}\cdot\left(v - \sqrt{v^2 - 2\cdot\mu_s\cdot g\cdot d_{drag}}\right)\right]^2$$

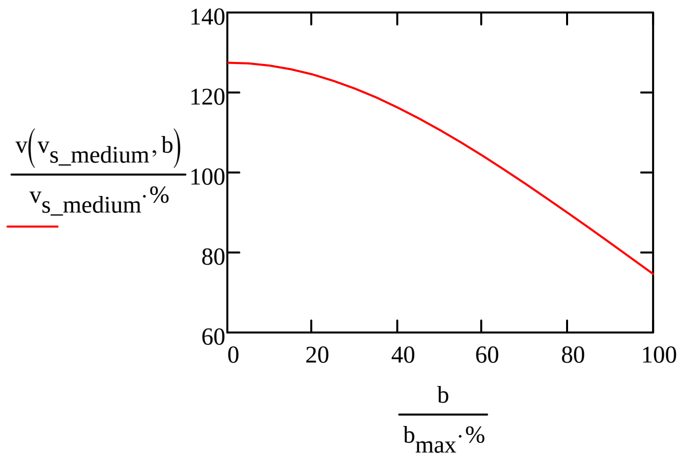
To account for energy loss in the tip-ball collision, we can use the following equations for ball speed and spin instead (from TP A.30), which account for tip inefficiency:

$\eta := 0.87$ typical cue tip efficiency

$$v(v_s, b) := v_s\cdot\frac{1 + \sqrt{\eta - \frac{1-\eta}{m_r}\cdot\left[1 + \frac{5}{2}\cdot\left(\frac{b}{R}\right)^2\right]}}{\left[1 + m_r + \frac{5}{2}\cdot\left(\frac{b}{R}\right)^2\right]}$$

$$\omega(v_s, b) := -\frac{5}{2}\cdot v(v_s, b)\cdot\frac{b}{R^2}$$

Here is how the cue ball speed varies with tip offset for a given cue speed, accounting for tip inefficiency:

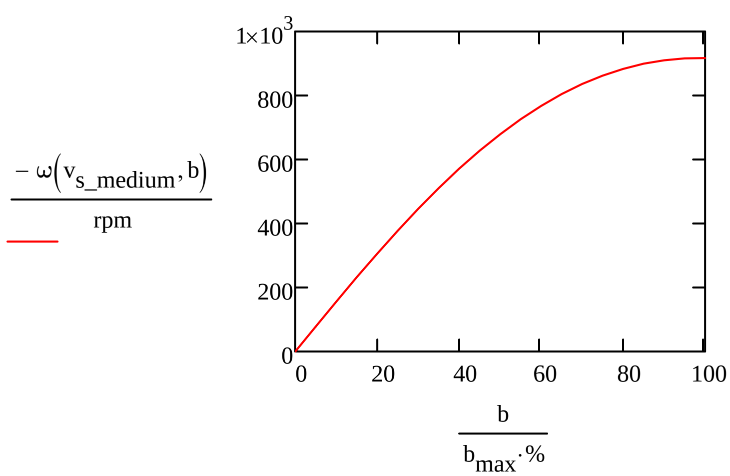


$$\frac{v(v_{s\_medium}, 0)}{v_{s\_medium}} = 127.452 \cdot \%$$

$$\frac{v(v_{s\_medium}, b_{max})}{v_{s\_medium}} = 74.628 \cdot \%$$

So with a center-ball hit, and a typical tip inefficiency, the CB speed is about 30% more than the cue speed; and at a maximum tip offset shot, the CB speed is about 75% (3/4) of the original cue speed.

Here is how the cue ball spin varies with tip offset for a given cue speed:



$$\frac{\omega(v_{s\_medium}, b_{max})}{\omega(v_{s\_medium}, 0.5b_{max})} = 134.983 \cdot \%$$

$$\frac{\omega(v_{s\_medium}, b_{max})}{\omega(v_{s\_medium}, 0.9b_{max})} = 100.728 \cdot \%$$

So, with a typical tip inefficiency, more tip offset gives you more spin, but the increase in spin isn't as great with increasing offset. The spin at maximum offset is only about 35% faster than the spin at half the maximum offset. Also, the spin at 90% offset is nearly equal to the spin at maximum (100%) offset.

To account for ball inelasticity, from TP A.5 (Equation 7),

$$v'' = \frac{(1 - e_b)}{2} \cdot v'$$

and to account for ball friction, from TP A.27

$$\mu(v) := 9.951 \cdot 10^{-3} + 0.108 \cdot e^{-1.088 \cdot \frac{v}{\frac{m}{s}}}$$

$$v_{rel}(v, \omega_x, \omega_z, \phi) := \sqrt{(v \cdot \sin(\phi) - R \cdot \omega_z)^2 + (R \cdot \omega_x \cdot \cos(\phi))^2}$$

$$\tan\theta(v, \omega_x, \omega_z, \phi) := \frac{\min\left(\frac{\mu(v_{rel}(v, \omega_x, \omega_z, \phi)) \cdot v \cdot \cos(\phi)}{v_{rel}(v, \omega_x, \omega_z, \phi)}, \frac{1}{7}\right) \cdot (v \cdot \sin(\phi) - R \cdot \omega_z)}{v \cdot \cos(\phi)}$$

$$\Delta\omega(v, \omega_x, \omega_z, \phi) := \frac{5}{2} \cdot \frac{v}{R} \cdot \tan\theta(v, \omega_x, \omega_z, \phi)$$

where $\Delta\omega$ is the change in spin experienced by both the CB and OB.

With this analysis, topspin in analogous to sidespin in TP A.27, and the throw is in the vertical direction. Therefore,

$\phi := 0$	$\omega_z = \omega'$	$\omega_x := 0$
no cut angle	(topspin)	no sidespin (English)

With topspin, $\omega_z > 0$ and $\Delta\omega < 0$. With bottomspin, $\omega_z < 0$ and $\Delta\omega > 0$.

For a follow shot, the CB spin, after impact, is:

$$\omega'' = \omega' + \Delta\omega$$

For a draw shot, the CB spin, after impact, is:

$$\omega'' = -(\omega' + \Delta\omega)$$

Now let's see how draw distance varies with cue speed, tip offset, shot distance, and table conditions. The following program calculates the draw distance for any case, and accounts for imperfect balls and an imperfect tip.

$$
\begin{array}{l}
d_{draw}(v_s, b, d_{drag}, \mu_s, \mu_r) := \\
\quad v \leftarrow v(v_s, b) \\
\quad \omega \leftarrow \omega(v_s, b) \\
\quad d_{stun} \leftarrow d_{stun}(v, \omega, \mu_s) \\
\quad \text{if } (d_{stun} < d_{drag}) \\
\qquad \text{"CB hits OB with topspin"} \\
\qquad d_{pre\_roll} \leftarrow d_{skid}(v, \omega, \mu_s) \\
\qquad \text{if } (d_{pre\_roll} < d_{drag}) \\
\qquad\quad \text{"CB rolls before OB contact"} \\
\qquad\quad v_{pre\_roll} \leftarrow v_{skid}(v, \omega) \\
\qquad\quad \text{d\_roll\_stop} \leftarrow d_{roll}(v_{pre\_roll}, \mu_r) \\
\qquad\quad \text{if } \left[\text{d\_roll\_stop} < (d_{drag} - d_{pre\_roll})\right] \\
\qquad\qquad \text{"CB doesn't reach OB"} \\
\qquad\qquad v' \leftarrow 0 \\
\qquad\quad \text{otherwise} \\
\qquad\qquad \text{"CB rolls into OB"} \\
\qquad\qquad v' \leftarrow v_{roll}\left[v_{pre\_roll}, (d_{drag} - d_{pre\_roll}), \mu_r\right] \\
\qquad\quad \omega' \leftarrow \dfrac{v'}{R} \\
\qquad \text{otherwise} \\
\qquad\quad \text{"CB slides into OB with partial roll"} \\
\qquad\quad \omega' \leftarrow \omega_{drag}(v, \omega, d_{drag}, \mu_s) \\
\qquad\quad v' \leftarrow v_{drag}(v, d_{drag}, \mu_s) \\
\qquad v'' \leftarrow \dfrac{(1 - e_b)}{2} \cdot v' \\
\qquad \omega'' \leftarrow -(\omega' + \Delta\omega(v', 0, \omega', 0)) \\
\qquad v''' \leftarrow v_{skid}(v'', -\omega'') \\
\qquad d_{draw} \leftarrow -\left(d_{skid}(v'', -\omega'', \mu_s) + d_{roll}(v''', \mu_r)\right)
\end{array}
$$

$$
\begin{array}{l}
\text{otherwise} \\
\quad \left| \begin{array}{l}
\text{"CB hits OB with backspin"} \\
v' \leftarrow v_{drag}\left(v, d_{drag}, \mu_s\right) \\
\omega' \leftarrow \omega_{drag}\left(v, \omega, d_{drag}, \mu_s\right) \\
\omega'' \leftarrow -(\omega' + \Delta\omega(v', 0, \omega', 0)) \\
v'' \leftarrow \dfrac{\left(1 - e_b\right)}{2} \cdot v' \\
v''' \leftarrow v_{skid}(-v'', \omega'') \\
d_{draw} \leftarrow d_{skid}\left(-v'', \omega'', \mu_s\right) + d_{roll}\left(v''', \mu_r\right)
\end{array} \right.
\end{array}
$$

