technical proof

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TP B.9
Draw shot spin vs. spin ratio

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
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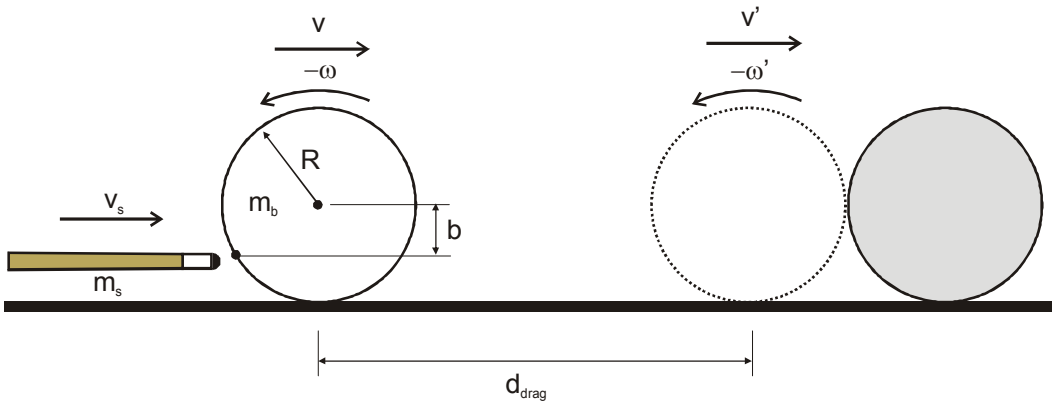


With a straight draw shot (with no cut angle), the amount of draw is determined solely by the amount of CB spin at contact with the OB (-ω'). This is the topic of TP B.8. Here we want to also look at the spin-to-speed ratio (-ω'R/v'). For some cases, it is more important to maximum spin-to-speed ratio instead of spin. Examples include shots where you don't want to hit the OB too hard (e.g., to leave the OB by a pocket if you can't or don't want to pocket it, or to increase the effective size of the pocket) while still maximizing draw (e.g., to achieve position for the next shot or to play a safety), and trying to keep the draw angle as narrow as possible when there is a cut angle (e.g., to avoid a ball or a pocket, or to get straight up or down the table better).

All of the data and equations and parameters below are from TP B.8.

Typical shot distances:

$d_{drag\_short} := 1 \cdot ft$ $d_{drag\_medium} := 4 \cdot ft$ $d_{drag\_long} := 8 \cdot ft$

Miscue limit and tip offset range:

$R := 1.125 \cdot in$ $b_{max} := \frac{R}{2} = 0.563 \cdot in$ $b := 0 \cdot in, \frac{b_{max}}{50} .. b_{max}$

Relevant physical parameters:

$\mu_s := 0.2$ typical ball-cloth coefficient of sliding friction

$\mu_r := 0.01$ typical ball-cloth coefficient of rolling resistance

$m_r := \frac{6}{19}$ typical ball-mass-to-cue-mass ratio (m_b/m_s):

$\eta := 0.87$ typical cue tip efficiency

$rps := \frac{rpm \cdot min}{sec}$ revs per second

$v_{slow} := 8 \cdot mph \qquad v_{medium} := 10 \cdot mph \qquad v_{fast} := 14 \cdot mph \qquad v_{power} := 19 \cdot mph$

$$v_s(v) := \frac{(1 + m_r)}{2} \cdot v$$

$v_{s\_slow} := v_s(v_{slow}) = 5.263 \cdot mph \qquad v_{s\_medium} := v_s(v_{medium}) = 6.579 \cdot mph$

$v_{s\_fast} := v_s(v_{fast}) = 9.211 \cdot mph \qquad v_{s\_power} := v_s(v_{power}) = 12.5 \cdot mph$

