

## TP A.28

### Throw plots for all types of shots

supporting:  
 “The Illustrated Principles of Pool and Billiards”  
<http://billiards.colostate.edu>  
 by David G. Alciatore, PhD, PE ("Dr. Dave")

originally posted: 4/3/2007    last revision: 9/21/2007

**Note: These results are based on the analysis in TP A.14.**

$$R := \frac{1.125 \cdot \text{in}}{\text{m}} \quad \text{ball radius converted to meters}$$

Model of friction based on Marlow data (Table 10 on p. 245 in "The Physics of Pocket Billiards," 1995)

$vd_1 := .1 \cdot \sin(45 \cdot \text{deg})$	$\mu d_1 := 0.11$
$vd_2 := 1 \cdot \sin(45 \cdot \text{deg})$	$\mu d_2 := 0.06$
$vd_3 := 10 \cdot \sin(45 \cdot \text{deg})$	$\mu d_3 := 0.01$

We can solve for the coefficients from the set of data above using:

initial guesses:

$$a := \mu d_3 \quad b := \mu d_1 - \mu d_3 \quad c := \frac{-1}{vd_2} \cdot \ln\left(\frac{\mu d_2 - a}{b}\right)$$

$$a = 0.01 \quad b = 0.1 \quad c = 0.98$$

Given

$$\mu d_1 = a + b \cdot e^{-c \cdot vd_1}$$

$$\mu d_2 = a + b \cdot e^{-c \cdot vd_2}$$

$$\mu d_3 = a + b \cdot e^{-c \cdot vd_3}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} := \text{Find}(a, b, c) \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 9.951 \times 10^{-3} \\ 0.108 \\ 1.088 \end{pmatrix}$$

$$\mu(v) := a + b \cdot e^{-c \cdot v}$$

this is the "model" of friction that will be used

MathCAD formulations of Equations 15 through 17, using the above formulation for friction:

$$v_{\text{rel}}(v, \omega_x, \omega_z, \phi) := \sqrt{(v \cdot \sin(\phi) - R \cdot \omega_z)^2 + (R \cdot \omega_x \cdot \cos(\phi))^2}$$

$$\theta_{\text{throw}}(v, \omega_x, \omega_z, \phi) := \text{atan} \left[ \frac{\min \left( \frac{\mu(v_{\text{rel}}(v, \omega_x, \omega_z, \phi)) \cdot v \cdot \cos(\phi)}{v_{\text{rel}}(v, \omega_x, \omega_z, \phi)}, \frac{1}{7} \right) \cdot (v \cdot \sin(\phi) - R \cdot \omega_z)}{v \cdot \cos(\phi)} \right]$$

cut angle range:

$$\phi := 0 \cdot \text{deg}, 1 \cdot \text{deg} .. 75 \cdot \text{deg}$$

various speeds (slow, medium, fast):

$$\begin{array}{lll} v_1 := 0.5 \cdot \frac{\text{m}}{\text{s}} & v_2 := 1.5 \cdot \frac{\text{m}}{\text{s}} & v_3 := 4.5 \cdot \frac{\text{m}}{\text{s}} \\ v_1 = 1.118 \text{ mph} & v_2 = 3.355 \text{ mph} & v_3 = 10.066 \text{ mph} \\ \underline{v_1} := \frac{v_1}{\frac{\text{m}}{\text{s}}} & \underline{v_2} := \frac{v_2}{\frac{\text{m}}{\text{s}}} & \underline{v_3} := \frac{v_3}{\frac{\text{m}}{\text{s}}} \end{array}$$

spin rate from percent English (from TP A.25):

$$\begin{aligned} \omega(v, \text{PE}) &= \text{SRF} \cdot \omega_{\text{roll}} = \frac{5}{4} \cdot \frac{v}{R} \cdot \text{PE} \\ \omega(v, \text{PE}) &:= \frac{5}{4} \cdot \frac{v}{R} \cdot \text{PE} \end{aligned}$$

throw angle equation variable info

$$\theta_{\text{throw}}(v, \omega_x, \omega_z, \phi)$$

$\omega_x, \omega_r$ : vertical plane spin rate (follow or draw)

$\omega_z, \omega_e$ : English spin rate

$\text{PE}_r$ : % topspin (roll)

$\text{PE}_e$ : % English (side spin)

**topspin:  $\omega_r, \text{PE}_r > 0$**

**outside English (OE):  $\omega_e, \text{PE}_e > 0$**

**bottom-spin:  $\omega_r, \text{PE}_r < 0$**

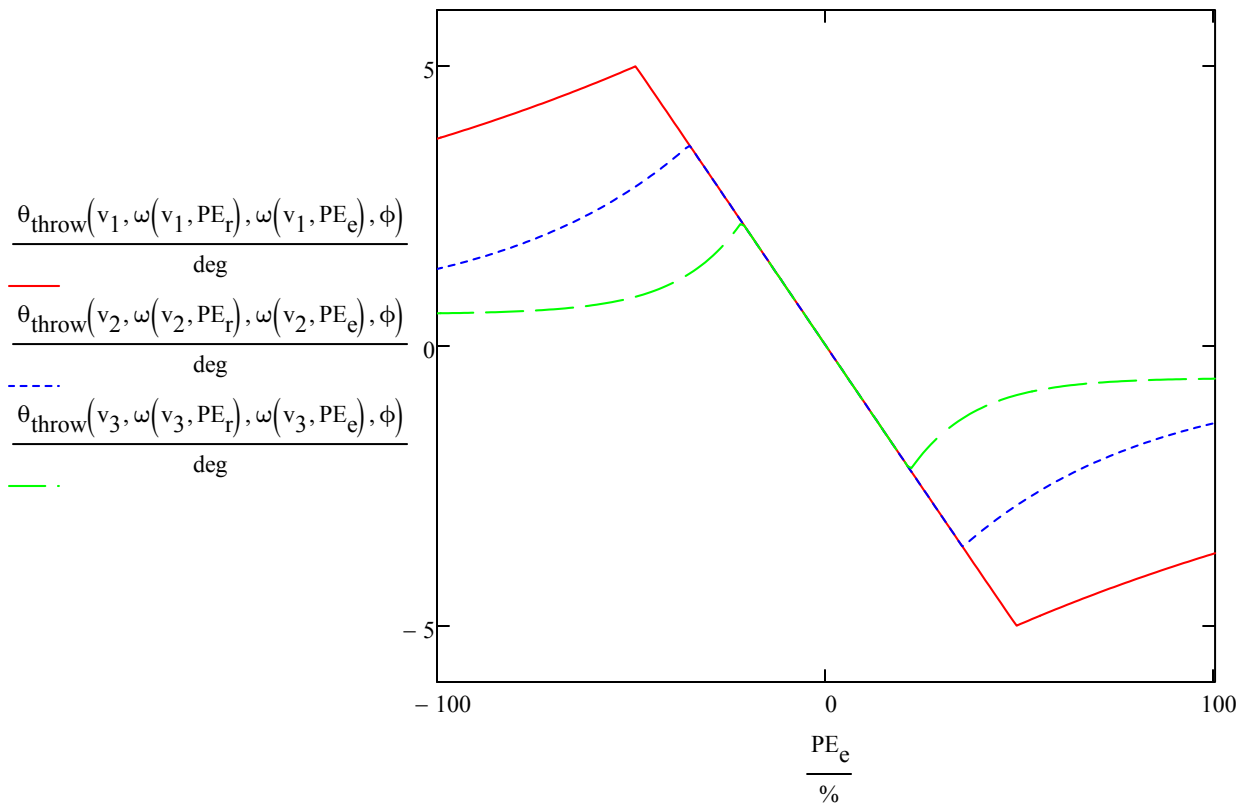
**inside English (IE):  $\omega_e, \text{PE}_e < 0$**

These plots show how the amount of throw varies with the amount of English for various cut angles, speeds, and amounts of vertical plane spin (draw or follow):

