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TP 4.4
Relationship between the amount of throw and cut angle

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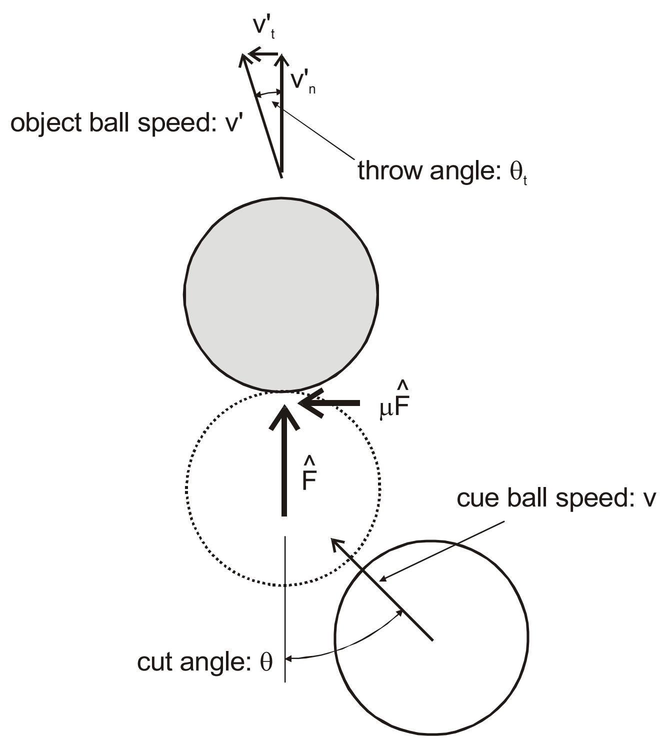

Object ball speed in the normal direction from the coefficient of restitution:

$$v'_n = e \cdot v \cdot \cos(\theta)$$

Assuming that all speed in the normal direction is delivered from the cue ball to the object ball, from linear impulse (F') and momentum:

$$F' = m \cdot v \cdot \cos(\theta) = \frac{m \cdot v'_n}{e}$$

From linear impulse and momentum in the tangent direction:

$$m \cdot v'_t = \mu \cdot F' = \frac{\mu \cdot m \cdot v'_n}{e}$$

so

$$v'_t = \frac{\mu \cdot v'_n}{e}$$

Therefore, the throw angle is given by:

$$\theta_t = \mathrm{atan}\left(\frac{v'_t}{v'_n}\right) = \mathrm{atan}\left(\frac{\mu}{e}\right)$$

μ and e both vary with speed and cut angle. The throw angle does not vary with speed significantly. The throw angle increases with cut angle. Here are typical values for a large cut angle shot:

$e := 0.92 \qquad \mu := 0.06$

$$\mathrm{atan}\left(\frac{\mu}{e}\right) = 3.731\ \mathrm{deg}$$

NOTE - The analysis above is a very simplified model. For a more thorough analysis that takes speed and spin effects into consideration, see TP A.14. The results in TP A.14 agree fairly closely with experimental data for various cases.