$v_{drag}(v, x, \mu_s) := \sqrt{v^2 - 2 \cdot \mu_s \cdot g \cdot x} \qquad \omega_{drag}(v, \omega, x, \mu_s) := \omega + \frac{5}{2 \cdot R} \cdot \left(v - \sqrt{v^2 - 2 \cdot \mu_s \cdot g \cdot x}\right)$

$$d_{stun}(v, \omega, \mu_s) := \frac{2 \cdot R \cdot (-\omega)}{5 \cdot \mu_s \cdot g} \left[v - \frac{(-\omega) \cdot R}{5}\right]$$

$d_{skid}(v, \omega, \mu_s) := sign\left(\frac{v}{R} - \omega\right) \cdot \frac{2}{49 \cdot \mu_s \cdot g} \cdot \left[6 \cdot v^2 - 5v \cdot R \cdot \omega - (R \cdot \omega)^2\right] \qquad v_{skid}(v, \omega) := \frac{5}{7} \cdot v + \frac{2}{7} \cdot R \cdot \omega$

$d_{roll}(v, \mu_r) := sign(v) \cdot \frac{v^2}{2 \cdot \mu_r \cdot g} \qquad v_{roll}(v, x, \mu_r) := \sqrt{v^2 - 2 \cdot \mu_r \cdot g \cdot x}$

$$v(v_s, b) := v_s \cdot \frac{1 + \sqrt{\eta - \frac{1 - \eta}{m_r} \cdot \left[1 + \frac{5}{2} \cdot \left(\frac{b}{R}\right)^2\right]}}{\left[1 + m_r + \frac{5}{2} \cdot \left(\frac{b}{R}\right)^2\right]} \qquad \omega(v_s, b) := -\frac{5}{2} \cdot v(v_s, b) \cdot \frac{b}{R^2}$$

Now let's see how CB spin and spin ratio, at OB contact, varies with cue speed, tip offset, shot distance, and table conditions.

SNR: **S**pin a**N**d spin **R**atio

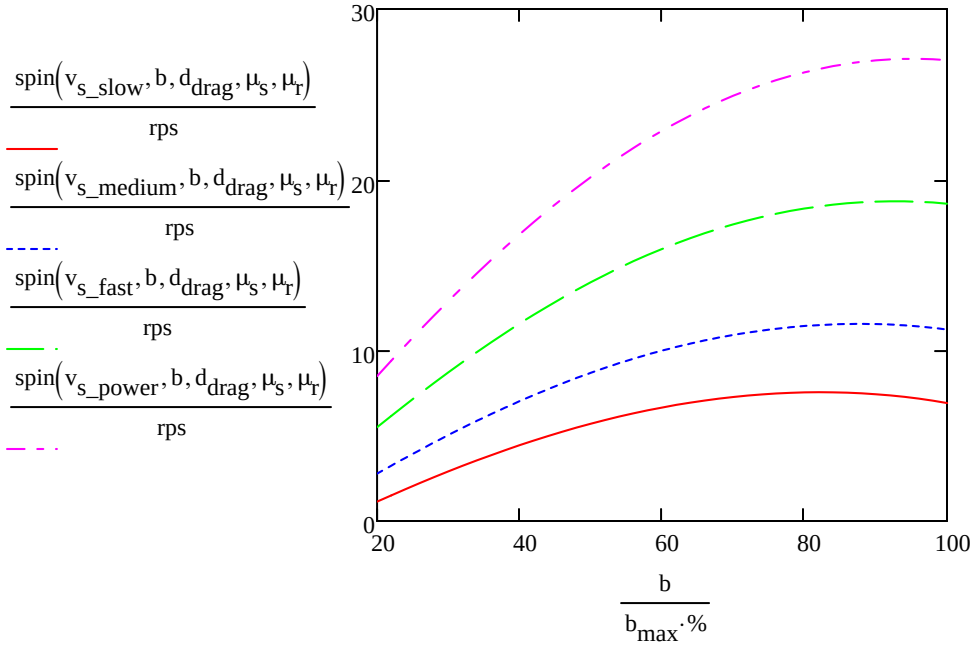
$$
\begin{aligned}
&\mathrm{SNR}(v_s, b, d_{drag}, \mu_s, \mu_r) := \\
&\quad v \leftarrow v(v_s, b) \\