$PE_r := 0\%$

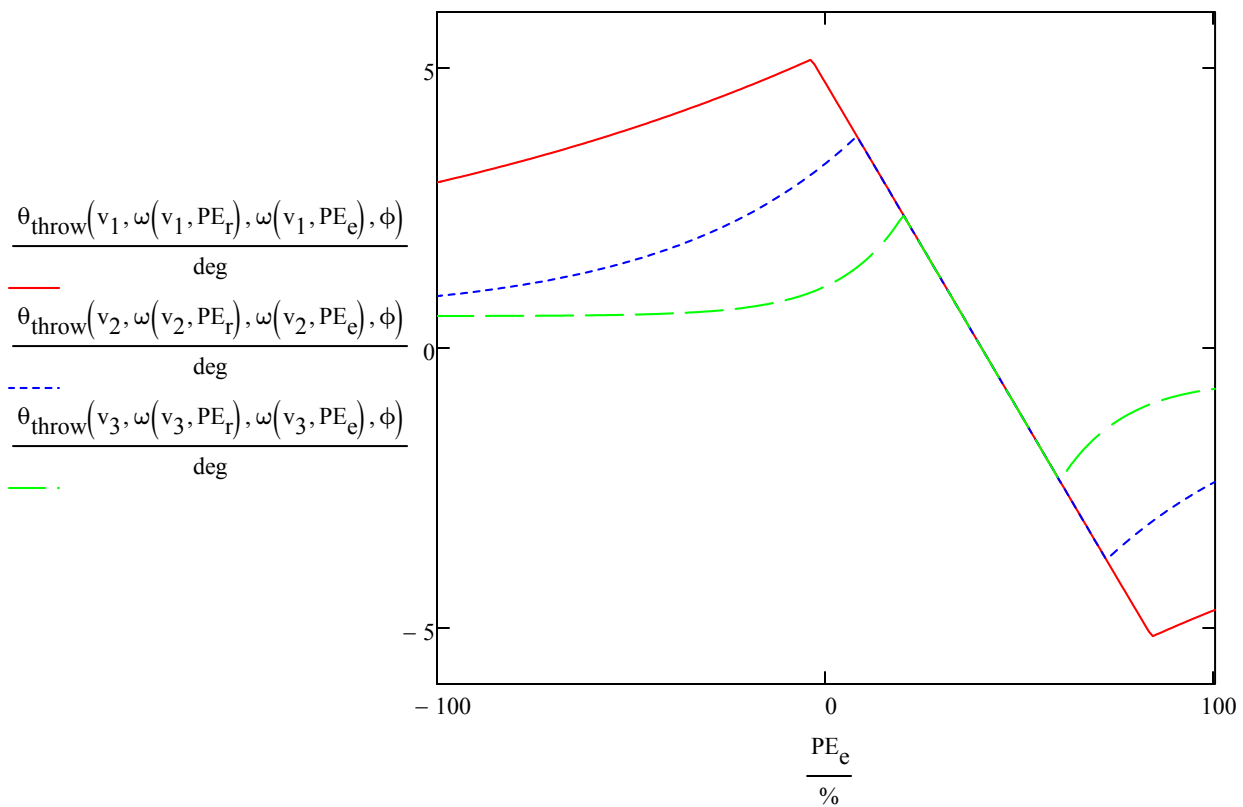
$\phi := 0\text{-deg}$

$PE_e := -100\%, -99\% \dots 100\%$

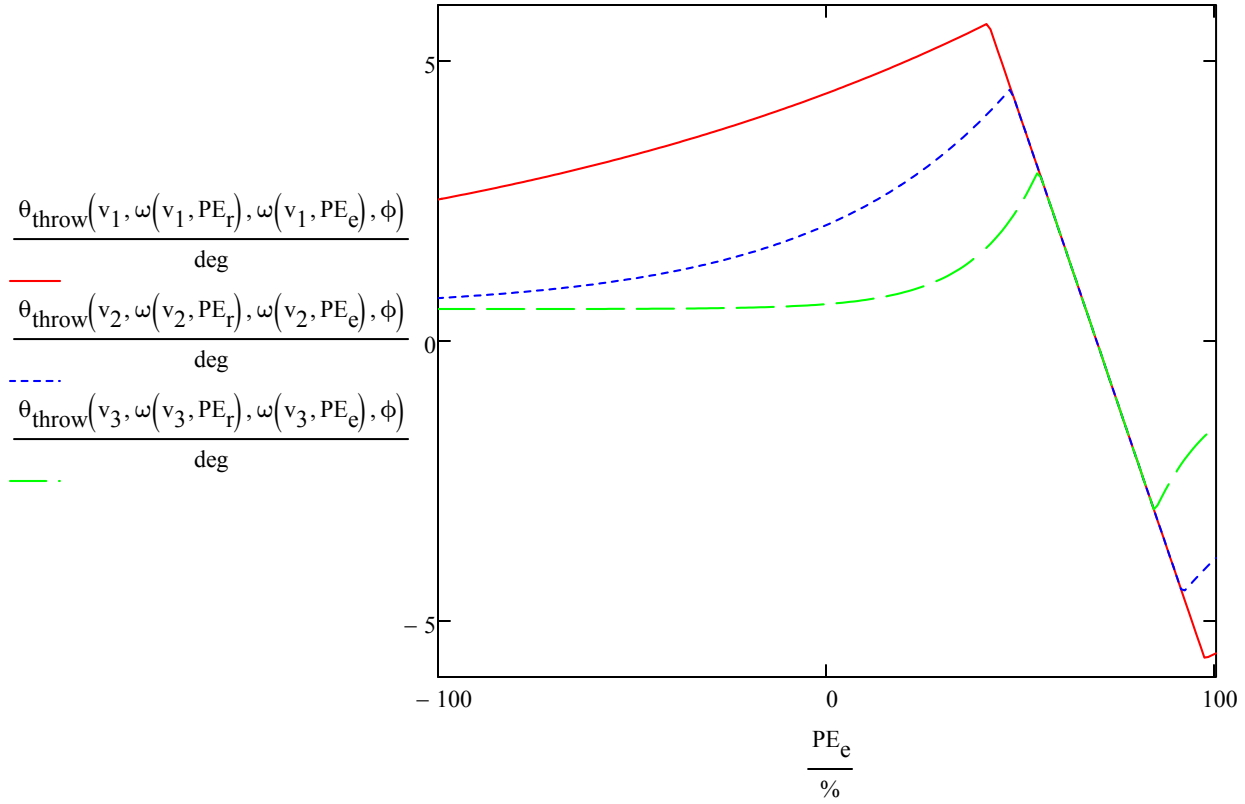


$PE_r := 0\%$

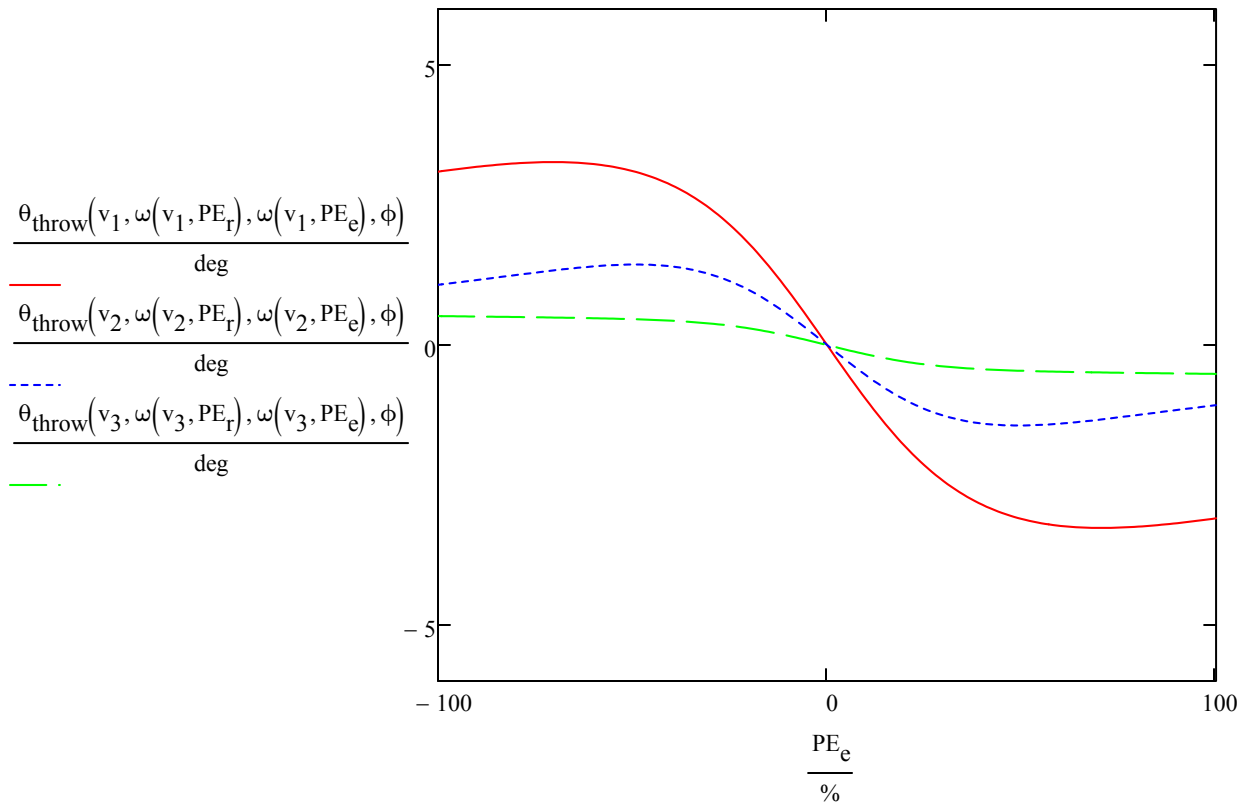
$\phi := 30\text{-deg}$



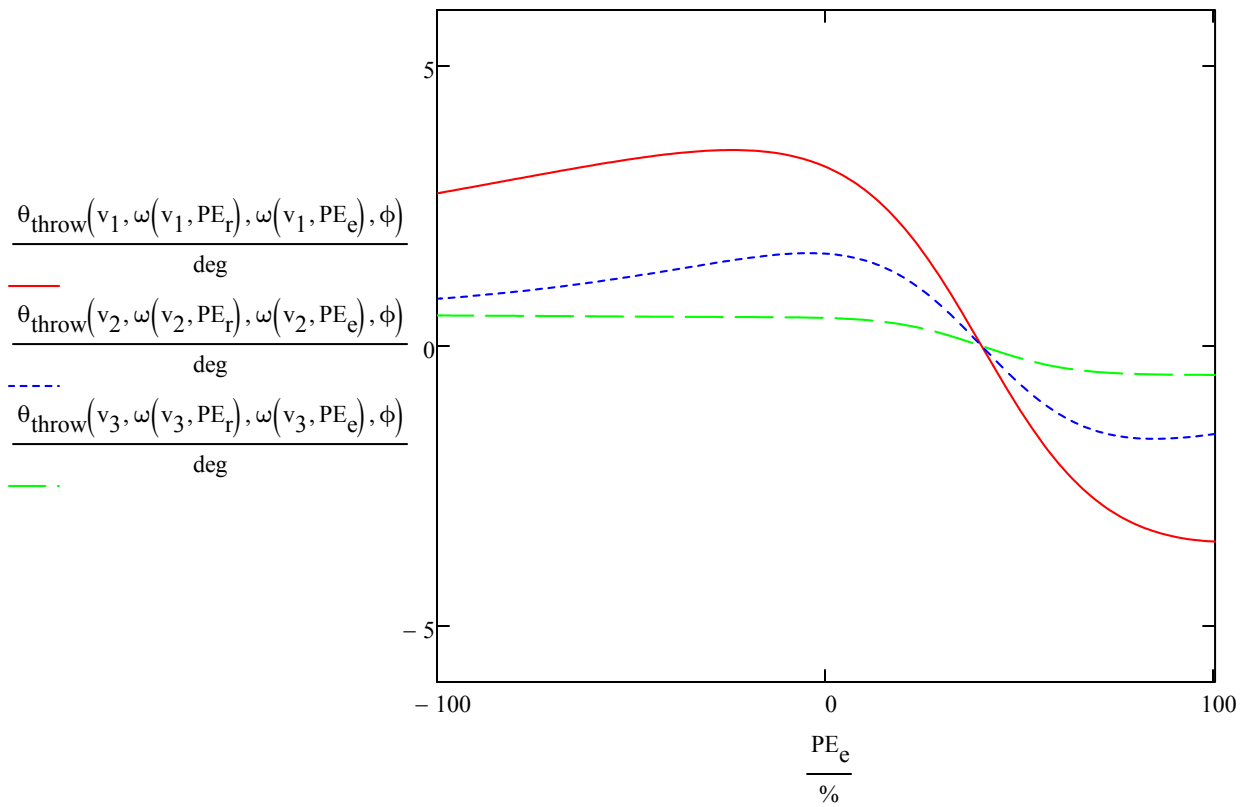
$PE_r := 0\%$      $\phi := 60\text{-deg}$



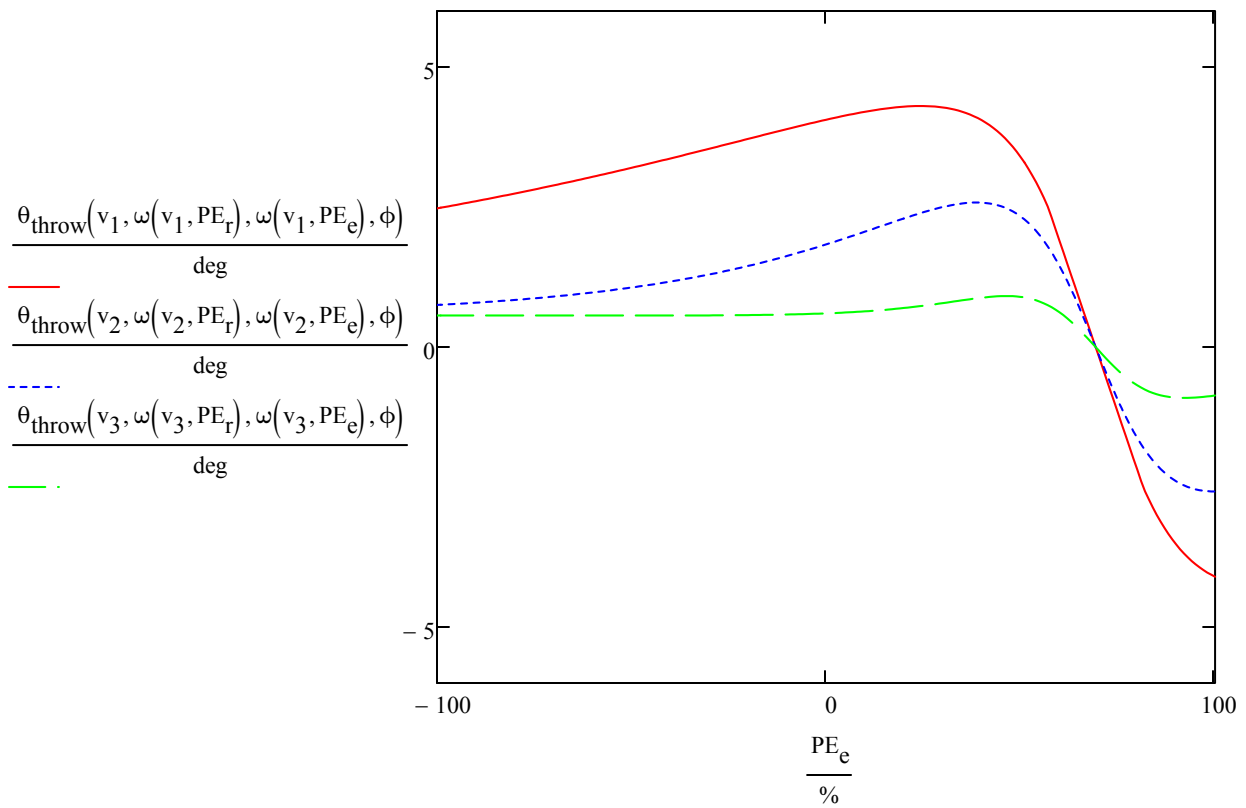
$PE_r := 50\%$      $\phi := 0\text{-deg}$



