TP A.15
Controlling the cue ball direction in a frozen cue ball shot

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu

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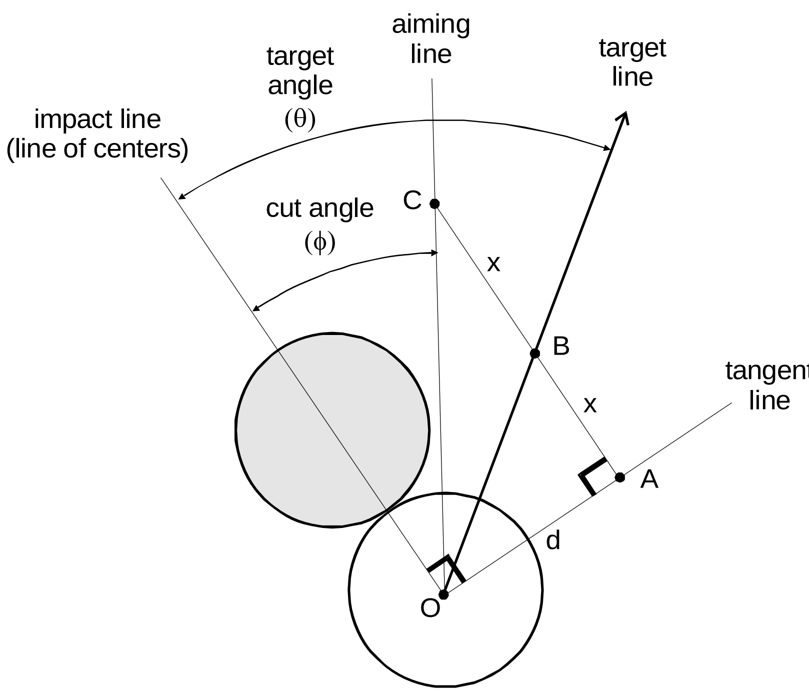


The cue ball is frozen to the object ball, and we want the cue ball to go in the target line direction (e.g., for a carom/billiard shot, break-up shot, or position play). Where do you aim the cue stick to have the cue ball head along the target line (i.e., what is the correct aiming line)? One possible, crude approximation (the "Dr. Dave Approximation") is to aim at 2/3 of the angle from the impact line to the target line. This can be expressed as:

$$\phi_{approx}(\theta) := \frac{2}{3}\cdot\theta$$

An alternative, and more accurate, geometric method was proposed by Bob Jewett (as described on page 117 of "Byrne's New Standard Book of Pool and Billiards"). It is illustrated in the diagram above. To use the method, you extend an imaginary line in the impact line direction from any point along the tangent line (line ABC). The necessary cue ball aiming line should be the same distance (x) away from the target line (segment BC) as the target line is from the tangent line (segment AB). Here's the math:

From triangle OAC,

$$\tan(90\cdot\text{deg} - \phi) = \frac{2x}{d} \qquad (1)$$

From triangle OAB,

$$\tan(90\cdot\text{deg} - \theta) = \frac{x}{d} \qquad (2)$$

Plugging Equation 2 into Equation 1, and solving for ϕ, gives:

$$\phi_{Jewett}(\theta) := 90\cdot\text{deg} - \text{atan}(2\cdot\tan(90\cdot\text{deg} - \theta))$$

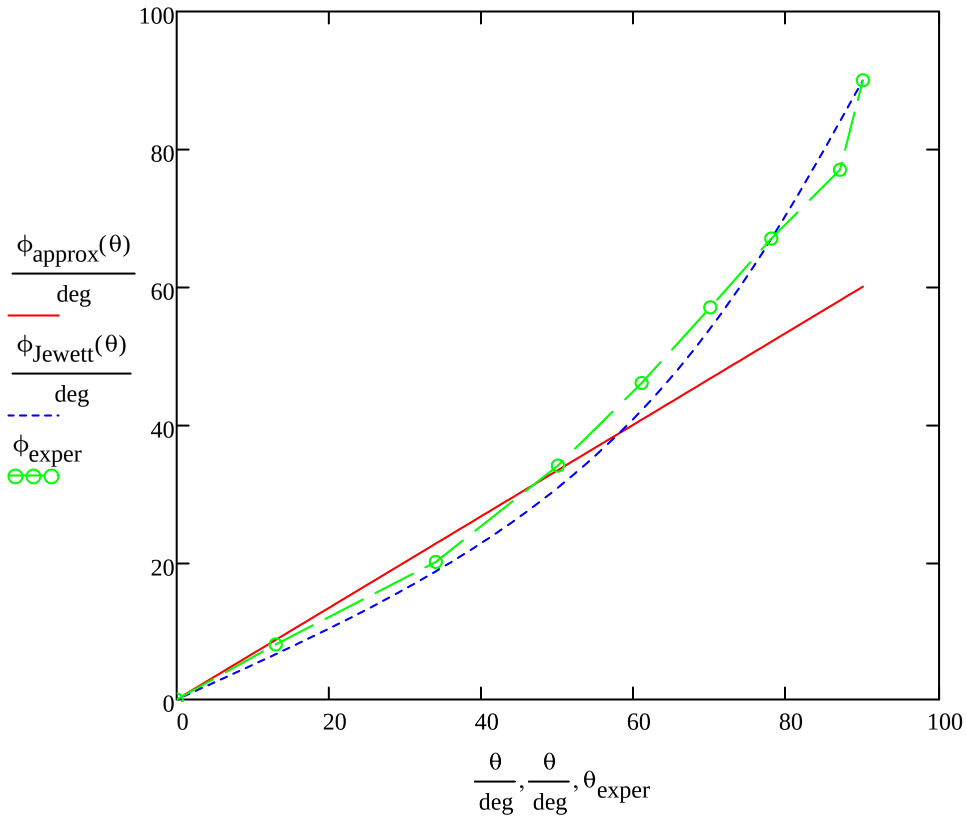
Nothing is better than hard experimental evidence, so I used a high-speed camera to shoot super-slow-motion video clip **HSV A.97** at **http://billiards.colostate.edu**. In the clip, the cut angle is gradually increased to see how the resulting target angle changes. Here are the data from the experiment:

$$M_{exper} := \begin{pmatrix} 0 & 0 \\ 13 & 8 \\ 34 & 20 \\ 50 & 34 \\ 61 & 46 \\ 70 & 57 \\ 78 & 67 \\ 87 & 77 \\ 90 & 90 \end{pmatrix}$$

$$\theta_{exper} := M_{exper}^{\langle 0 \rangle} \qquad \phi_{exper} := M_{exper}^{\langle 1 \rangle}$$

Now, lets compare the two aiming methods to the experimental results:

$$\theta := 1 \cdot deg, 2 \cdot deg .. 90 \cdot deg$$



As you can see, both methods match the experimental results fairly closely over a fairly wide range of target angles. However, the Jewett interference system does match the data better at large target angles.