&\quad \omega \leftarrow \omega(v_s, b) \\
&\quad d_{stun} \leftarrow d_{stun}(v, \omega, \mu_s) \\
&\quad \text{if } (d_{stun} < d_{drag}) \\
&\qquad \text{"CB hits OB with topspin"} \\
&\qquad d_{pre\_roll} \leftarrow d_{skid}(v, \omega, \mu_s) \\
&\qquad \text{if } (d_{pre\_roll} < d_{drag}) \\
&\qquad\quad \text{"CB rolls before OB contact"} \\
&\qquad\quad v_{pre\_roll} \leftarrow v_{skid}(v, \omega) \\
&\qquad\quad d\_roll\_stop \leftarrow d_{roll}(v_{pre\_roll}, \mu_r) \\
&\qquad\quad \text{if } [d\_roll\_stop < (d_{drag} - d_{pre\_roll})] \\
&\qquad\qquad \text{"CB doesn't reach OB"} \\
&\qquad\qquad v' \leftarrow 0 \\
&\qquad\quad \text{otherwise} \\
&\qquad\qquad \text{"CB rolls into OB"} \\
&\qquad\qquad v' \leftarrow v_{roll}[v_{pre\_roll}, (d_{drag} - d_{pre\_roll}), \mu_r] \\
&\qquad\quad \omega' \leftarrow \frac{v'}{R} \\
&\qquad \text{otherwise} \\
&\qquad\quad \text{"CB slides into OB with partial roll"} \\
&\qquad\quad \omega' \leftarrow \omega_{drag}(v, \omega, d_{drag}, \mu_s) \\
&\qquad\quad v' \leftarrow v_{drag}(v, d_{drag}, \mu_s) \\
&\quad \text{otherwise} \\
&\qquad \text{"CB hits OB with backspin"} \\
&\qquad v' \leftarrow v_{drag}(v, d_{drag}, \mu_s) \\
&\qquad \omega' \leftarrow \omega_{drag}(v, \omega, d_{drag}, \mu_s) \\
&\quad \begin{pmatrix} -\omega' & \frac{v'}{R} \end{pmatrix}^T
\end{aligned}
$$

