technical proof

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TP 3.4
Margin of error based on distance and cut angle

from:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu

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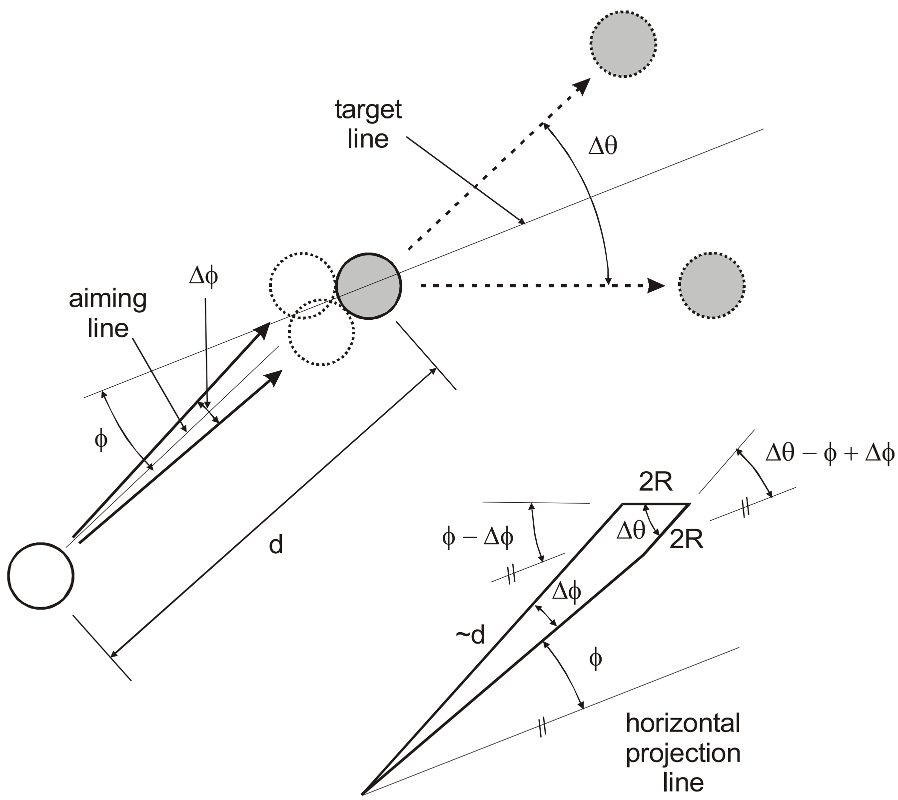


ball radius: $R := 1.125$ ϕ: cut angle θ: object ball leaving angle

d: distance between cue and object balls

Assume: d >> R

From the bottom right portion of the figure above, equating the vertical components of the loop (perpendicular to the horizontal project line) gives:

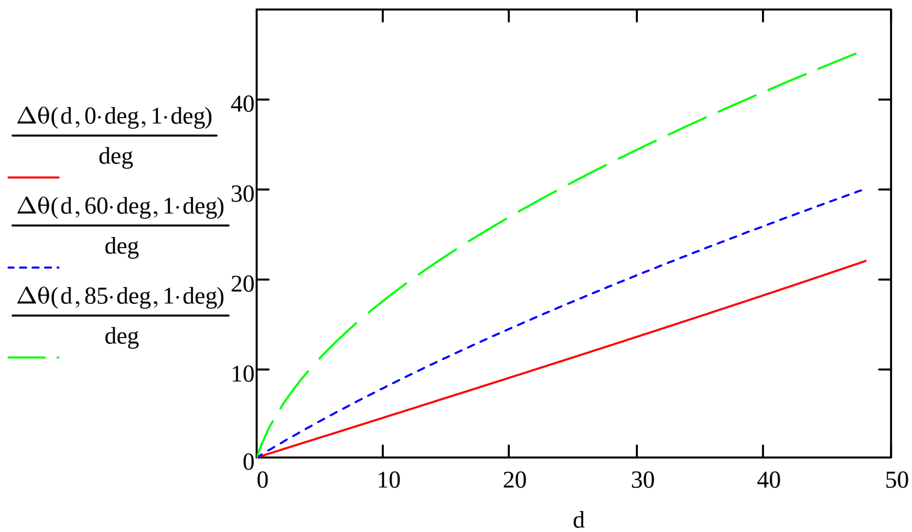
$$d \cdot \sin(\Delta\phi) = 2 \cdot R \cdot \sin(\Delta\theta - \phi + \Delta\phi) + 2 \cdot R \cdot \sin(\phi - \Delta\phi)$$

