technical proof

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# TP 3.2
# Ball speeds and distances after stun-shot impact

from:
"The Illustrated Principles of Pool and Billiards"
**http://billiards.colostate.edu**
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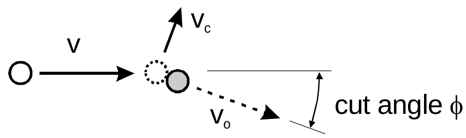
**For a rolling cue ball shot, see TP A.16.**



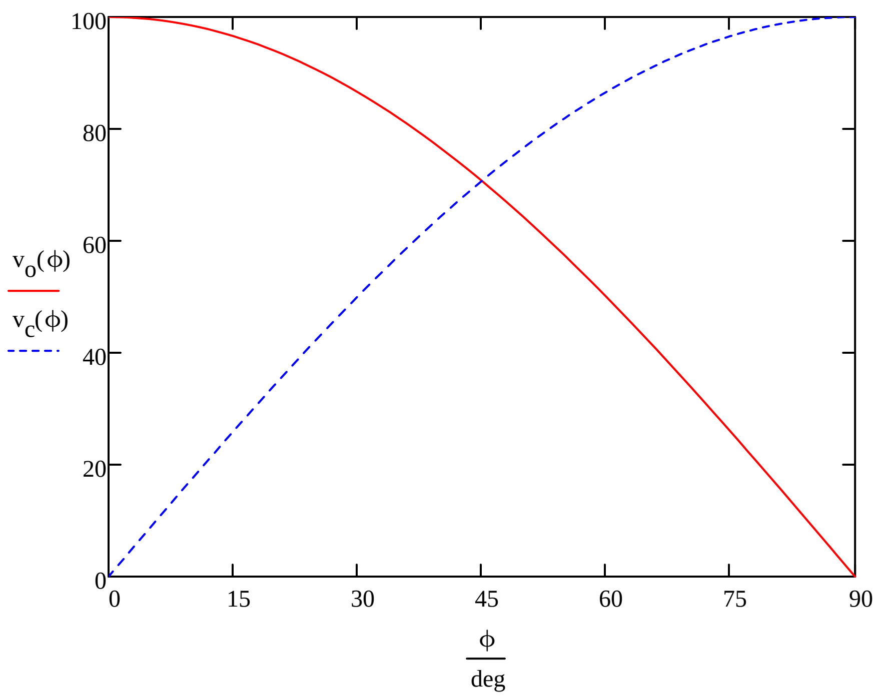

From conservation of momentum (see TP 3.1),

$v := 100$ incoming speed, scaled to 100

$v_o(\phi) := v \cdot \cos(\phi)$ post-impact object ball speed

$v_c(\phi) := v \cdot \sin(\phi)$ post-impact cue ball speed

$\phi := 0 \cdot deg, 1 \cdot deg .. 90 \cdot deg$




$v_o(20 \cdot deg) = 93.969$ $v_c(45 \cdot deg) = 70.711$ $v_o(70 \cdot deg) = 34.202$

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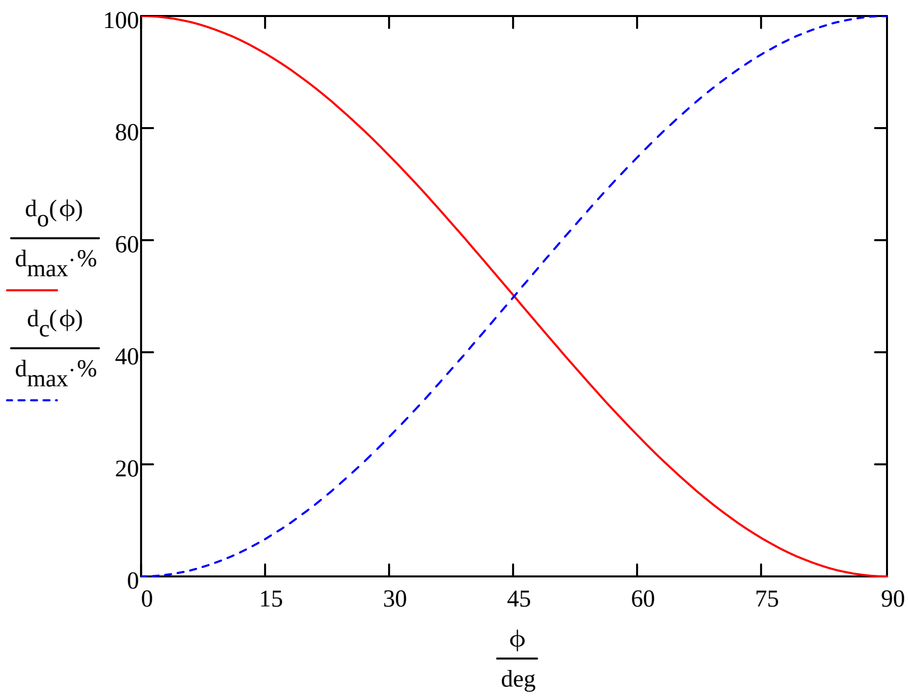
From TP 4.1, the final ball speeds (after rolling develops) are 5/7 of the initial sliding speeds. Also, with constant rolling resistance, the travel distance is proportional to the square of the speed (as in TP 4.1), so the approximate relative distances traveled by each ball, compared to the distance the stunned cue ball would travel without collision (dmax), is:

$$d_o(\varphi) := \left(\frac{5}{7}\cdot v_o(\varphi)\right)^2 \qquad d_c(\varphi) := \left(\frac{5}{7}\cdot v_c(\varphi)\right)^2 \qquad d_{max} := d_c(90\cdot deg)$$