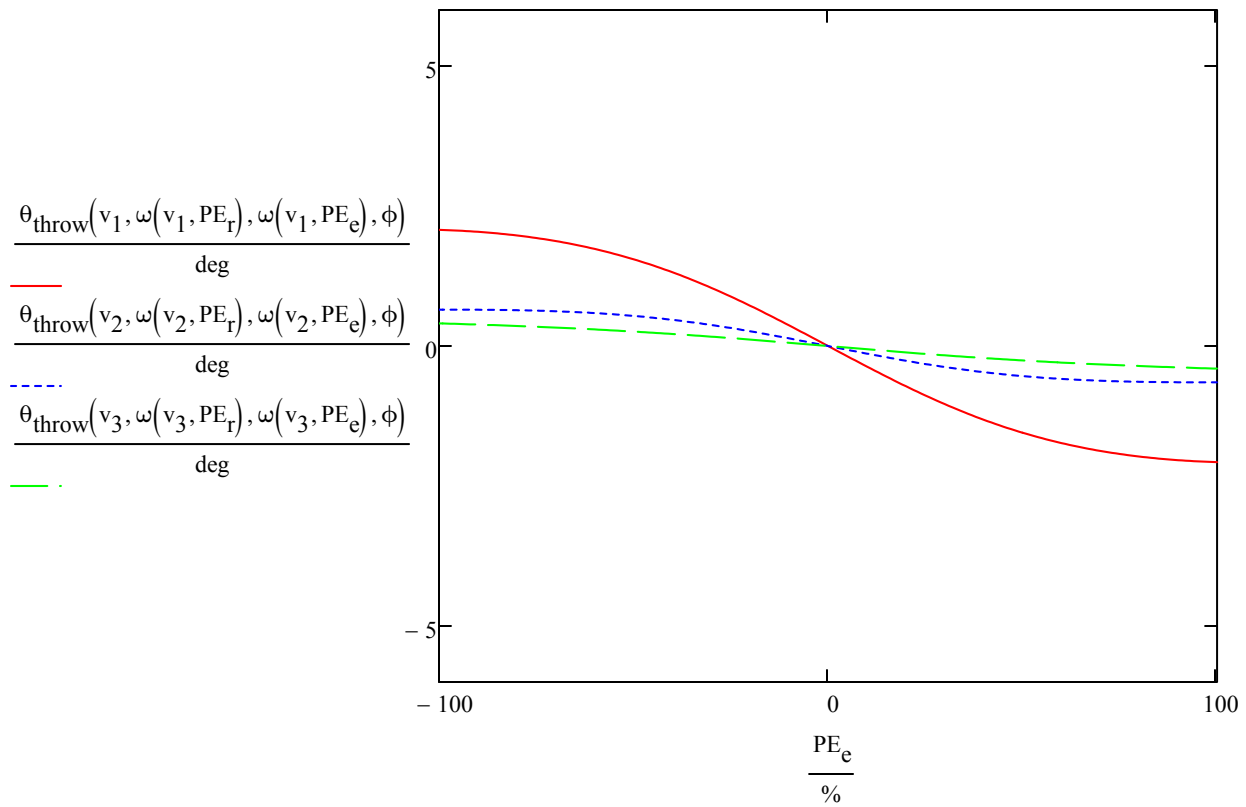
$PE_e := 50\%$     $\phi := 30\text{-deg}$



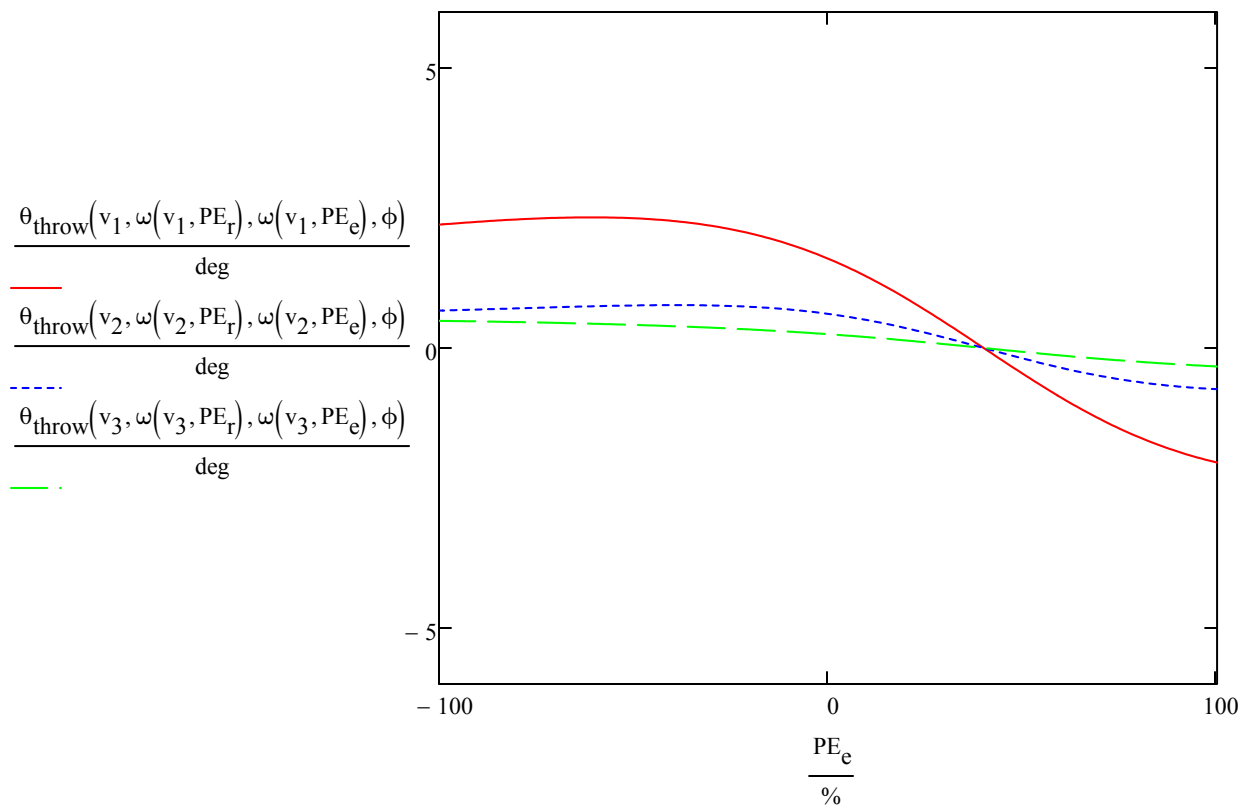
$PE_e := 50\%$     $\phi := 60\text{-deg}$



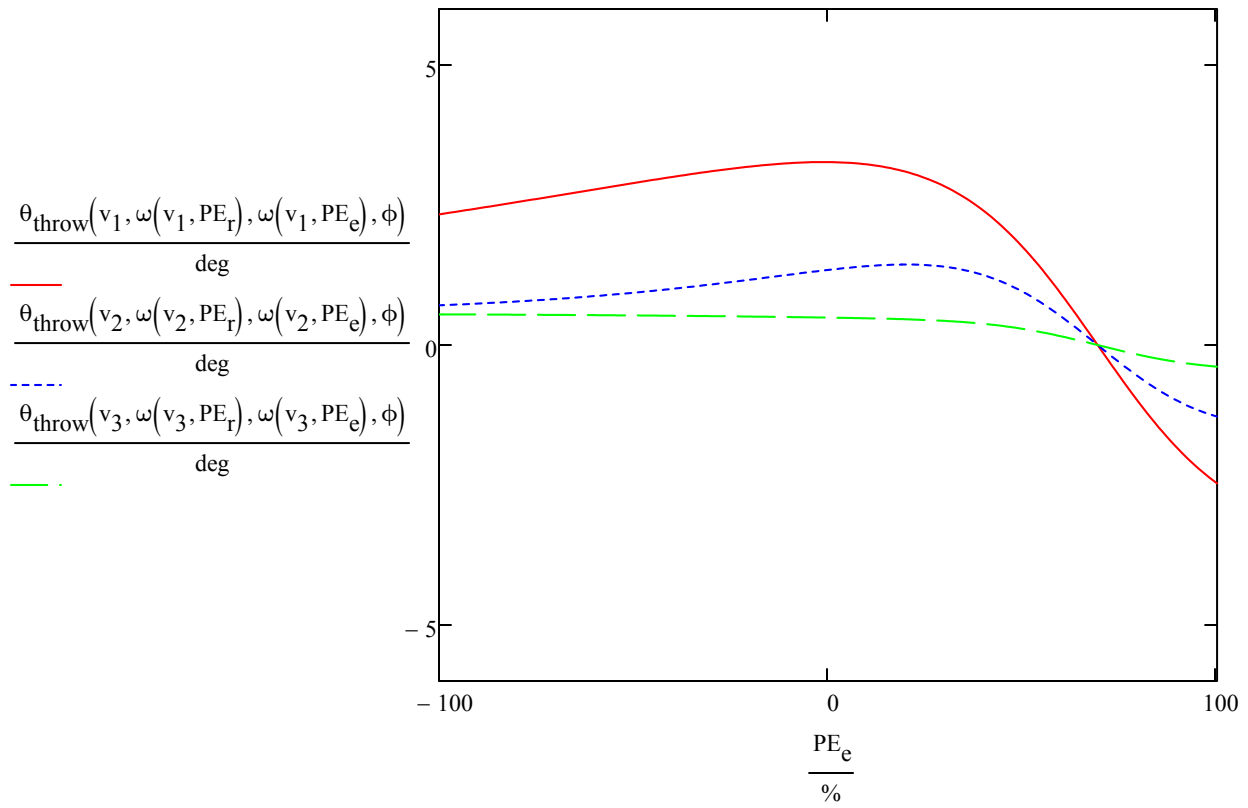
$\overline{PE}_r := 100\%$     $\phi := 0\text{-deg}$



$\overline{PE}_r := 100\%$     $\phi := 30\text{-deg}$



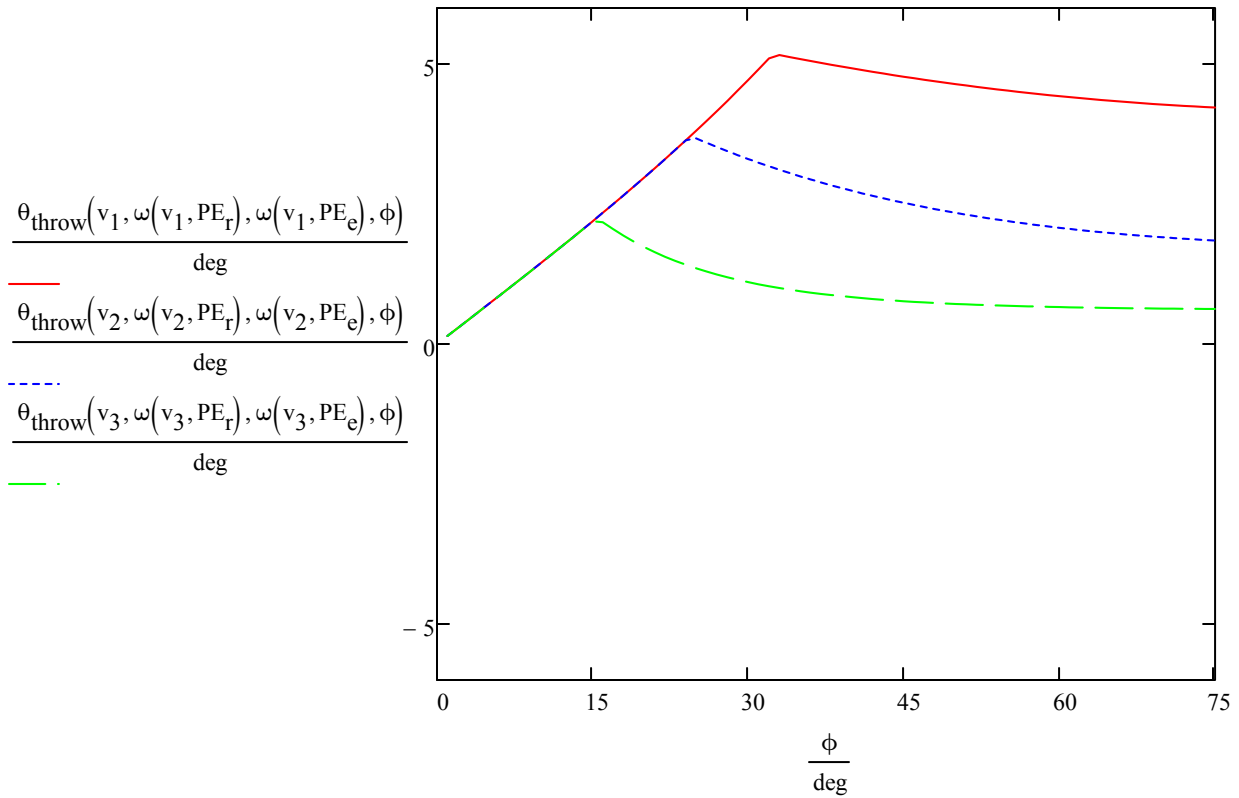
$PE_r := 100\%$     $\phi := 60\text{-deg}$