$$
\mathrm{spin}(v_s, b, d_{drag}, \mu_s, \mu_r) := \mathrm{SNR}(v_s, b, d_{drag}, \mu_s, \mu_r)_0 \qquad \mathrm{ratio}(v_s, b, d_{drag}, \mu_s, \mu_r) := \frac{\mathrm{SNR}(v_s, b, d_{drag}, \mu_s, \mu_r)_0}{\mathrm{SNR}(v_s, b, d_{drag}, \mu_s, \mu_r)_1}
$$

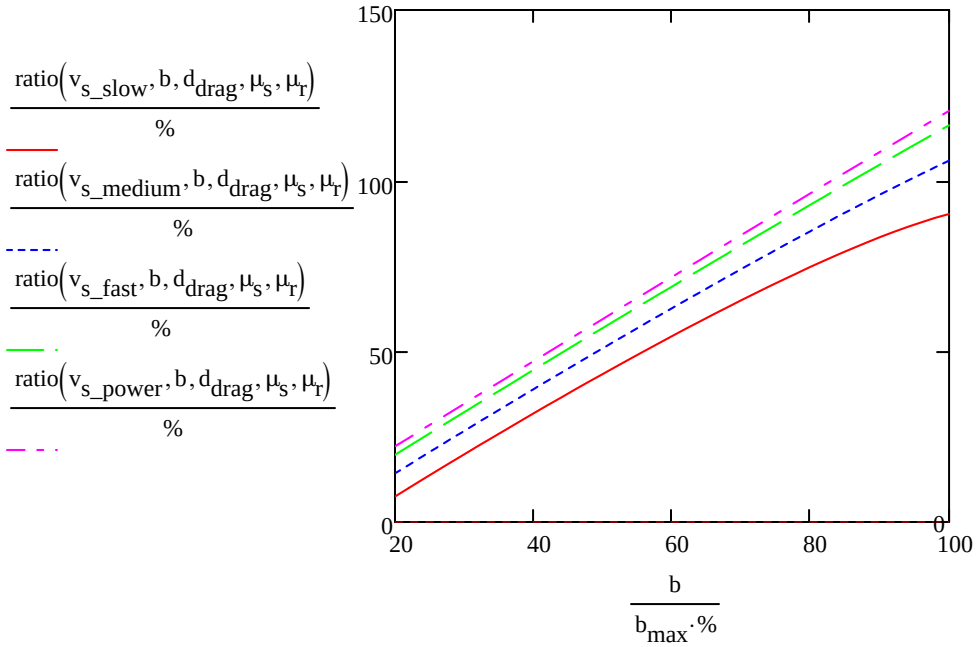
Spin and spin ratio vs. tip offset for various cue speeds:

$d_{drag} := d_{drag\_short}$ $\quad b_{min} := .2 \cdot b_{max}$ $\quad b := b_{min}, b_{min} + \frac{b_{max}}{50} .. b_{max}$

A

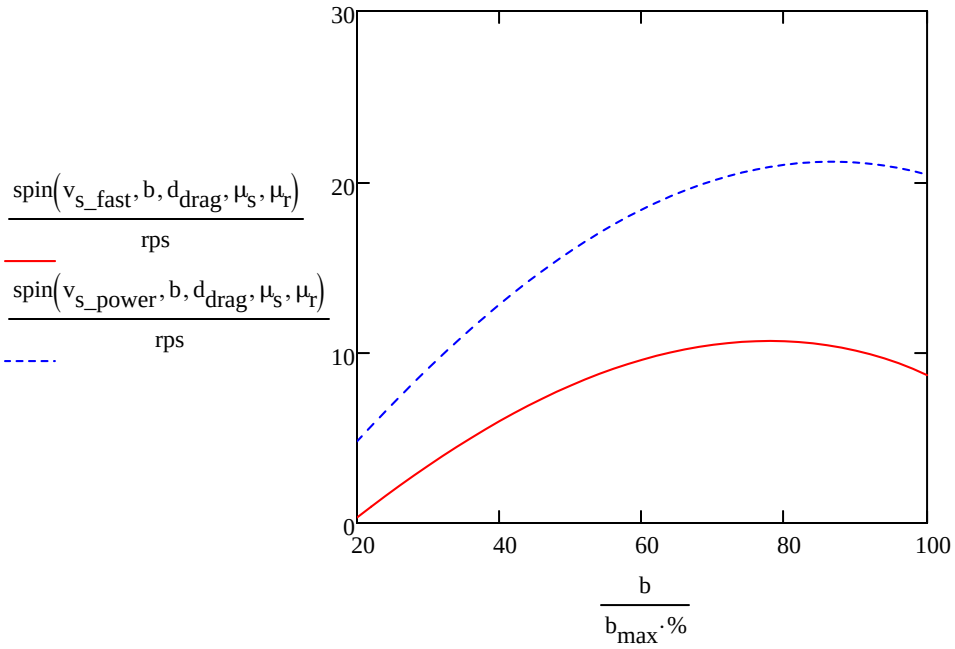


B

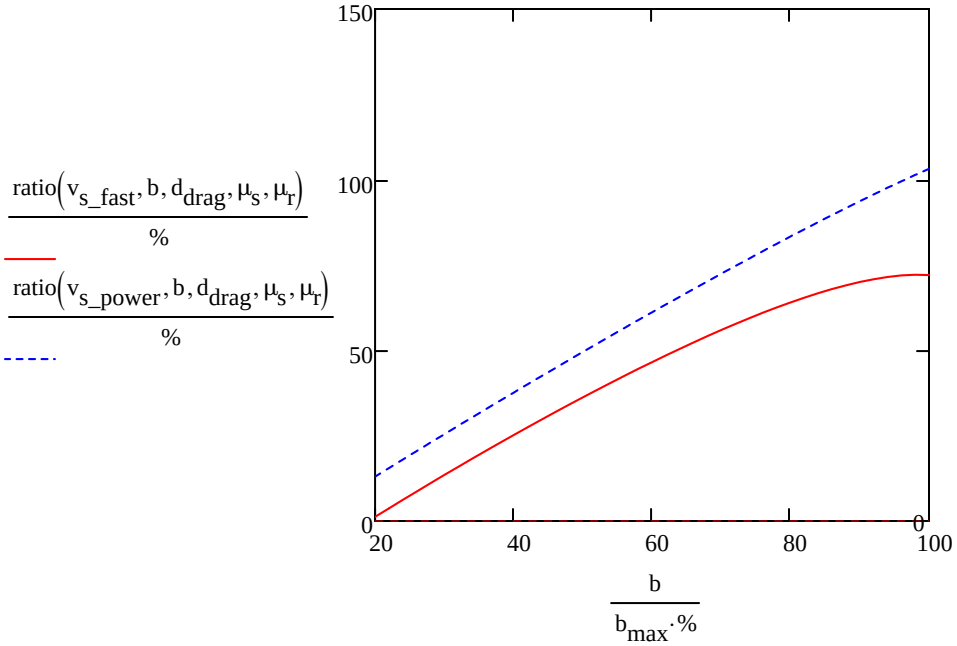


$d_{drag} := d_{drag\_medium}$ $b_{min} := .2 \cdot b_{max}$ $b := b_{min}, b_{min} + \frac{b_{max}}{50} .. b_{max}$

C



D

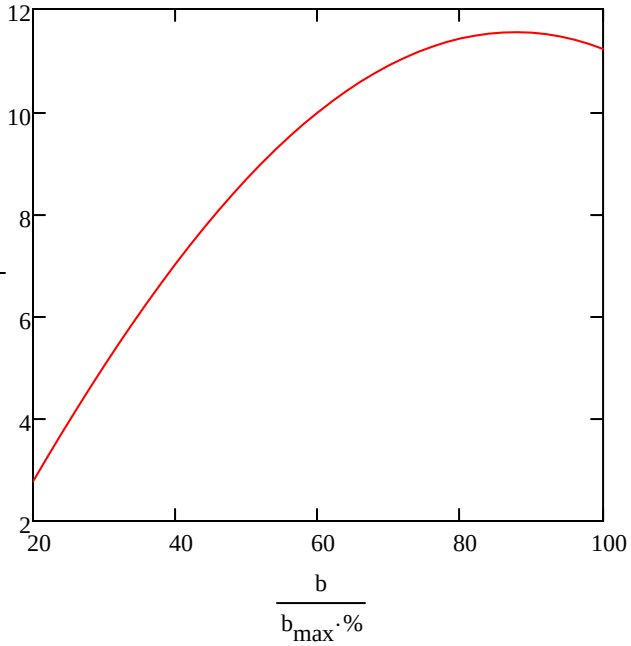


Spin and spin ratio vs. tip offset for various shot (drag) distances:

$v_s := v_{s\_medium}$

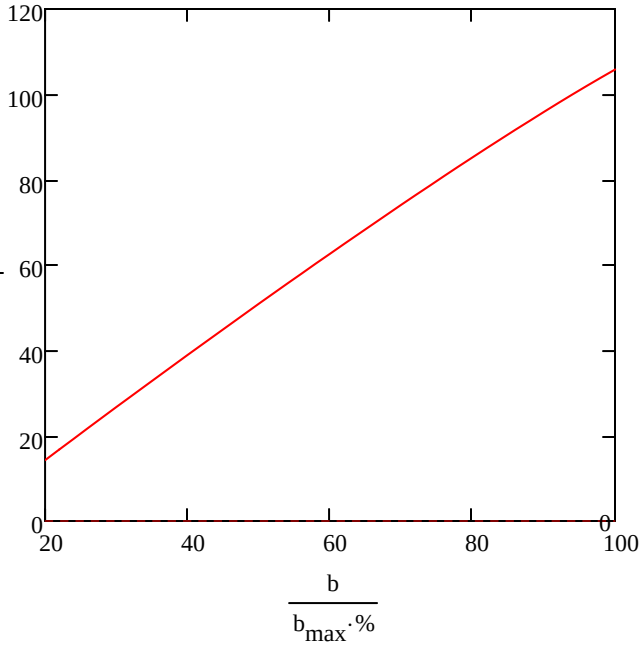
E

$\frac{spin(v_s, b, d_{drag\_short}, \mu_s, \mu_r)}{rps}$



F

$\frac{ratio(v_s, b, d_{drag\_short}, \mu_s, \mu_r)}{\%}$

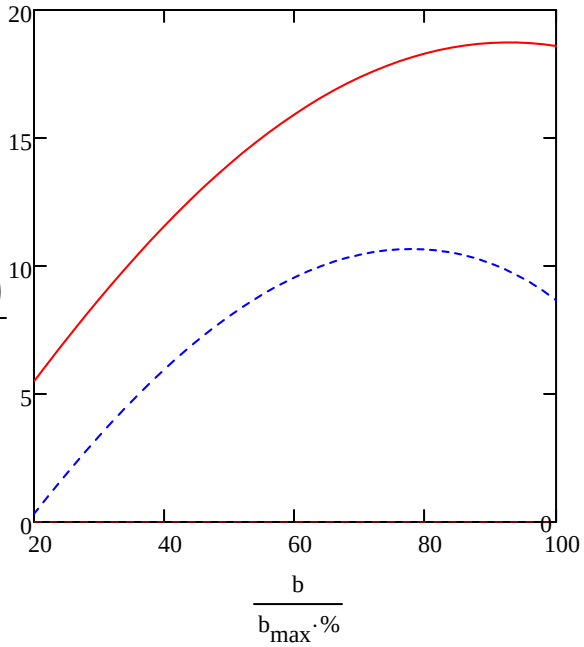


$v_s := v_{s\_fast}$

G

$$\frac{spin(v_s, b, d_{drag\_short}, \mu_s, \mu_r)}{rps}$$

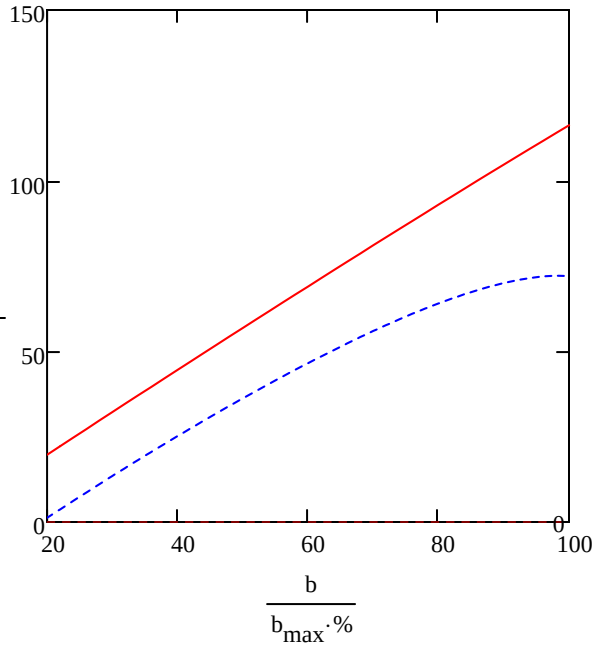
$$\frac{spin(v_s, b, d_{drag\_medium}, \mu_s, \mu_r)}{rps}$$