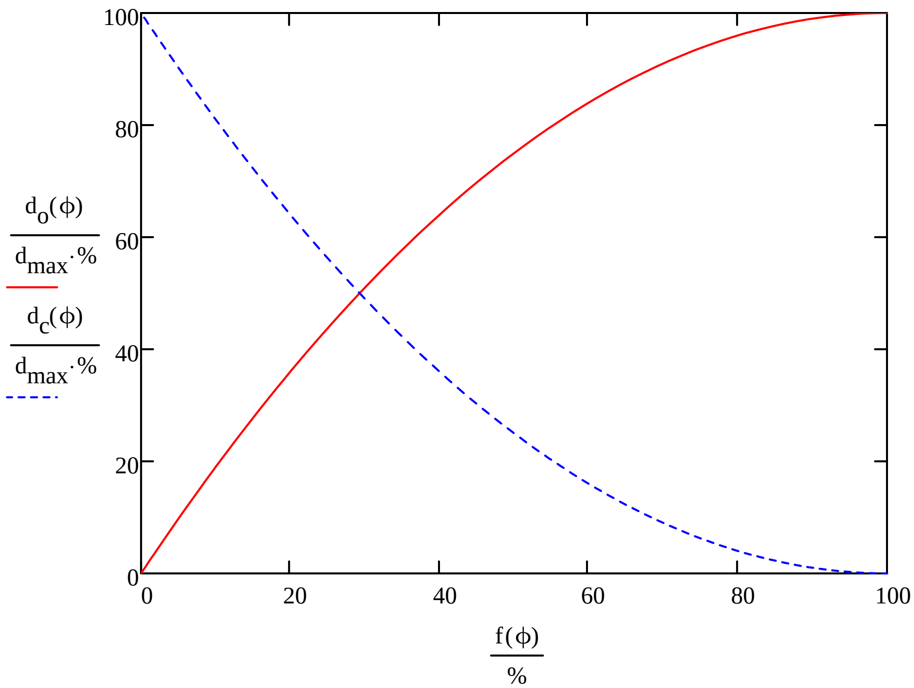



**At a 45-degree cut angle, the post-impact speeds are equal and about 70% of the initial cue ball speed (see the graph on the previous page). For the same cut angle, the post-impact ball-travel distances are also equal and 50% of the distance the stunned cue ball would travel if there were no collision (see the graph above).**

From TP 3.3, the cut angle and ball-hit fraction are related by:

$$f(\phi) := 1 - \sin(\phi)$$

so the graph above can also be expressed in terms of ball-hit fraction:




A 45-degree cut angle is between a 1/4-ball and 1/3-ball hit:

$$\frac{1}{4} = 25\cdot\% \qquad f(45\cdot deg) = 29.289\cdot\% \qquad \frac{1}{3} = 33.333\cdot\%$$

**Here are travel distance ratios for various ball-hit fractions and cut angles:**

$f(\phi) := 1 - \sin(\phi)$ $\qquad$ $\phi(f) := \mathrm{asin}(1 - f)$

| | | | approximate OB:CB distance ratio: |
|---|---|---|---|
| **full-ball hit:** | | | |
| $f_x := 1$ | $\varphi_x := \phi(f_x) = 0 \cdot deg$ | CB stops in place, OB travels same distance CB would have | |
| **3/4-ball hit:** | | | |
| $f_x := \frac{3}{4}$ | $\varphi_x := \phi(f_x) = 14.5 \cdot deg$ | $\frac{d_o(\varphi_x)}{d_c(\varphi_x)} = 15$ | 15 : 1 |
| **1/2-ball hit (30-degree cut):** | | | |
| $f_x := \frac{1}{2}$ | $\varphi_x := \phi(f_x) = 30 \cdot deg$ | $\frac{d_o(\varphi_x)}{d_c(\varphi_x)} = 3$ | 3 : 1 |
| **45-degree cut:** | | | |
| $f_x := f(45 \cdot deg) = 29.289 \cdot \%$ | $\varphi_x := 45 \cdot deg$ | $\frac{d_o(\varphi_x)}{d_c(\varphi_x)} = 1$ | 1 : 1 |
| **1/4-ball hit:** | | | |
| $f_x := \frac{1}{4}$ | $\varphi_x := \phi(f_x) = 48.6 \cdot deg$ | $\frac{d_o(\varphi_x)}{d_c(\varphi_x)} = 0.78$ | 1.5 : 2 |
| **3/16-ball hit (between 1/4 and 1/8):** | | | |
| $f_x := \frac{3}{16}$ | $\varphi_x := \phi(f_x) = 54.3 \cdot deg$ | $\frac{d_o(\varphi_x)}{d_c(\varphi_x)} = 0.51$ | 1 : 2 |
| **1/8-ball hit:** | | | |
| $f_x := \frac{1}{8}$ | $\varphi_x := \phi(f_x) = 61 \cdot deg$ | $\frac{d_o(\varphi_x)}{d_c(\varphi_x)} = 0.31$ | 1 : 3 |