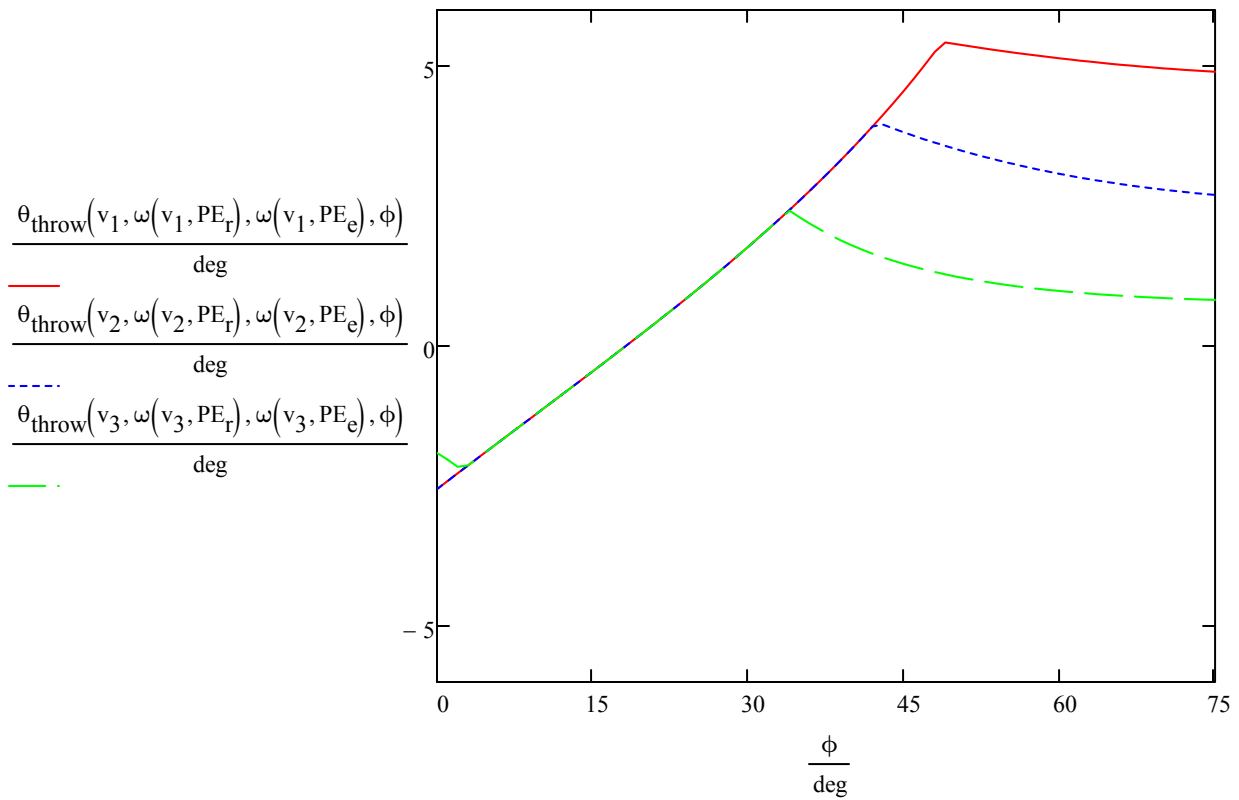
The remaining plots show how throw varies with cut angle for various speed shots with various amounts of English and vertical plane spin (follow or draw):