Typical shot distances:

$d_{drag\_short} := 1 \cdot ft$ $\qquad$ $d_{drag\_medium} := 4 \cdot ft$ $\qquad$ $d_{drag\_long} := 8 \cdot ft$

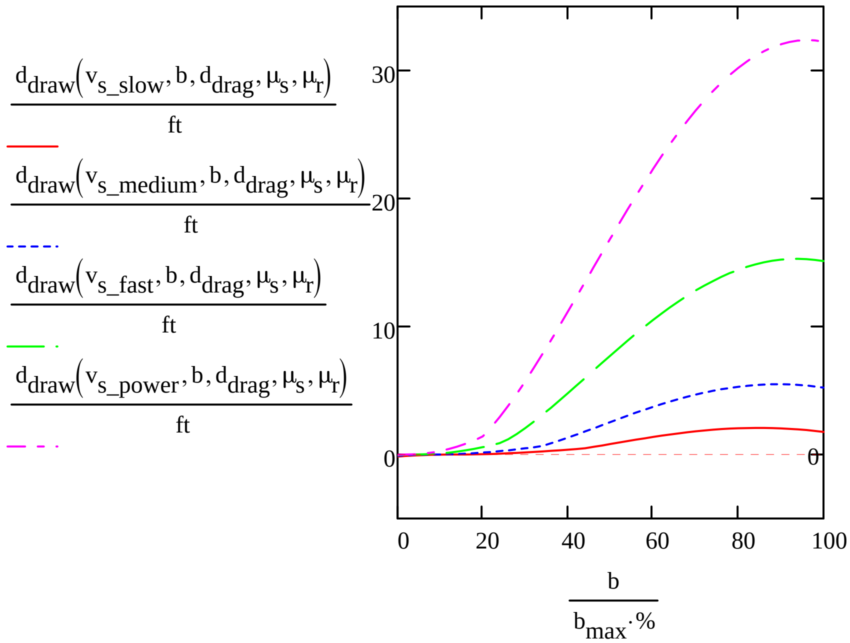
Valid range of tip offset (for no miscue): $b := 0 \cdot in, \dfrac{b_{max}}{50} .. b_{max}$

NOTE - In all of the plots below, a negative d_{draw} implies follow and a zero implies a stop shot or the CB not making it to the OB.

Draw distance vs. tip offset for various cue speeds:

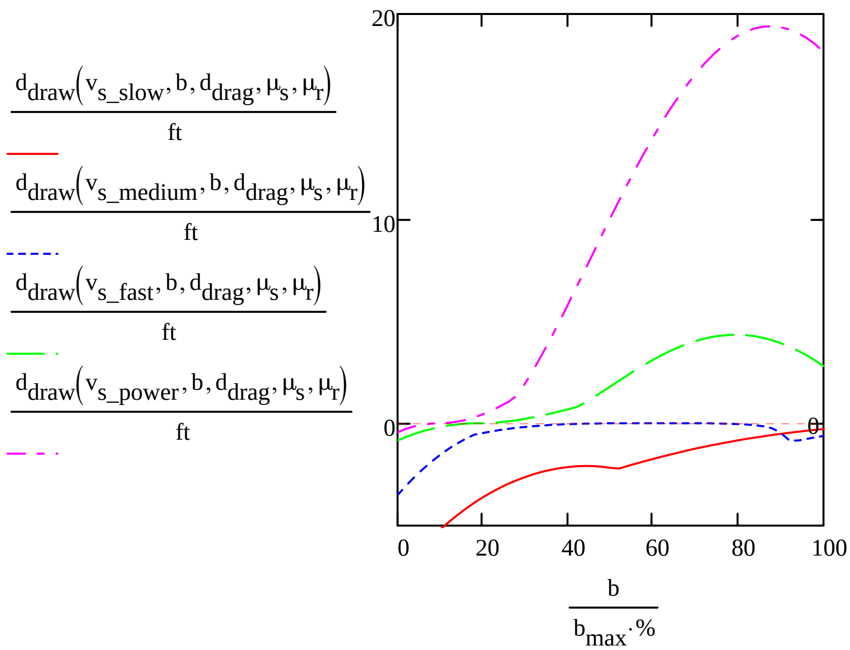
$d_{drag} := d_{drag\_short}$

A



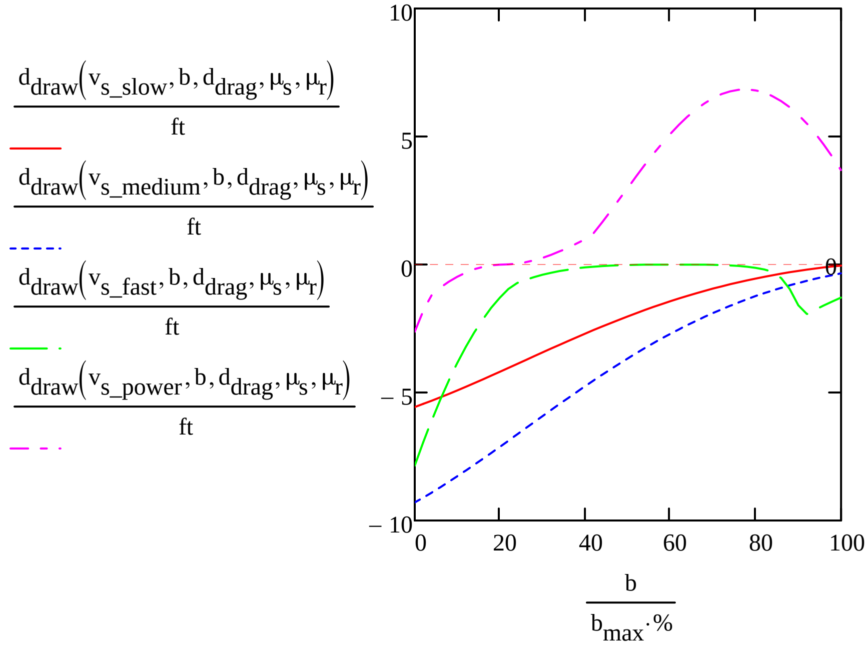
$d_{drag} := d_{drag\_medium}$

B



$d_{drag} := d_{drag\_long}$

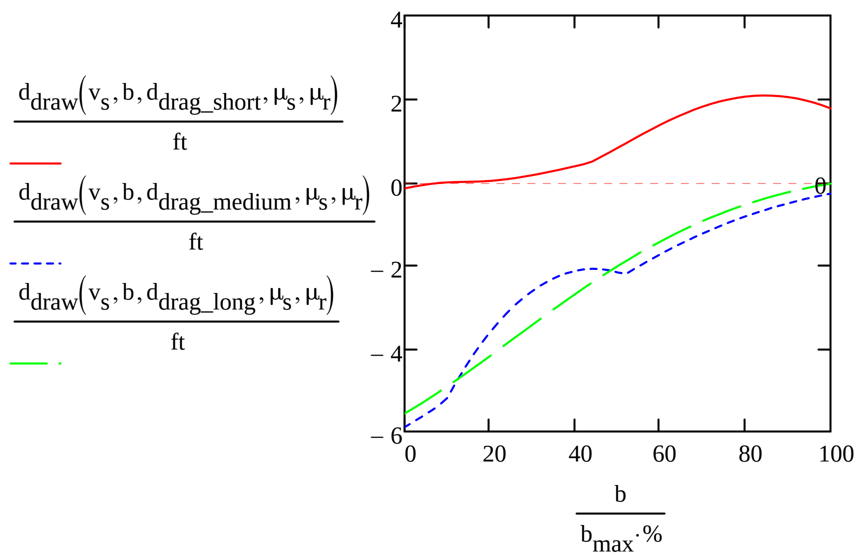
C



Draw distance vs. tip offset for various shot (drag) distances:

