technical proof technical proof

TP 4.1
Distance required for normal roll to develop

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
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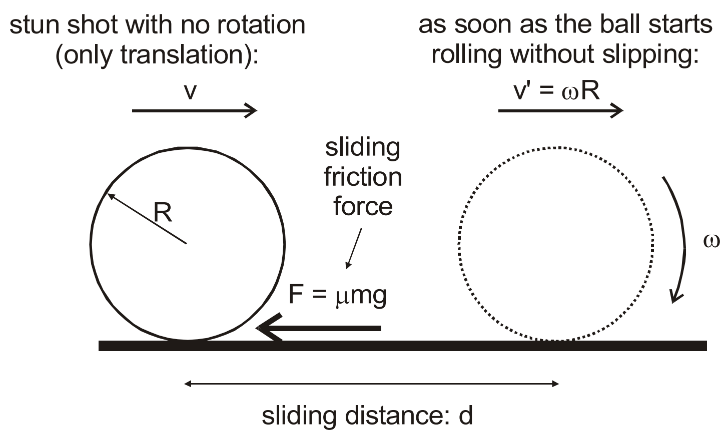

ball mass: m ball moment of inertia about its center: $I := \frac{2}{5}\cdot m\cdot R^2$

t: time for sliding to stop over distance d

Constant linear acceleration (negative implies deceleration) caused by the friction force:

$$a = -\frac{F}{m} = -\mu\cdot g$$

Linear speed when sliding stops

$$v' = v + a\cdot t = v - \mu\cdot g\cdot t$$

Constant angular acceleration caused by the moment of the friction force about the ball center:

$$\alpha = \frac{F\cdot R}{I} = \frac{\mu\cdot m\cdot g\cdot R}{\frac{2}{5}\cdot m\cdot R^2} = \frac{5\cdot\mu\cdot g}{2\cdot R}$$

Angular speed when sliding stops:

$$\omega = \alpha \cdot t = \frac{5 \cdot \mu \cdot g}{2 \cdot R} \cdot t$$

At time t, the ball is rolling without slipping, so:

$$v' = \omega \cdot R$$

$$v - \mu \cdot g \cdot t = \frac{5 \cdot \mu \cdot g}{2} \cdot t$$

$$\mu \cdot g \cdot t \cdot \left(\frac{5}{2} + 1 \right) = v$$

$$t = \frac{2 \cdot v}{7 \cdot \mu \cdot g}$$

The distance and time are related with the following constant acceleration relation:

$$d = v \cdot t + \frac{1}{2} \cdot a \cdot t^2$$

$$d = \frac{2 \cdot v^2}{7 \cdot \mu \cdot g} - \frac{1}{2} \cdot \mu \cdot g \cdot \left[\frac{4 \cdot v^2}{49 \cdot (\mu \cdot g)^2} \right] = \frac{v^2}{\mu \cdot g} \cdot \left(\frac{2}{7} - \frac{2}{49} \right) = \frac{12 \cdot v^2}{49 \cdot \mu \cdot g}$$

Interestingly, the final ball speed (after sliding stops and rolling begins) is independent of table and ball conditions:

$$v' = v - \mu \cdot g \cdot t = v - \mu \cdot g \cdot \frac{2 \cdot v}{7 \cdot \mu \cdot g} = v \left(1 - \frac{2}{7} \right) = \frac{5}{7} \cdot v$$

The final ball speed, for an initially sliding ball, is always 5/7 of the initial speed!