$PE_r := 0\%$       $PE_e := 0\%$

$\phi := 0\text{-deg}, 1\text{-deg}.. 75\text{-deg}$

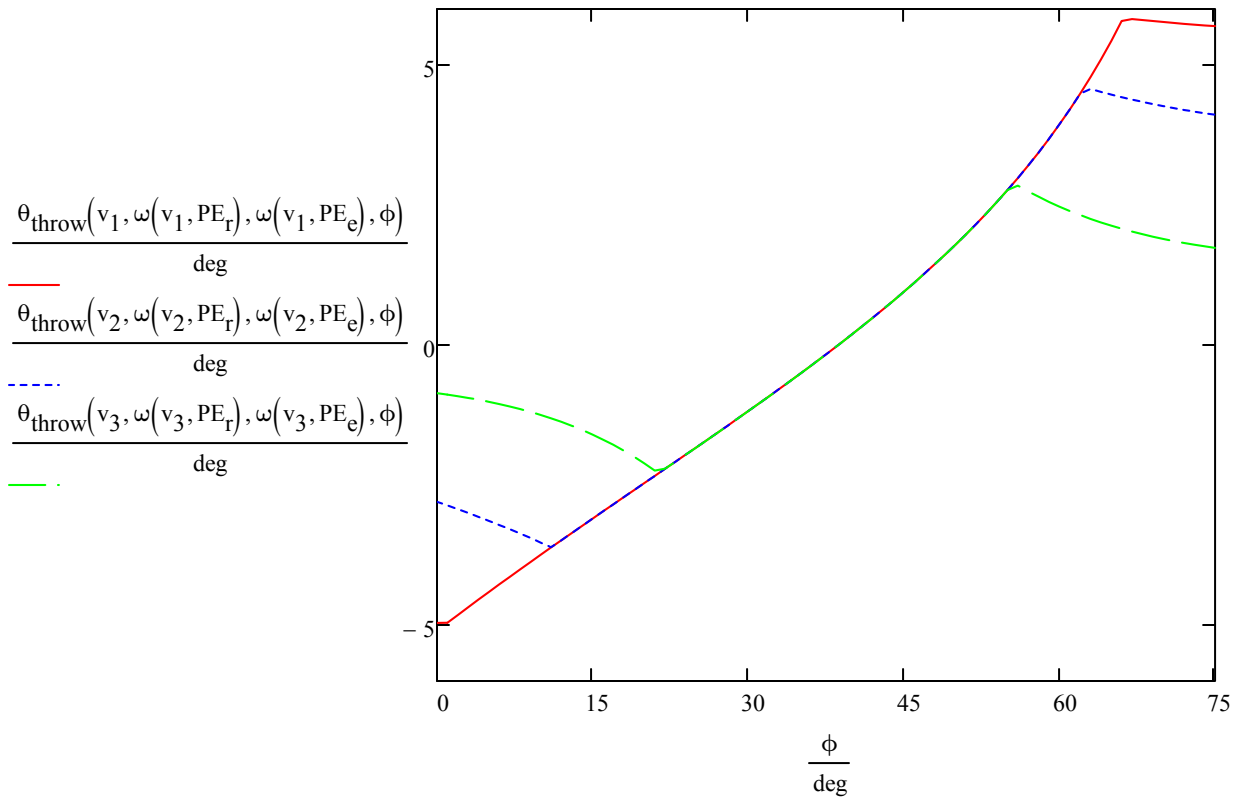


$PE_r := 0\%$       $PE_e := 25\%$

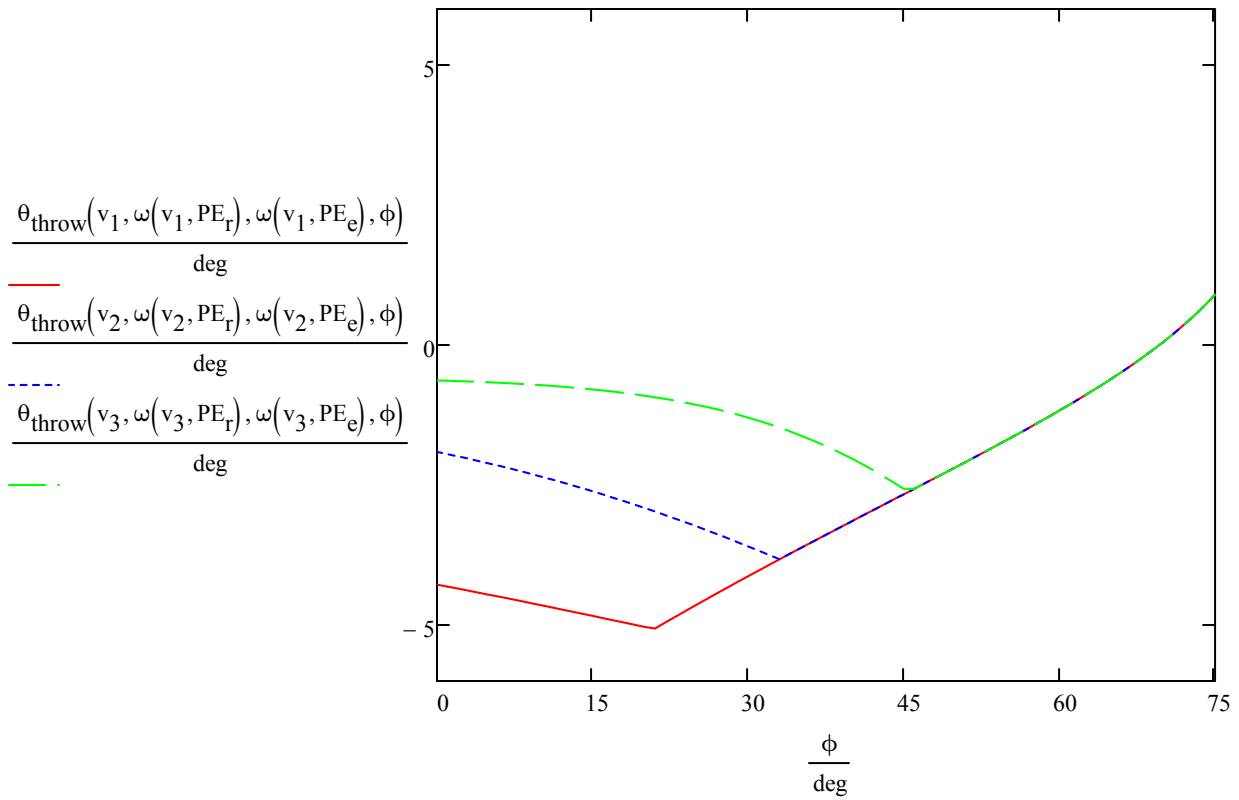




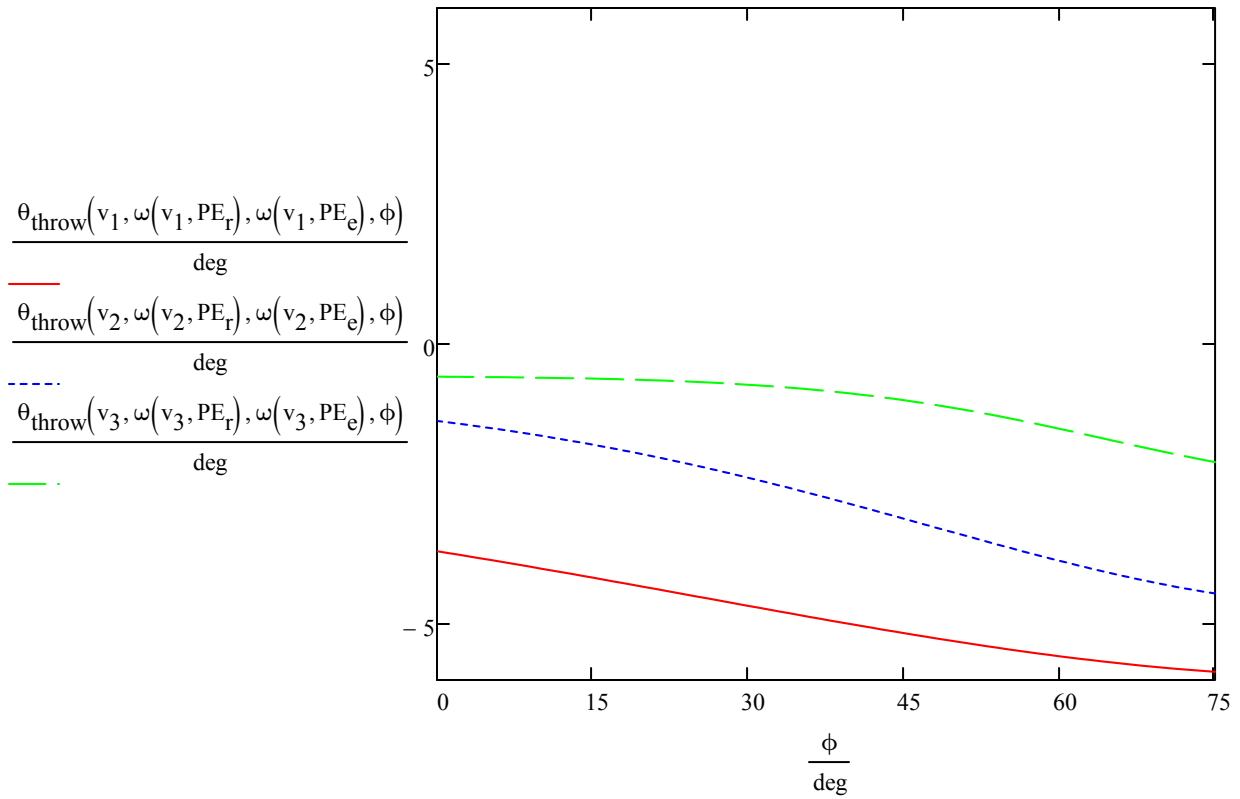
$PE_r := 0\%$        $PE_e := 50\%$



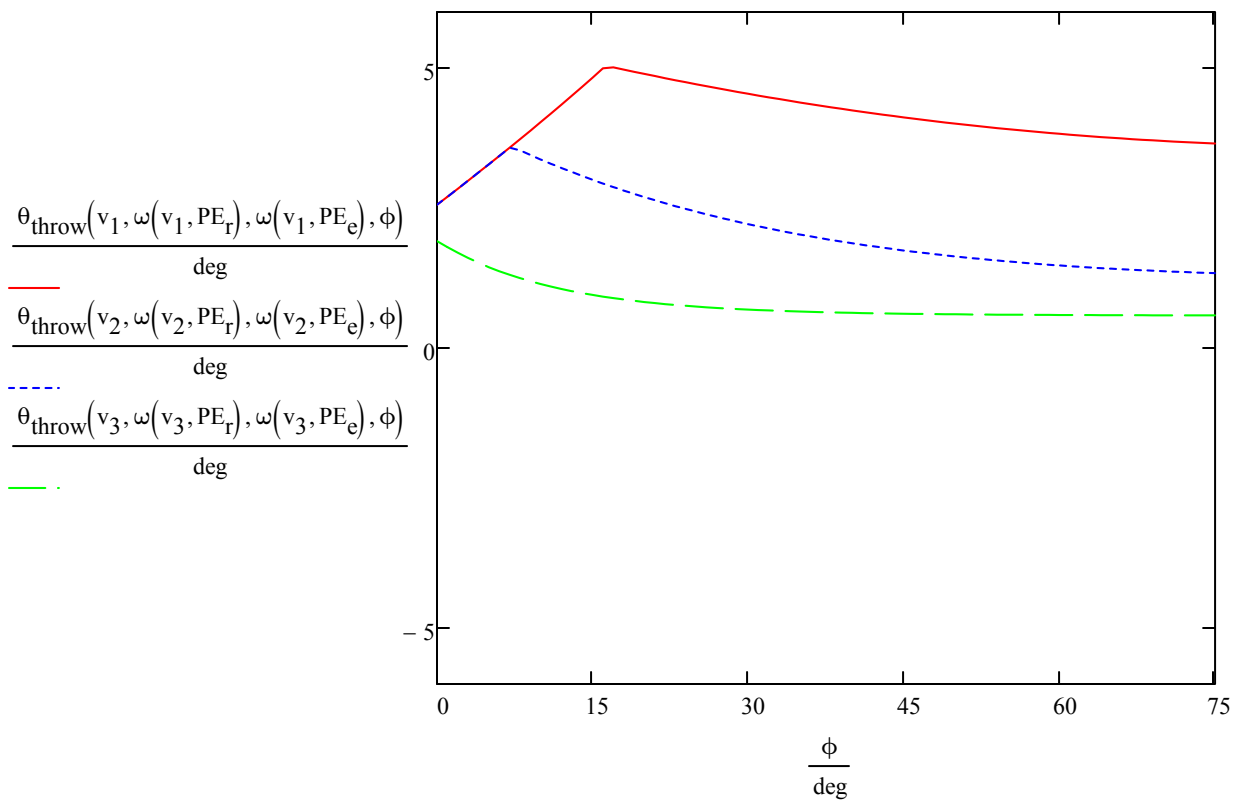
$PE_r := 0\%$        $PE_e := 75\%$



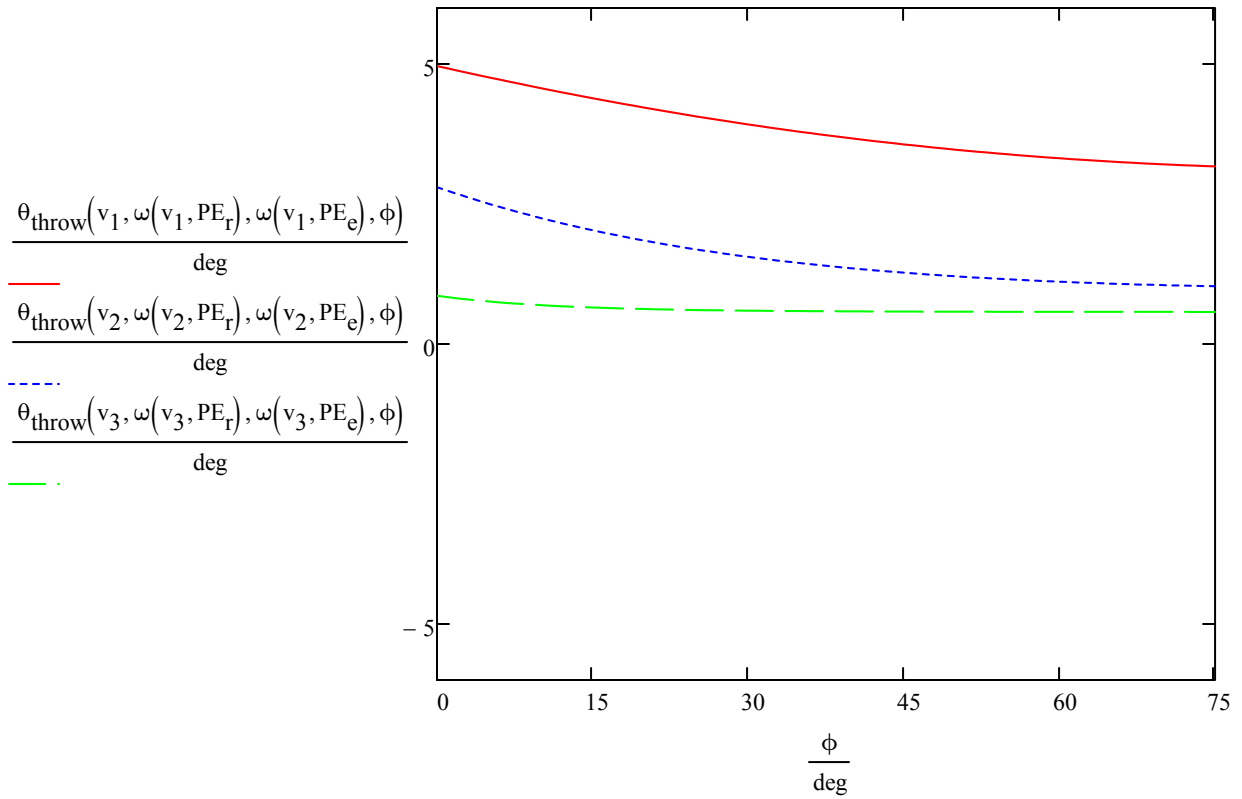
$PE_r := 0\%$        $PE_e := 100\%$



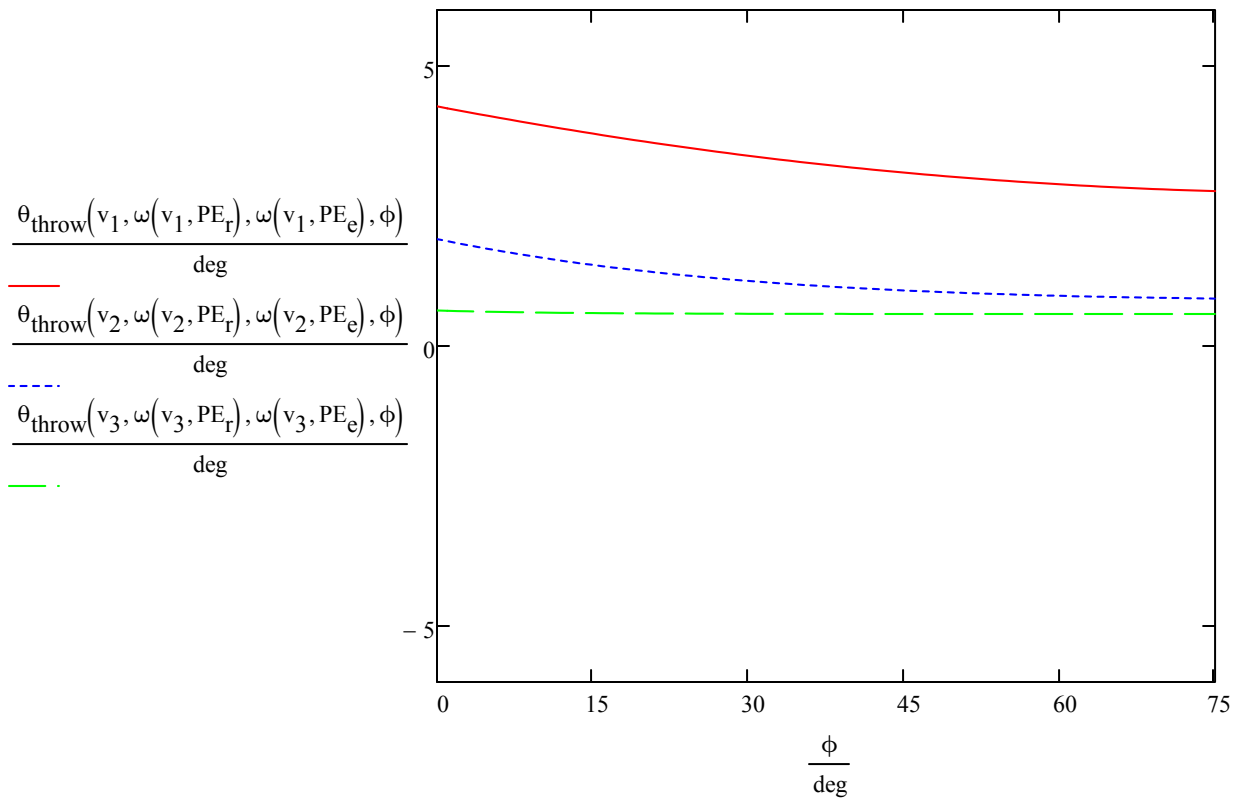
$PE_r := 0\%$        $PE_e := -25\%$



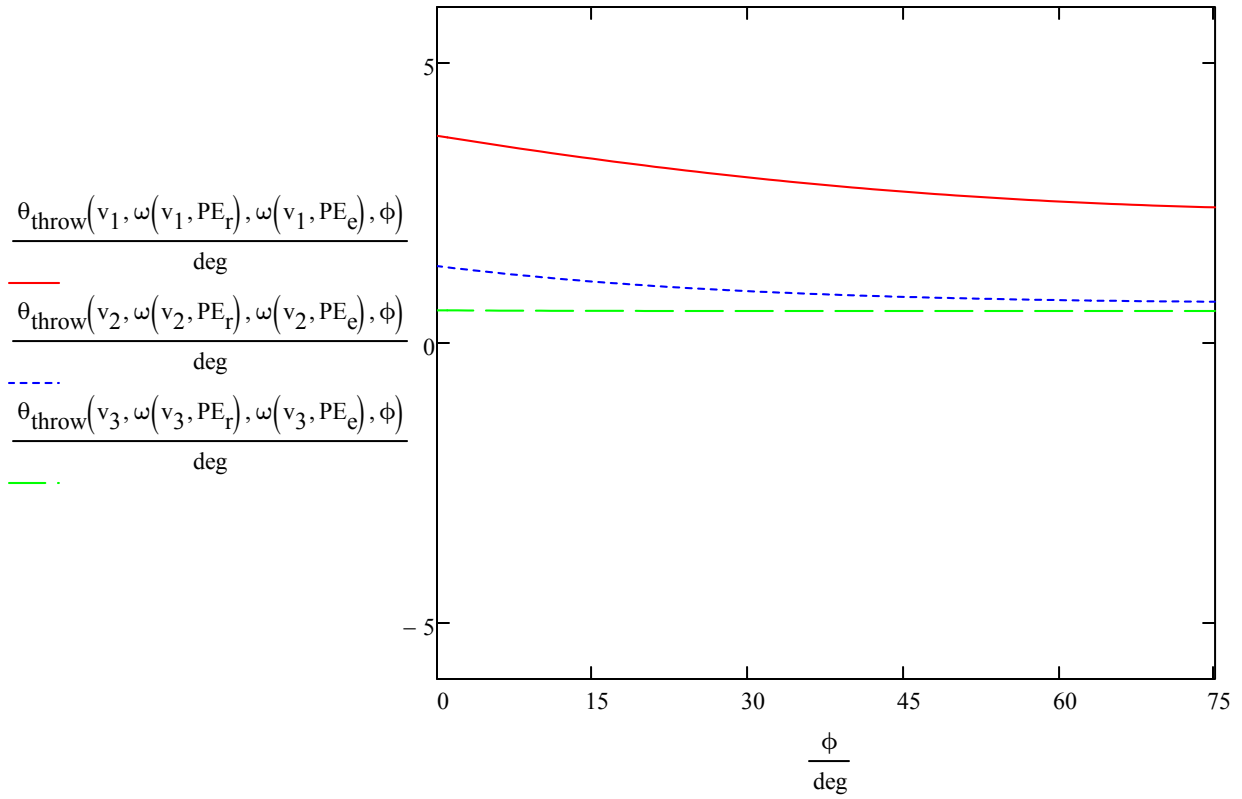
$\underline{PE}_r := 0\%$       $\underline{PE}_e := -50\%$