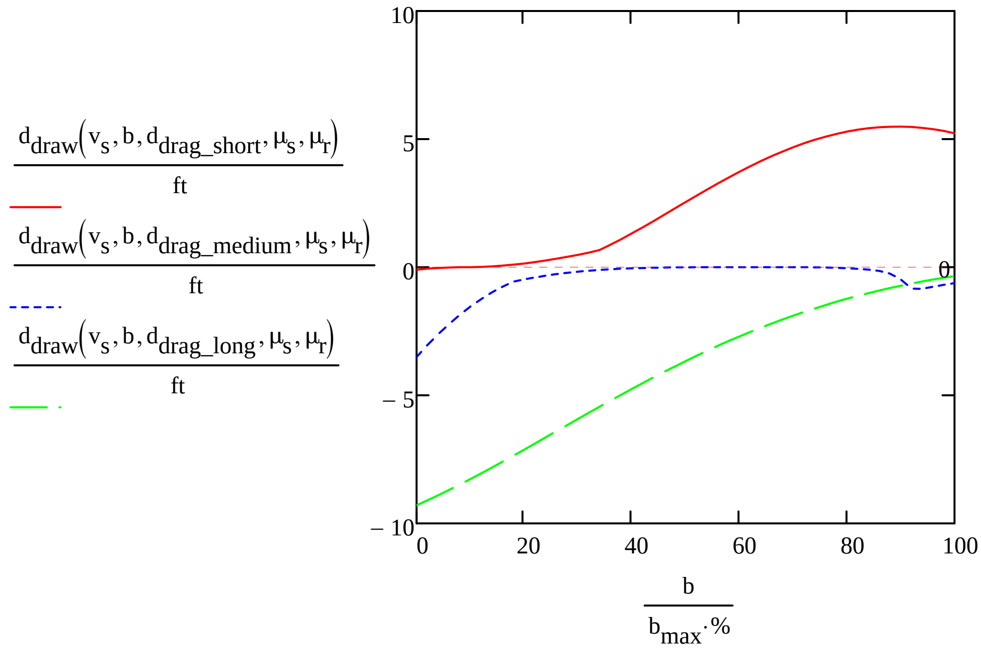
$v_s := v_{s\_slow}$

D



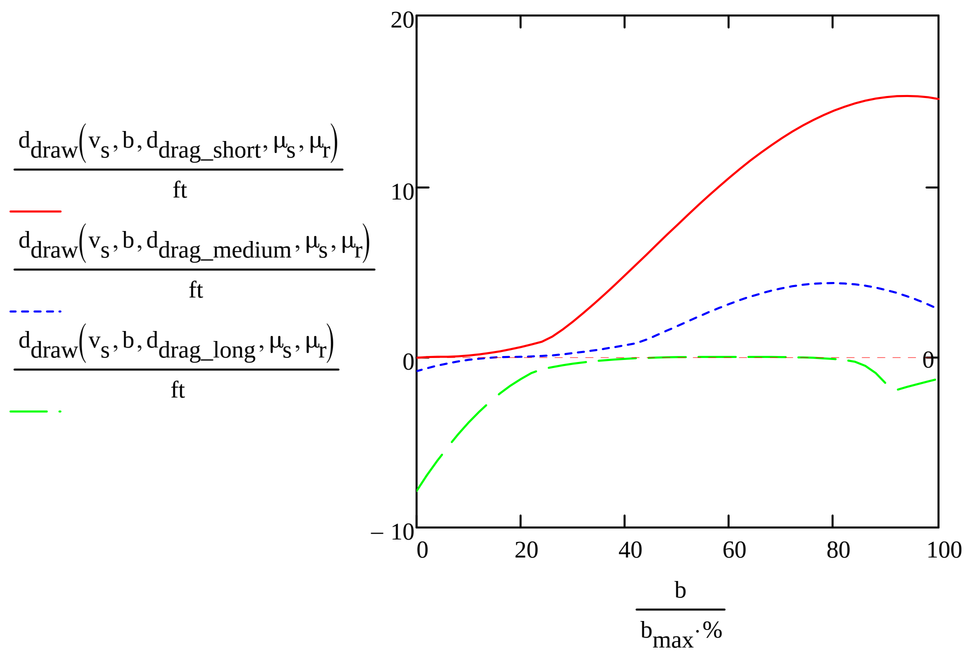
$v_s := v_{s\_medium}$

E

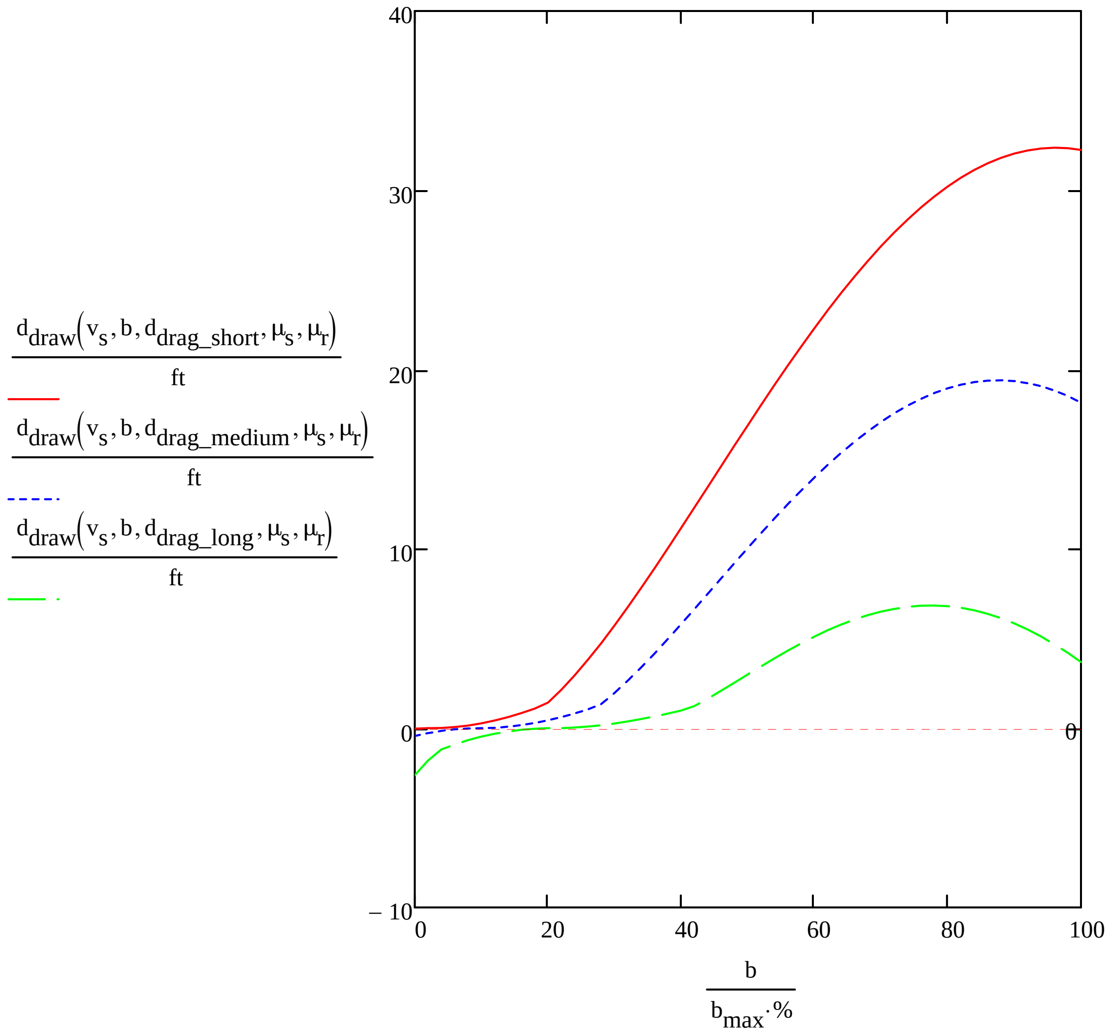


$v_s := v_{s\_fast}$

F



$v_s := v_{s\_power}$



$\frac{d_{draw}(v_s, b, d_{drag\_short}, \mu_s, \mu_r)}{ft}$
$\frac{d_{draw}(v_s, b, d_{drag\_medium}, \mu_s, \mu_r)}{ft}$
$\frac{d_{draw}(v_s, b, d_{drag\_long}, \mu_s, \mu_r)}{ft}$
40
30
20
10
0
−10
0
20
40
60
80
100
$\frac{b}{b_{max} \cdot \%}$