H

$$\frac{ratio(v_s, b, d_{drag\_short}, \mu_s, \mu_r)}{\%}$$

$$\frac{ratio(v_s, b, d_{drag\_medium}, \mu_s, \mu_r)}{\%}$$



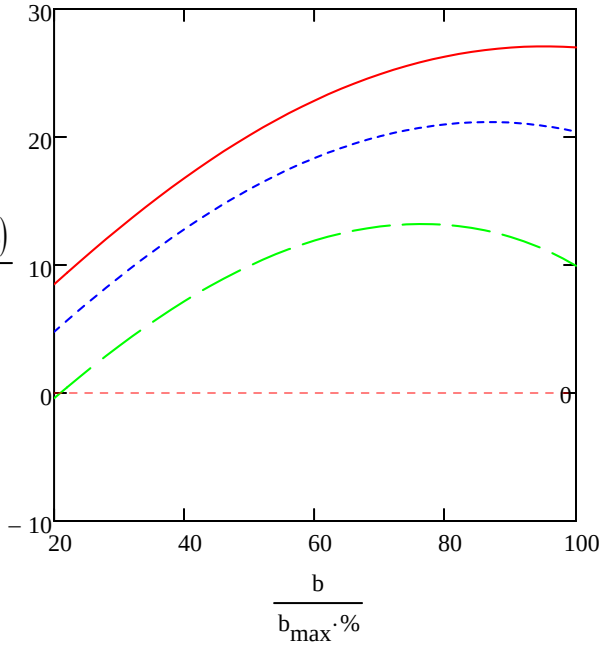
$v_s := v_{s\_power}$

I

$$\frac{spin(v_s, b, d_{drag\_short}, \mu_s, \mu_r)}{rps}$$

$$\frac{spin(v_s, b, d_{drag\_medium}, \mu_s, \mu_r)}{rps}$$

$$\frac{spin(v_s, b, d_{drag\_long}, \mu_s, \mu_r)}{rps}$$

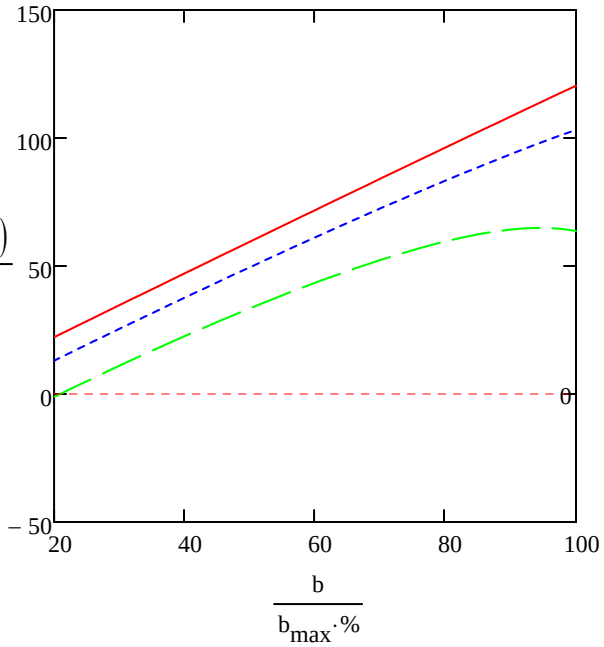


J

$$\frac{ratio(v_s, b, d_{drag\_short}, \mu_s, \mu_r)}{\%}$$

$$\frac{ratio(v_s, b, d_{drag\_medium}, \mu_s, \mu_r)}{\%}$$

$$\frac{ratio(v_s, b, d_{drag\_long}, \mu_s, \mu_r)}{\%}$$



Useful conclusions from the graphs above, which are consistent with generally accepted "best practices" and good-player intuition:

1. Generally, **to get more spin** at contact with the OB, you must **hit the CB harder and lower**. However, **as you approach the miscue limit**, you get a **smaller gain in spin**. And **for longer drag distance shots**, **the amount of spin actually decreases as you approach the miscue limit** (see Graphs A and I). See TP B.8 for more info and results.

2. **More tip offset results in a greater spin-to-speed ratio** at OB contact (except for long drag shots, especially if conditions are "sticky") (see Graphs B and J). So to get a better spin-to-speed ratio, hit as low as you can on the CB without being at too high a risk of miscuing.