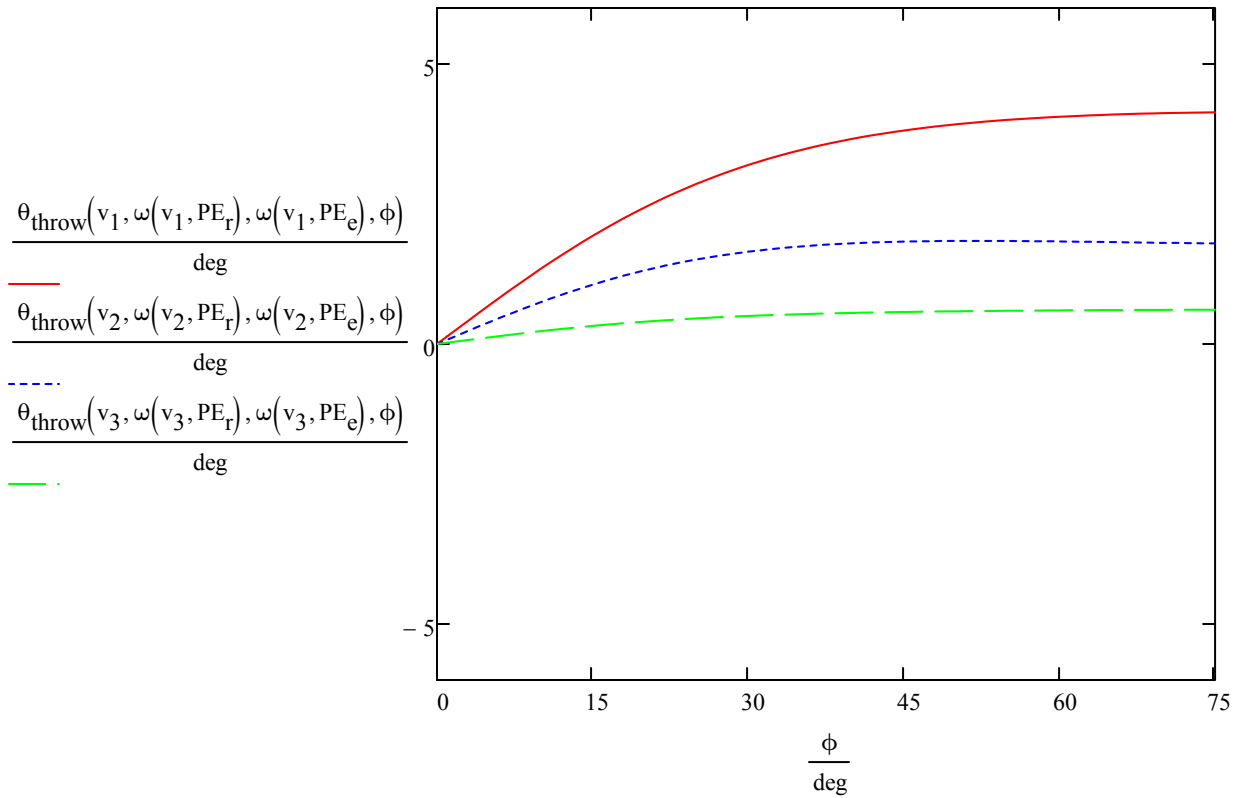
$\underline{PE}_r := 0\%$       $\underline{PE}_e := -75\%$