G

Sensitivity analysis for cue speed and tip offset for various draw distances:

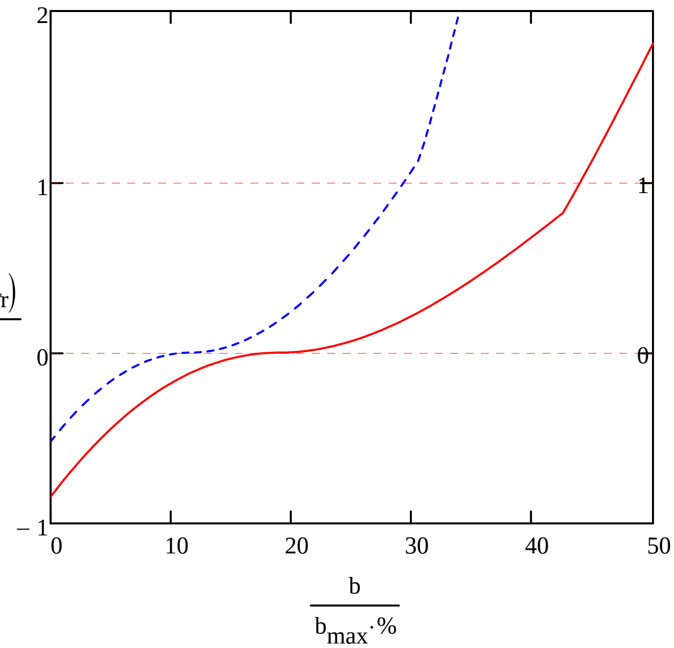
$d_{drag} := d_{drag\_medium}$

$b := 0, 0.5\% \cdot b_{max} .. 50 \cdot \% \, b_{max}$

H

$$\frac{d_{draw}(v_{s\_fast}, b, d_{drag}, \mu_s, \mu_r)}{ft}$$

$$\frac{d_{draw}(1.25 v_{s\_fast}, b, d_{drag}, \mu_s, \mu_r)}{ft}$$



$d_{drag} := d_{drag\_short}$

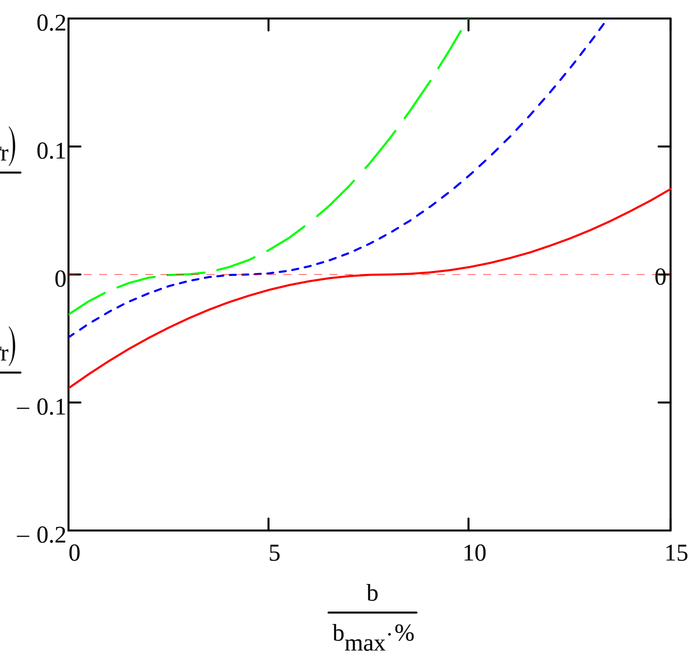
$b := 0, 0.5\% \cdot b_{max} .. 15 \cdot \% \, b_{max}$

I

$$\frac{d_{draw}(0.75 v_{s\_fast}, b, d_{drag}, \mu_s, \mu_r)}{ft}$$

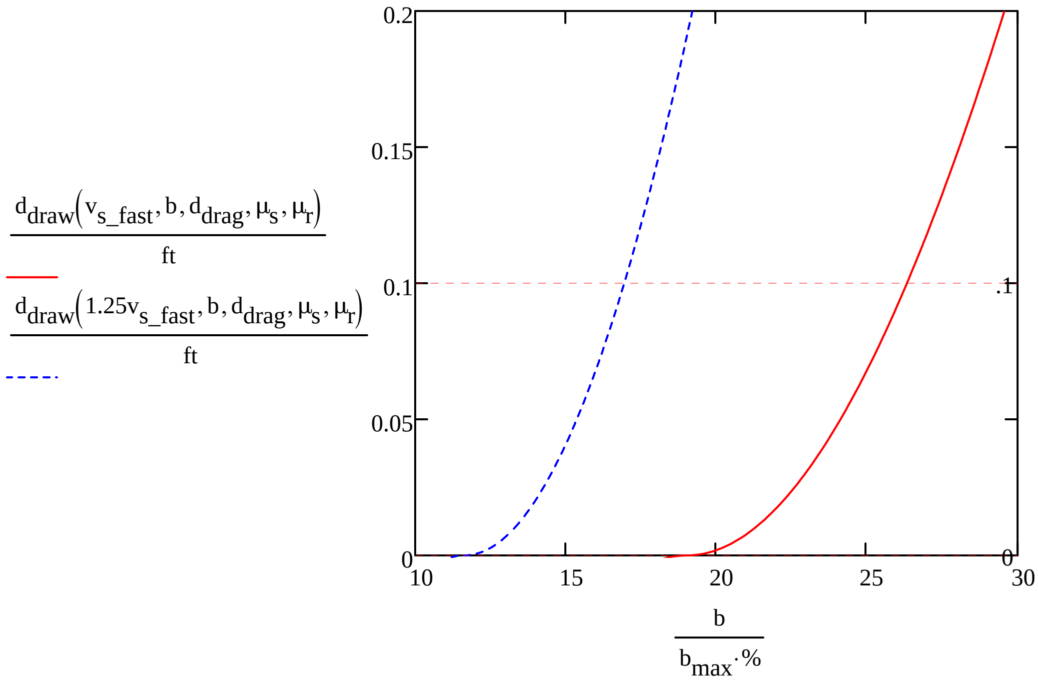
$$\frac{d_{draw}(v_{s\_fast}, b, d_{drag}, \mu_s, \mu_r)}{ft}$$

$$\frac{d_{draw}(1.25 v_{s\_fast}, b, d_{drag}, \mu_s, \mu_r)}{ft}$$



$d_{drag} := d_{drag\_medium}$ $\qquad b := 10\% \cdot b_{max}, 10.1\% \cdot b_{max} .. 30 \cdot \% \, b_{max}$

J

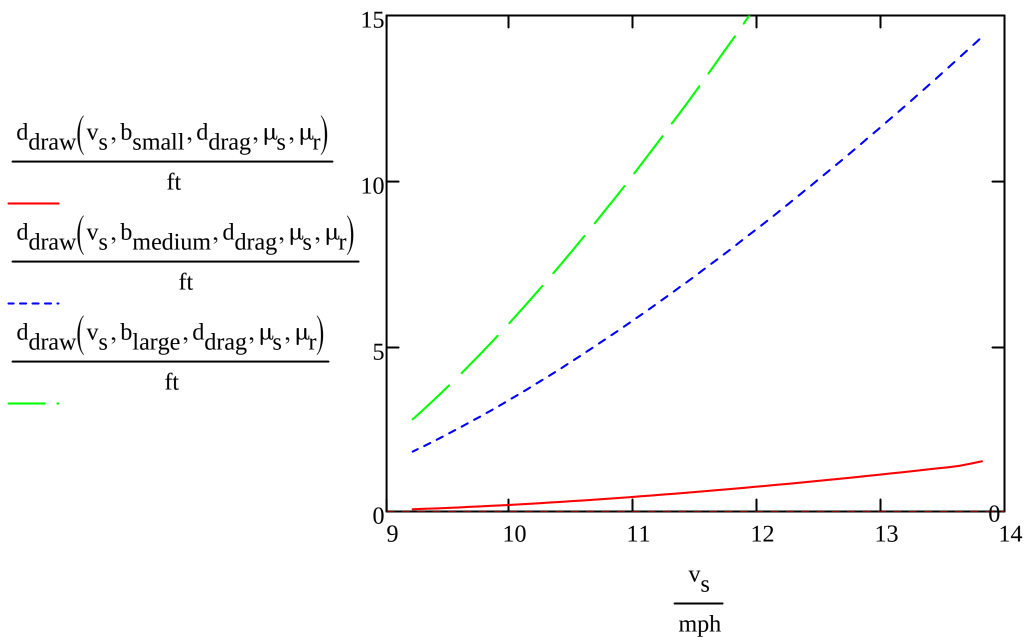


$d_{drag} := d_{drag\_medium}$

$v_{s\_low} := v_{s\_fast}$ $\qquad v_{s\_high} := 1.5 \cdot v_{s\_fast}$ $\qquad v_s := v_{s\_low}, v_{s\_low} + \frac{v_{s\_high} - v_{s\_low}}{50} .. v_{s\_high}$