Therefore,

$$\Delta\theta(d,\phi,\Delta\phi) := \phi - \Delta\phi + \operatorname{asin}\left(\frac{d}{2\cdot R}\cdot\sin(\Delta\phi) - \sin(\phi - \Delta\phi)\right)$$

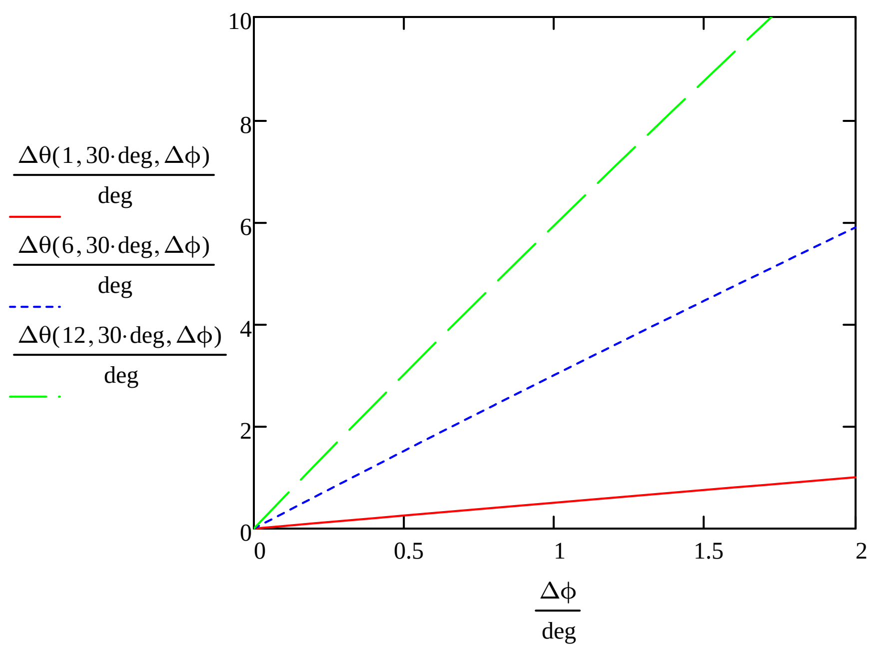
The following plot shows how the angular error in OB motion (Δθ) varies with distance between the CB and OB (d) at different cut angles (ϕ) for a given angular margin for error (Δϕ) in CB motion (1 deg):

$$d := 0, 0.1 .. 48$$



The following plot shows how the angular error in OB motion (Δθ) varies with CB angle margin for error (Δϕ) at different distances between the CB and OB (d) for a 1/2-ball hit (30 deg cut angle):

$$\Delta\phi := 0\cdot deg, 0.1\cdot deg .. 2\cdot deg$$



Example: *What is the allowable object ball (OB) angle error and required cue ball (CB) angle accuracy for a slow, 30-degree cut angle (half-ball hit) shot straight into a corner pocket (i.e., the angle to the pocket is 0 degrees) if the distances between the CB and OB and the OB and pocket are both about 4 diamonds (about 4.5' = 54" on a 9' table)?*

From Figure 3.41 in the book, the approximate allowable object ball angle error is:

$$\Delta\theta_{allowable} := 2 \cdot deg$$

The distance between the CB and OB is:

$$d := 54$$

and the cut angle for a half-ball hit is:

$$\phi := 30 \cdot deg$$

Therefore, the required cue ball angle accuracy is:

initial guess: $\Delta\phi_{required} := 0.1 \cdot deg$

Given

$$\Delta\theta\left(d, \phi, \Delta\phi_{required}\right) = \Delta\theta_{allowable}$$