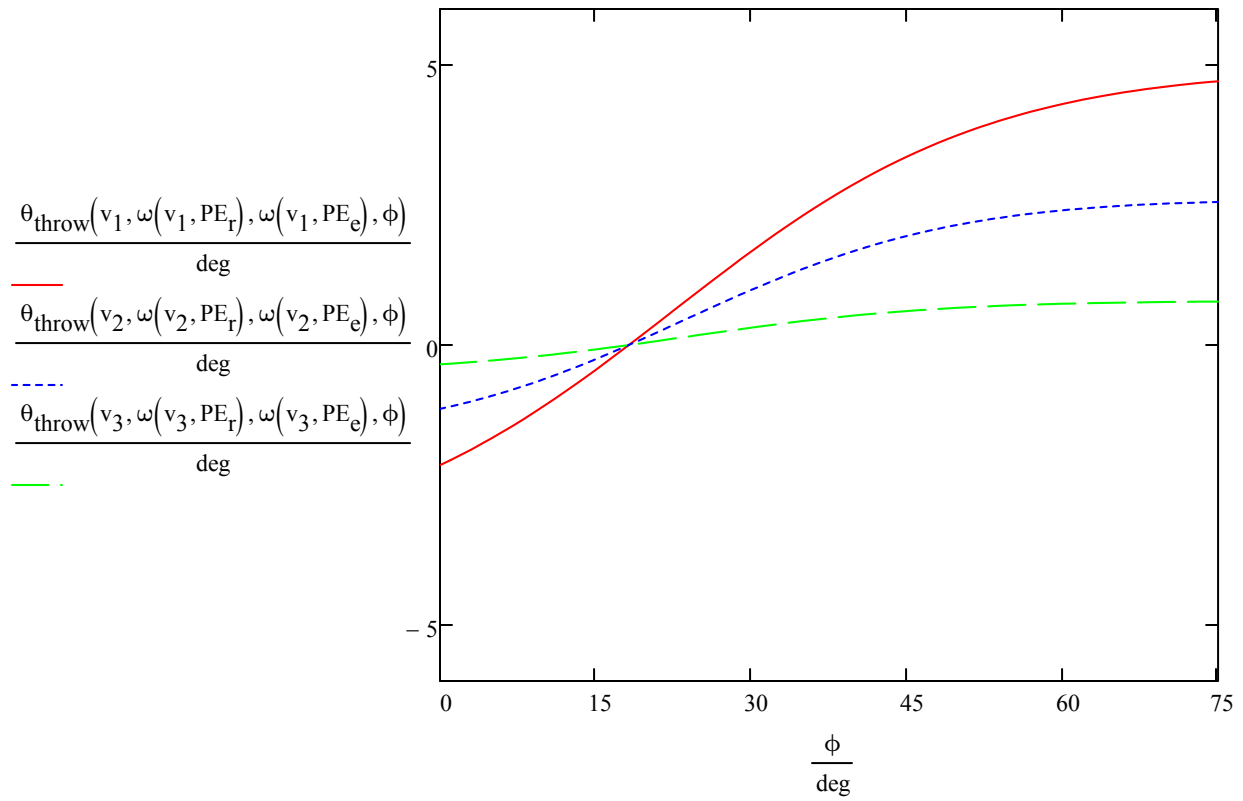
$PE_r := 0\%$       $PE_e := -100\%$



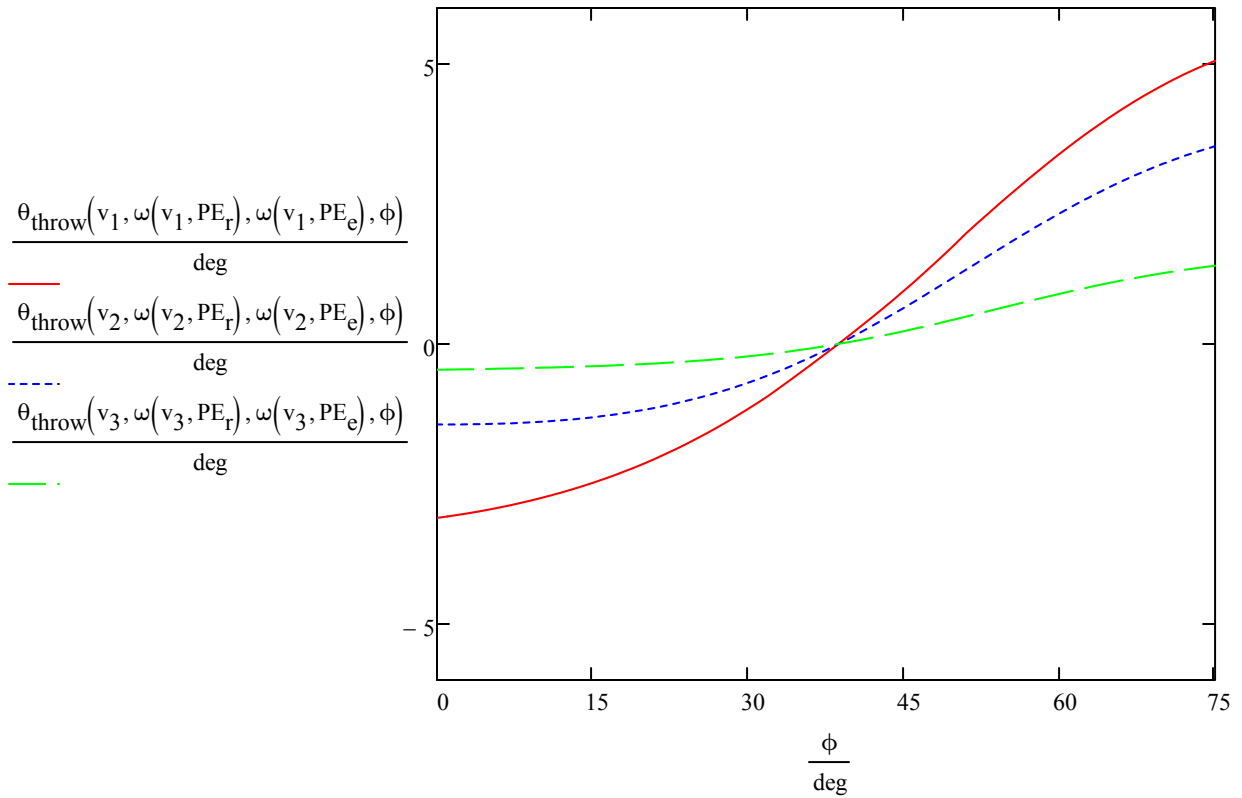
$PE_r := 50\%$       $PE_e := 0\%$



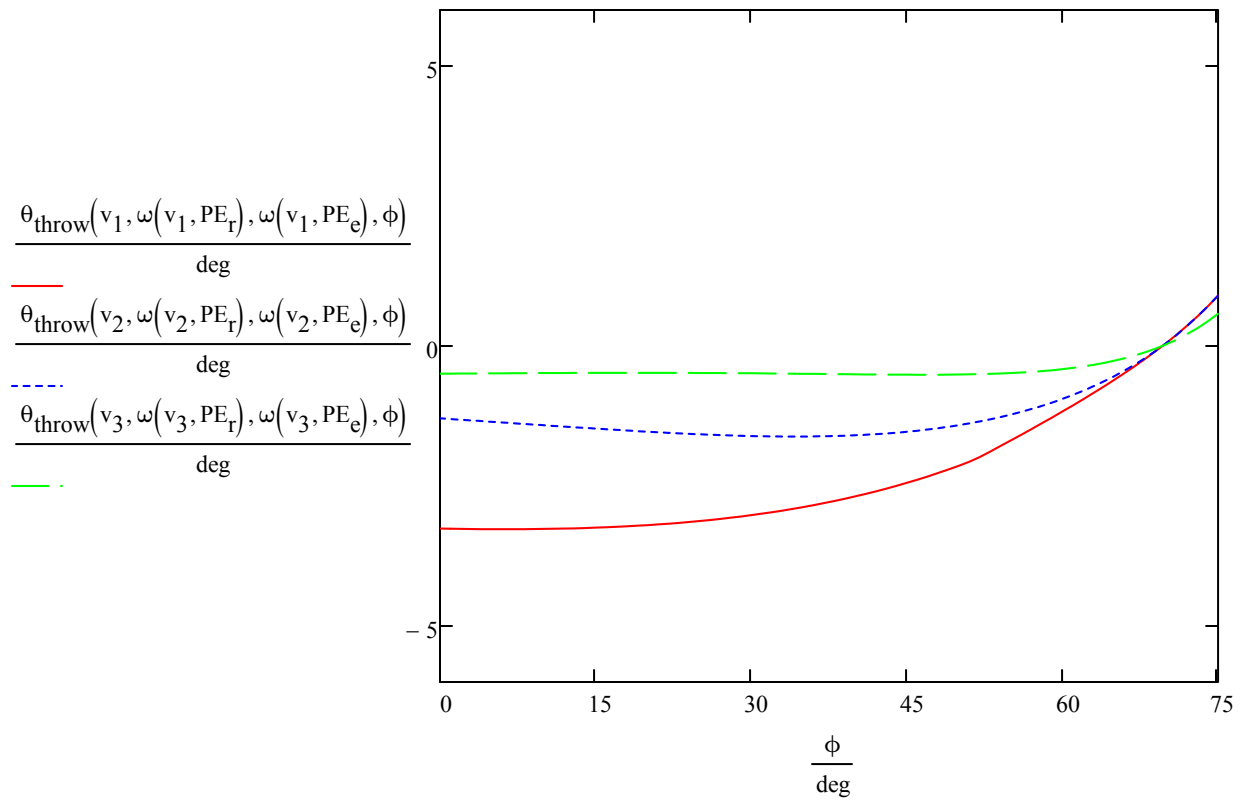
$\overline{PE}_r := 50\%$      $\overline{PE}_e := 25\%$



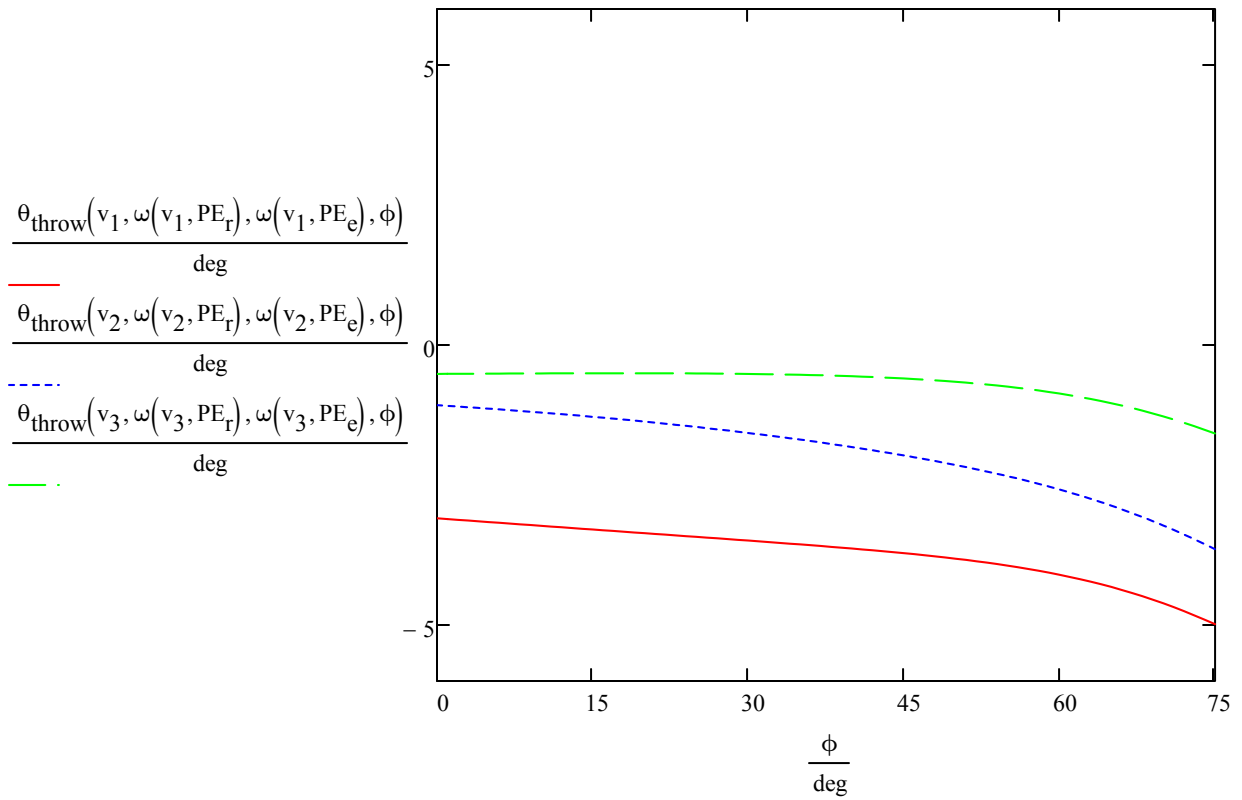
$\overline{PE}_r := 50\%$      $\overline{PE}_e := 50\%$



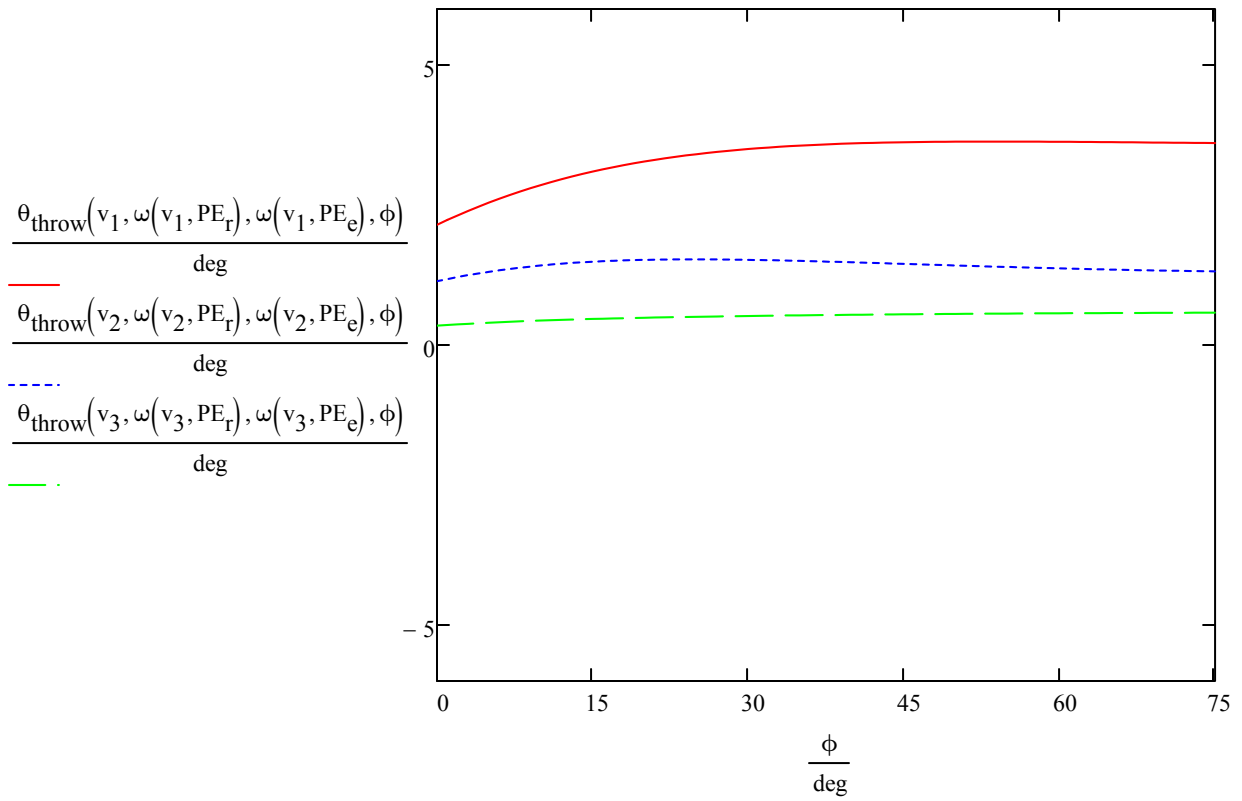
$\overline{PE}_r := 50\%$      $\overline{PE}_e := 75\%$



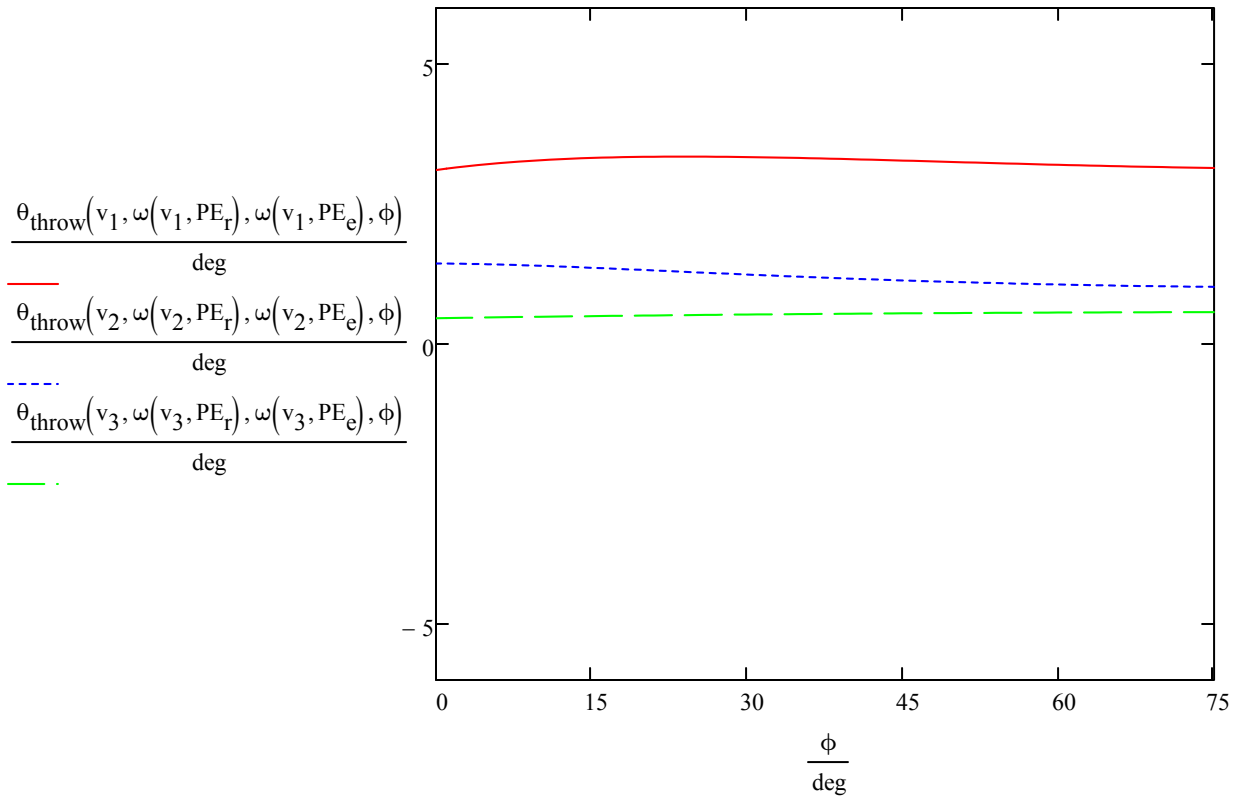
$\overline{PE}_r := 50\%$      $\overline{PE}_e := 100\%$