$b_{small} := 25\% \cdot b_{max}$ $\qquad b_{medium} := 50\% \cdot b_{max}$ $\qquad b_{large} := 100 \cdot \% \cdot b_{max}$

K



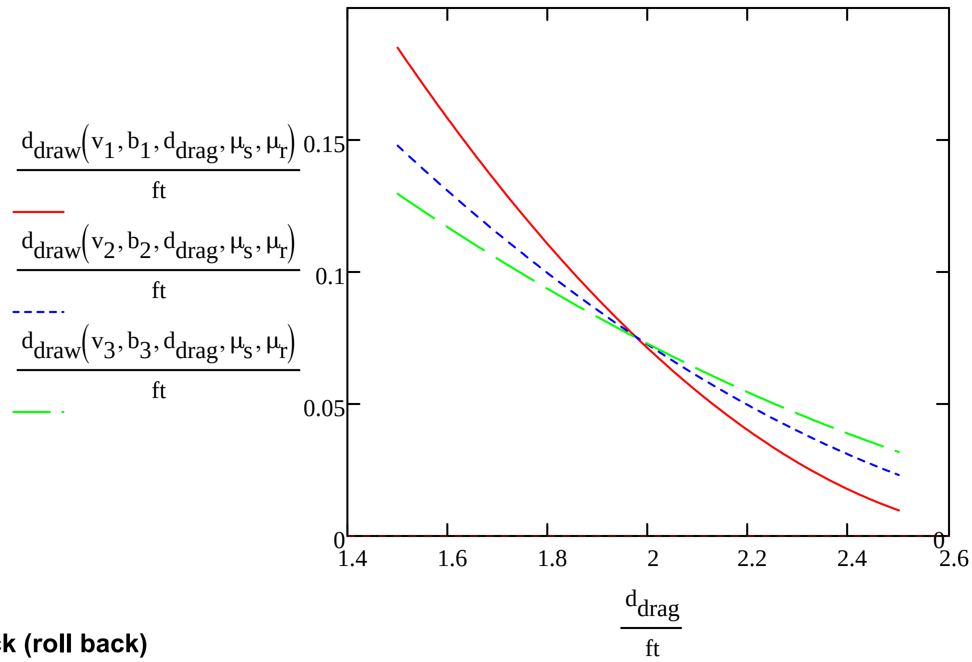
$v_1 := 0.75 \cdot v_{s\_fast}$ $\qquad v_2 := v_{s\_fast}$ $\qquad v_3 := 1.25 \cdot v_{s\_fast}$

$b_1 := 24.6\% \cdot b_{max}$ $\qquad b_2 := 14.5\% \cdot b_{max}$ $\qquad b_3 := 10\% \cdot b_{max}$

stun-through (roll through)
stun-follow shot:

$d_{drag} := 1.5 \cdot ft, 1.51 \cdot ft .. 2.5 \cdot ft$

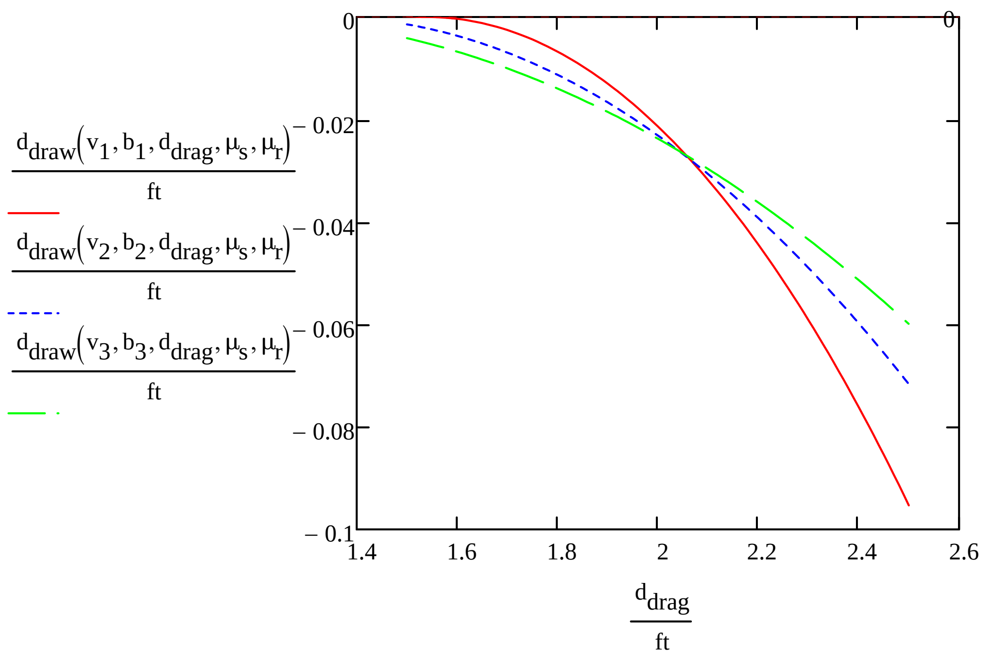
L



stun-back (roll back)
stun-draw shot:

$b_1 := 12.5\% \cdot b_{max}$ $\qquad b_2 := 5.9\% \cdot b_{max}$ $\qquad b_3 := 3.2\% \cdot b_{max}$

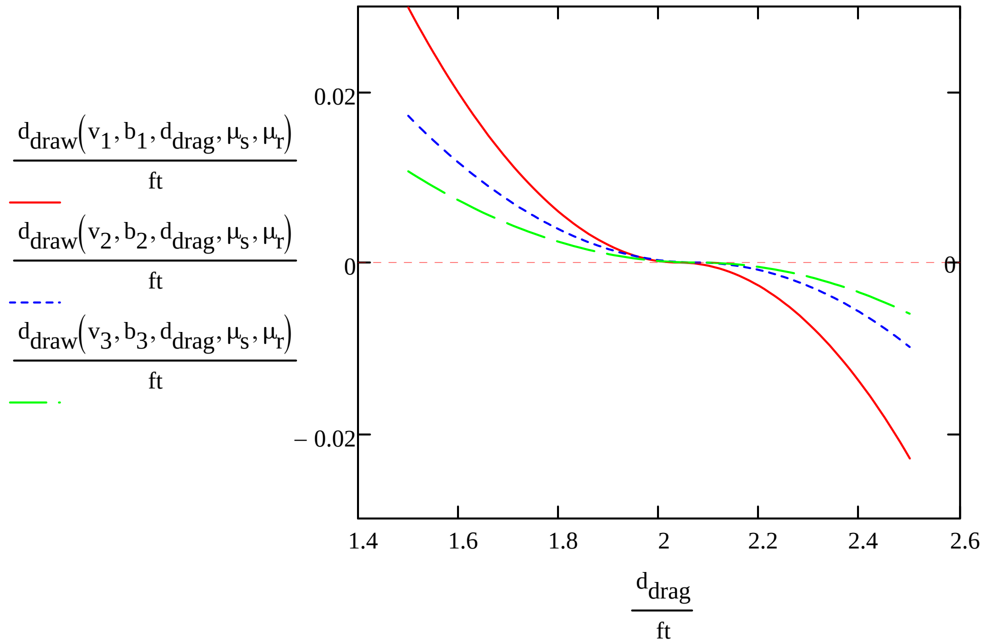
M



stop shot:

$b_1 := 17\% \cdot b_{max}$ $\quad$ $b_2 := 9.3\% \cdot b_{max}$ $\quad$ $b_3 := 5.87\% \cdot b_{max}$

N

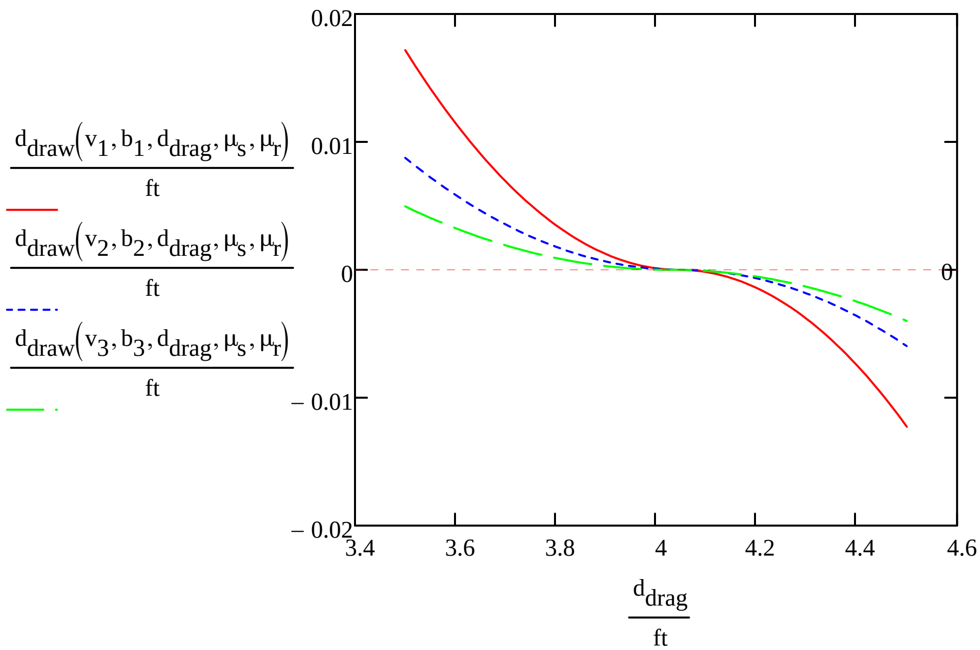


$d_{drag} := 3.5 \cdot ft, 3.51 \cdot ft .. 4.5 \cdot ft$

$v_1 := 0.75 \cdot v_{s\_power}$ $\quad$ $v_2 := v_{s\_power}$ $\quad$ $v_3 := 1.25 \cdot v_{s\_power}$

$b_1 := 18.5\% \cdot b_{max}$ $\quad$ $b_2 := 9.9\% \cdot b_{max}$ $\quad$ $b_3 := 6.2\% \cdot b_{max}$

O



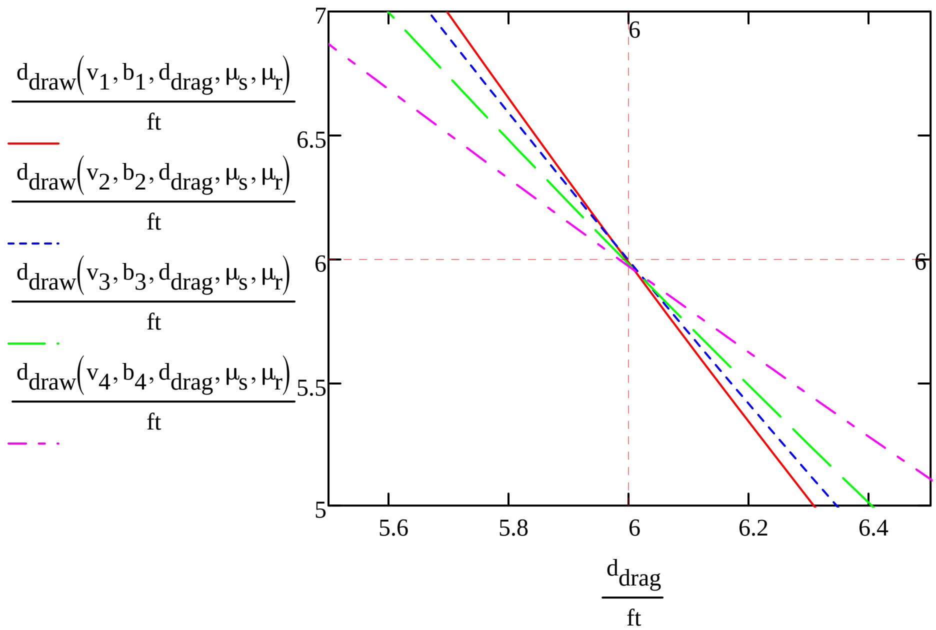
Let's look at how draw distance varies with shot distance, speed, and tip offset for a fairly powerful draw shot, where the OB is 6 feet away and you want to draw the CB 6 feet back to its original location:

$d_{drag} := 6 \cdot ft \qquad d_{drag} := 0.75 \cdot d_{drag}, 0.76 \cdot d_{drag} .. 1.25 \cdot d_{drag}$

$v_1 := 11.65 \cdot mph \qquad v_2 := 11.25 \cdot mph \qquad v_3 := 11.1 \cdot mph \qquad v_4 := 12.42 \cdot mph$

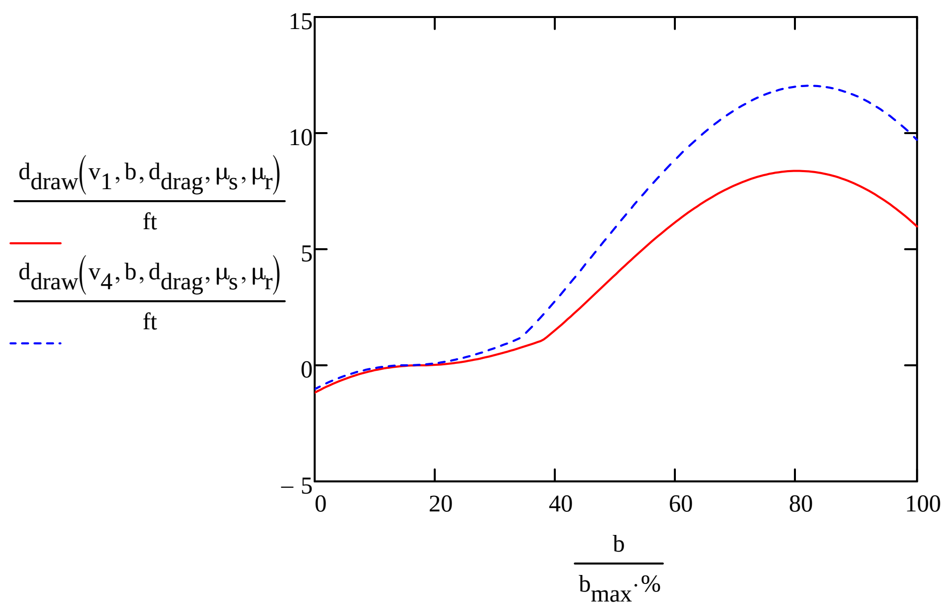
$b_1 := b_{max} \qquad b_2 := .9 \cdot b_{max} \qquad b_3 := .75 \cdot b_{max} \qquad b_4 := .5 \cdot b_{max}$

P



$d_{drag} := 6 \cdot ft \qquad b := 0, 0.5\% \cdot b_{max} .. 100 \cdot \% \, b_{max}$