$$\text{Find}\left(\Delta\phi_{required}\right) = 0.073\ deg$$

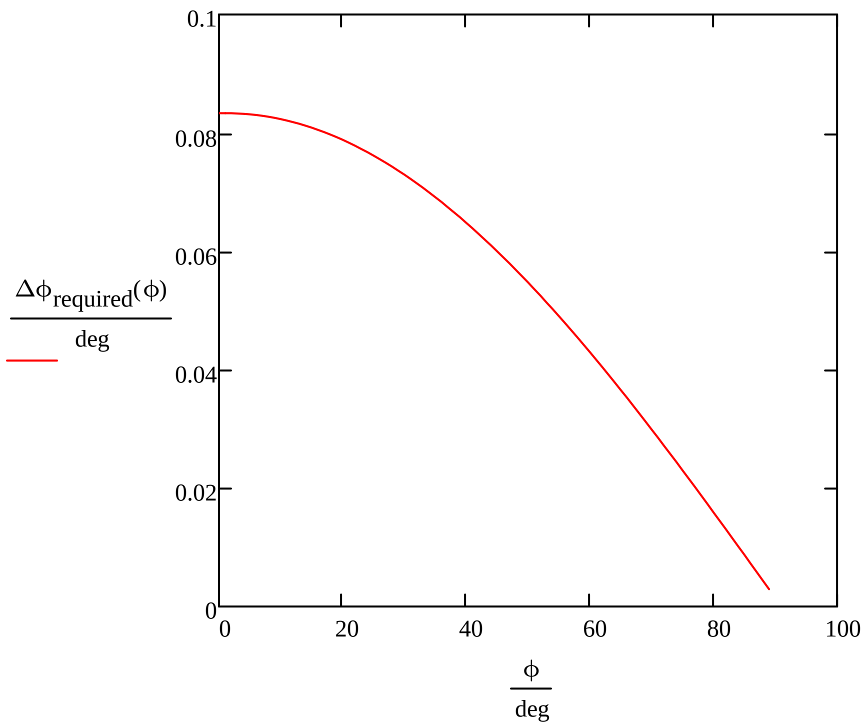
So the margin for error for the CB in this shot is very small (less than 0.1 degree!)

Allowable cue ball margin for error for various cut angles for the example shot above:

$\Delta\phi := 0.01 \cdot deg$ initial guess

$\Delta\phi_{required}(\phi) := root\left(\Delta\theta(d, \phi, \Delta\phi) - \Delta\theta_{allowable}, \Delta\phi\right)$

$\phi := 0 \cdot deg, 1 \cdot deg .. 90 \cdot deg$



$\Delta\phi_{required}(0 \cdot deg) = 0.083\, deg$

$\Delta\phi_{required}(30 \cdot deg) = 0.073\, deg$

$\Delta\phi_{required}(60 \cdot deg) = 0.043\, deg$

$\left(\frac{\Delta\phi_{required}(0 \cdot deg)}{\Delta\phi_{required}(30 \cdot deg)}\right) = 1.142$

a straight-in shot is 1.15X (15%) easier than a 30 degree cut angle shot.

$\left(\frac{\Delta\phi_{required}(0 \cdot deg)}{\Delta\phi_{required}(60 \cdot deg)}\right) = 1.939$

a straight-in shot is 1.97X (97%) easier than a 60 degree cut angle shot.

NOTE- If the effective size of the pocket (s) is known (e.g., from TP 3.5-3.8), the allowable angular error in the OB direction ($\Delta\theta$), for a given distance to the pocket (d_p), can be calculated with:

$$\Delta\theta = 2 \cdot \tan^{-1}\left(\frac{\frac{s}{2}}{d_p}\right)$$

A simple way to estimate the effective size of a pocket is to subtract 2 inches (a little less than a ball diameter) from the point-to-point pocket-size measurement (p). For example, for the shot above, with a 4.5" pocket, with the OB 4.5 feet from the pocket:

$$p := 4.5 \cdot in \qquad d_p := 4.5 \cdot ft$$

$$s_{approx} := p - 2 \cdot in = 2.5\,in$$

$$\Delta\theta_{approx} := 2 \cdot atan\left(\frac{\frac{s_{approx}}{2}}{d_p}\right) = 2.7\,deg$$