Q



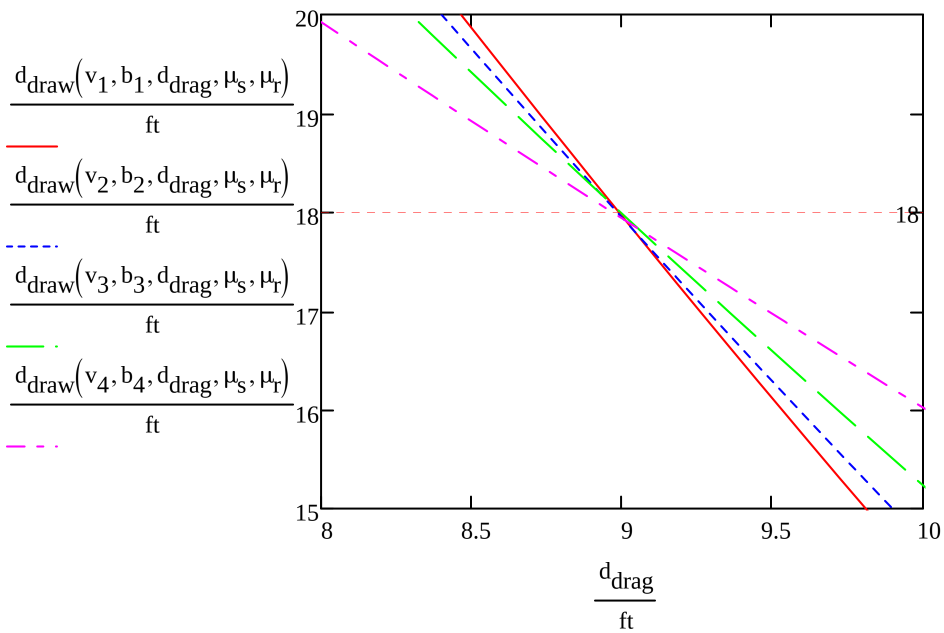
Let's look at how draw distance varies with shot distance, speed, and tip offset for an example draw shot, where the OB is 9 feet away and you want to draw the CB the equivalent of 18 feet back:

$d_{drag} := 9 \cdot ft \qquad d_{drag} := 0.75 \cdot d_{drag}, 0.76 \cdot d_{drag} .. 1.25 \cdot d_{drag}$

$v_1 := 15.7 \cdot mph \qquad v_2 := 15.26 \cdot mph \qquad v_3 := 15.24 \cdot mph \qquad v_4 := 17.45 \cdot mph$

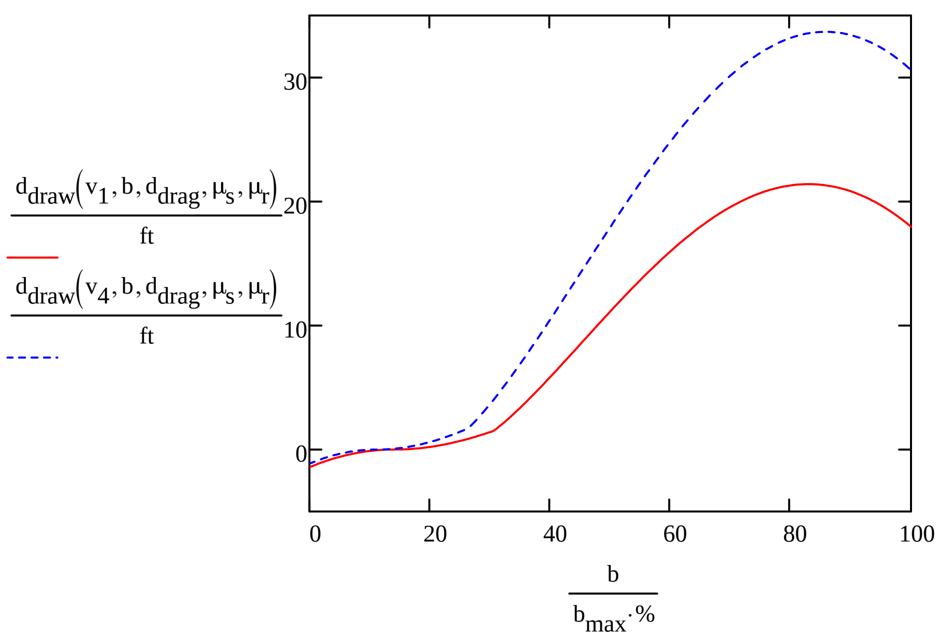
$b_1 := b_{max} \qquad b_2 := .9 \cdot b_{max} \qquad b_3 := .75 \cdot b_{max} \qquad b_4 := .5 \cdot b_{max}$

R



$d_{drag} := 9 \cdot ft \qquad b := 0, 0.5\% \cdot b_{max} .. 100 \cdot \% \, b_{max}$

S



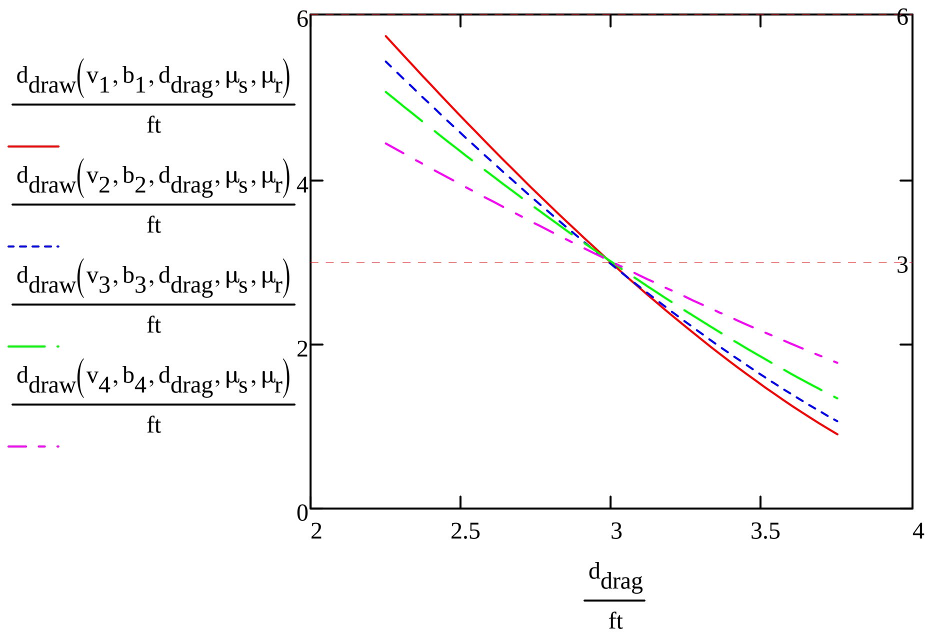
Let's look at how draw distance varies with shot distance, speed, and tip offset for a more modest draw shot, where the OB is 3 feet away and you want to draw the CB 3 feet back to its original location:

$$d_{drag} := 3 \cdot ft \qquad d_{drag} := 0.75 \cdot d_{drag}, 0.76 \cdot d_{drag} .. 1.25 \cdot d_{drag}$$

$$v_1 := 8.3 \cdot mph \qquad v_2 := 8.02 \cdot mph \qquad v_3 := 7.95 \cdot mph \qquad v_4 := 8.98 \cdot mph$$

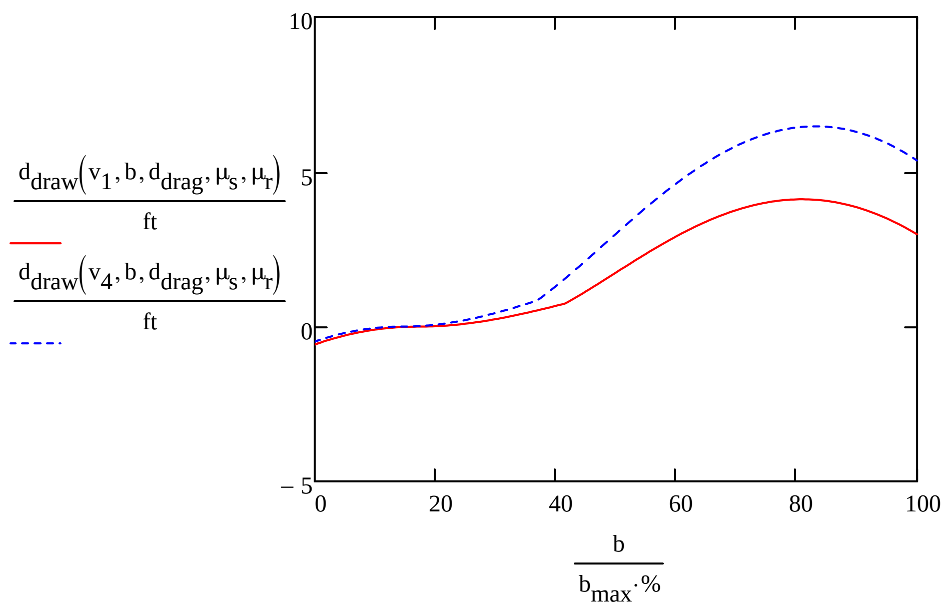
$$b_1 := b_{max} \qquad b_2 := .9 \cdot b_{max} \qquad b_3 := .75 \cdot b_{max} \qquad b_4 := .5 \cdot b_{max}$$

T



$$d_{drag} := 3 \cdot ft \qquad b := 0, 0.5\% \cdot b_{max} .. 100 \cdot \% \, b_{max}$$

U



Now let's look at the effects of cloth conditions (a sticky cloth vs. a slick cloth):

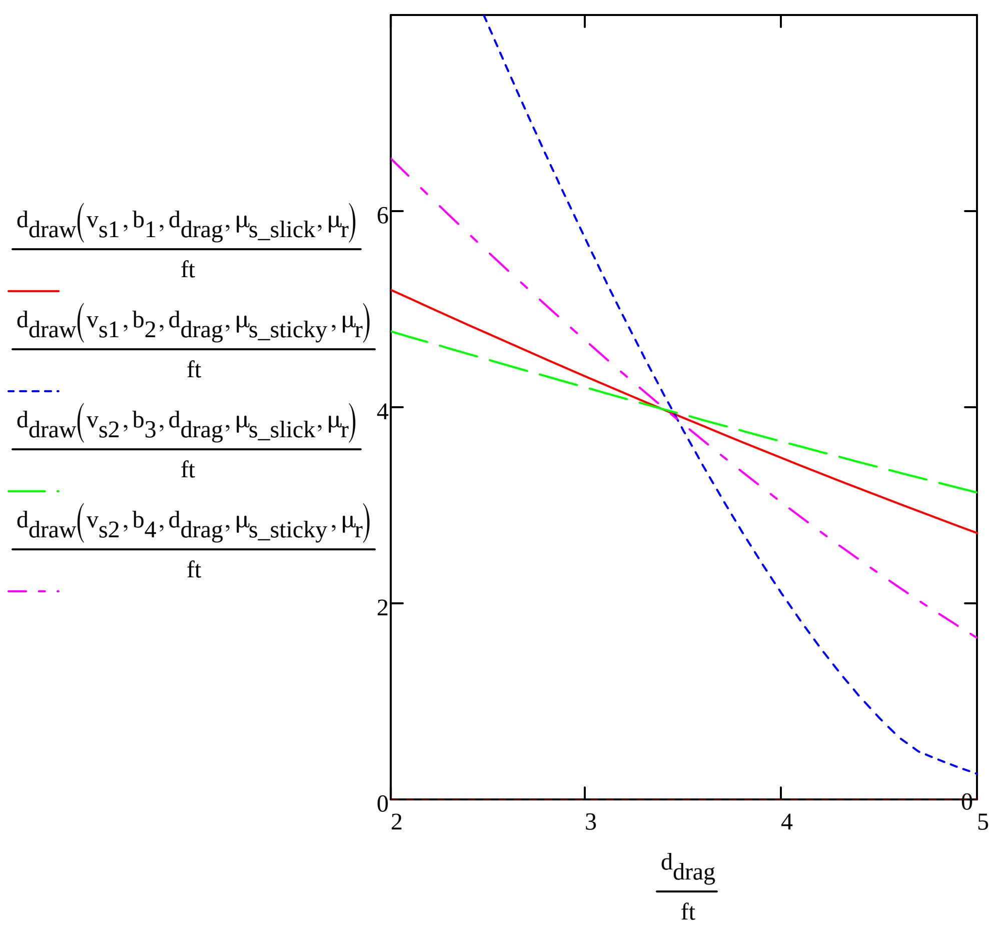
$\mu_{s\_slick} := 0.1 \qquad \mu_{s\_sticky} := 0.3$

$v_{s1} := 10 \cdot mph \qquad v_{s2} := 15 \cdot mph$

$b_1 := 36.6\% \cdot b_{max} \qquad b_2 := 80\% \cdot b_{max} \qquad b_3 := 22.9\% \cdot b_{max} \qquad b_4 := 30.1\% \cdot b_{max}$

$d_{drag} := 2 \cdot ft, 2.1 \cdot ft .. 5ft$

V



Optimal tip offset for maximum draw for different speeds and drag distances:

$v_s := v_{s\_power}$ $\quad$ $d_{drag} := d_{drag\_long}$ $\quad$ $b := .75 \cdot b_{max}$ (initial guess)

$$b\_opt(b) := d_{draw}(v_s, b, d_{drag}, \mu_s, \mu_r)$$

Given

$b > 0$

$b < b_{max}$

$b\_opt := \text{Maximize}(b\_opt, b) = 0.436 \cdot in$

$\frac{b\_opt}{b_{max}} = 77.48 \cdot \%$ $\quad$ $d_{draw}(v_s, b\_opt, d_{drag}, \mu_s, \mu_r) = 6.84 \cdot ft$

Here are some example values:

$v_{s\_slow} = 5.263 \cdot mph$ $\quad$ $d_{drag\_short} = 1 \cdot ft$ $\quad$ $b\_opt := 82.3\%$ $\quad$ $d\_draw := 2.6ft$

$v_{s\_medium} = 6.579 \cdot mph$ $\quad$ $d_{drag\_short} = 1 \cdot ft$ $\quad$ $b\_opt := 87.7\%$ $\quad$ $d\_draw := 6.2ft$

$v_{s\_fast} = 9.211 \cdot mph$ $\quad$ $d_{drag\_short} = 1 \cdot ft$ $\quad$ $b\_opt := 92.7\%$ $\quad$ $d\_draw := 16.2ft$

$v_{s\_power} = 12.5 \cdot mph$ $\quad$ $d_{drag\_short} = 1 \cdot ft$ $\quad$ $b\_opt := 95.2\%$ $\quad$ $d\_draw := 33.9ft$

$v_{s\_medium} = 6.579 \cdot mph$ $\quad$ $d_{drag\_medium} = 4 \cdot ft$ $\quad$ $b\_opt := 61.8\%$ $\quad$ $d\_draw := 0.0ft$

$v_{s\_fast} = 9.211 \cdot mph$ $\quad$ $d_{drag\_medium} = 4 \cdot ft$ $\quad$ $b\_opt := 77.9\%$ $\quad$ $d\_draw := 5.3ft$

$v_{s\_power} = 12.5 \cdot mph$ $\quad$ $d_{drag\_medium} = 4 \cdot ft$ $\quad$ $b\_opt := 87.7\%$ $\quad$ $d\_draw := 24.6ft$

$v_{s\_fast} = 9.211 \cdot mph$ $\quad$ $d_{drag\_long} = 8 \cdot ft$ $\quad$ $b\_opt := 61.1\%$ $\quad$ $d\_draw := 0.0ft$

$v_{s\_power} = 12.5 \cdot mph$ $\quad$ $d_{drag\_long} = 8 \cdot ft$ $\quad$ $b\_opt := 76.3\%$ $\quad$ $d\_draw := 8.1ft$

Useful conclusions from all of the plots above, which are consistent with generally accepted "best practices" and good-player intuition:

1. Generally, **to get more draw**, you must **hit the cue ball harder** and lower (see graphs A and G). No big surprise here!

2. More tip offset does not produce significantly more draw as you approach the miscue limit; so, generally, **it is advisable to not hit too close to the miscue limit** (see graphs A and G).

3. With larger drag distances, and for a given maximum cue speed, **max draw occurs at less than maximum tip offset** (at about 70%-80% tip offset). In other words, you don't get more power draw by hitting closer to the miscue limit (see Graphs G, Q, S, and U, and see the data on the previous page).

4. In general, with a draw shot with a medium desired draw distance, a **slower cue speed with more tip offset** will result in **better draw distance control** than a faster cue speed with less offset (see the slopes of the curves in Graphs A and G at a given draw distance).

5. Stop shots are much less sensitive to tip offset position than draw shots are. In other words, **CB position is much easier to control with a stop shot**, as compared to a draw shot (see Graphs G, H, I, N, and O).

6. For a **short stop shot**, **slower speed** offers slightly better control (see the overall slopes of the curves in Graph I). For **longer stop shots**, **faster speed** appears to offer slightly better control (see Graphs N and O).

7. For **stun-through** (small controlled follow) and **stun-back** (small controlled draw), a firmer hit **closer to center** offers better CB control (see Graphs L and M).

8. It is much **easier to control draw** distance **on a new, slick cloth** than it is on a "sticky" cloth, especially with lower-speed shots (see Graph T). The statement assumes the player is equally well "adjusted" to each cloth condition. Any player will need to adjust when playing under different cloth conditions.

9. It is **easier to draw the ball on slick cloth**, and **faster cloth allows for greater draw distances**.