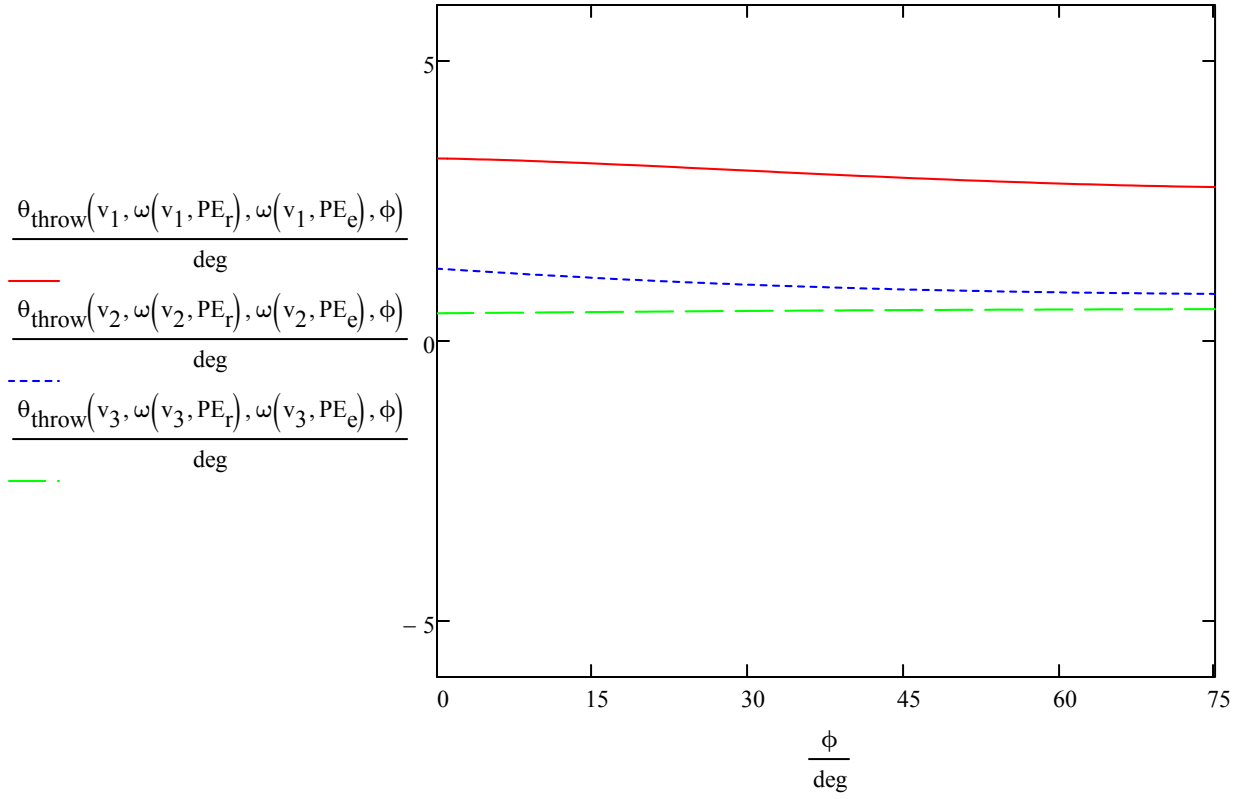
$\overline{PE}_r := 50\%$      $\overline{PE}_e := -25\%$



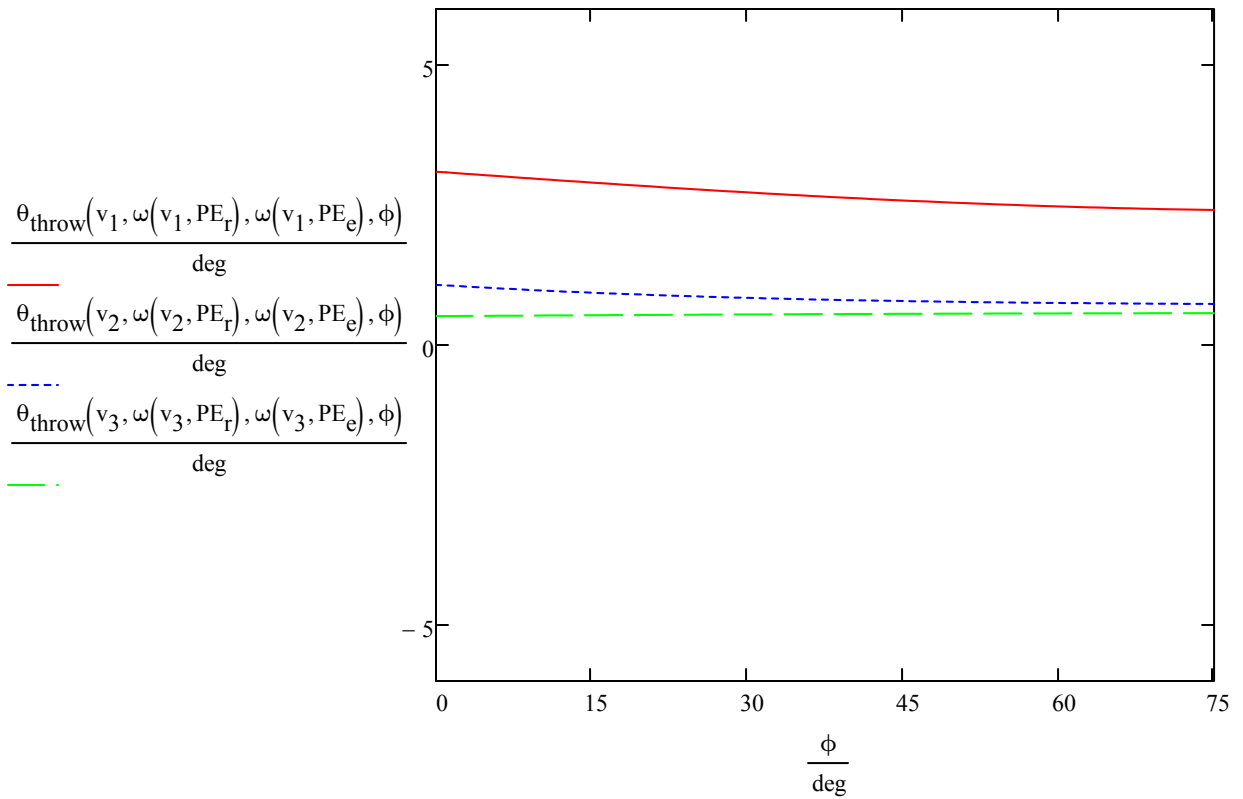
$\overline{PE}_r := 50\%$      $\overline{PE}_e := -50\%$



$\overline{PE}_r := 50\%$      $\overline{PE}_e := -75\%$

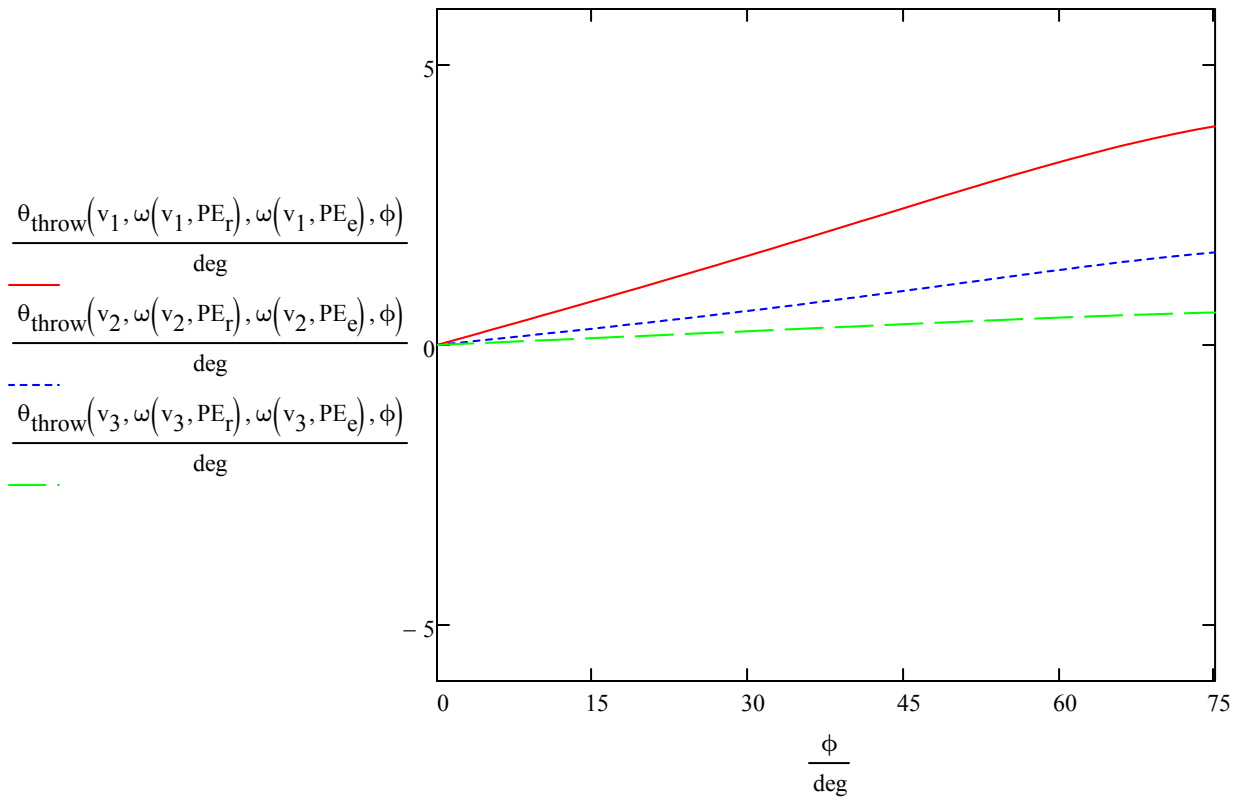


$\overline{PE}_r := 50\%$      $\overline{PE}_e := -100\%$

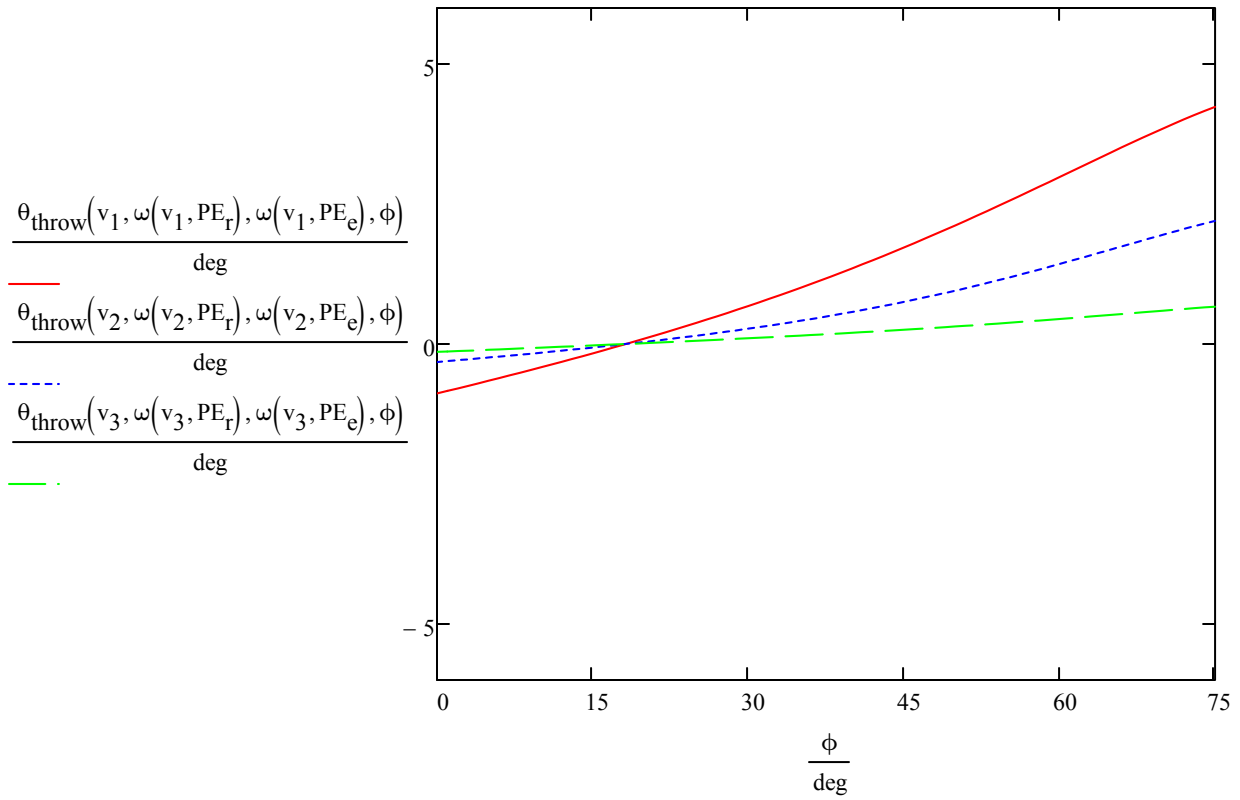




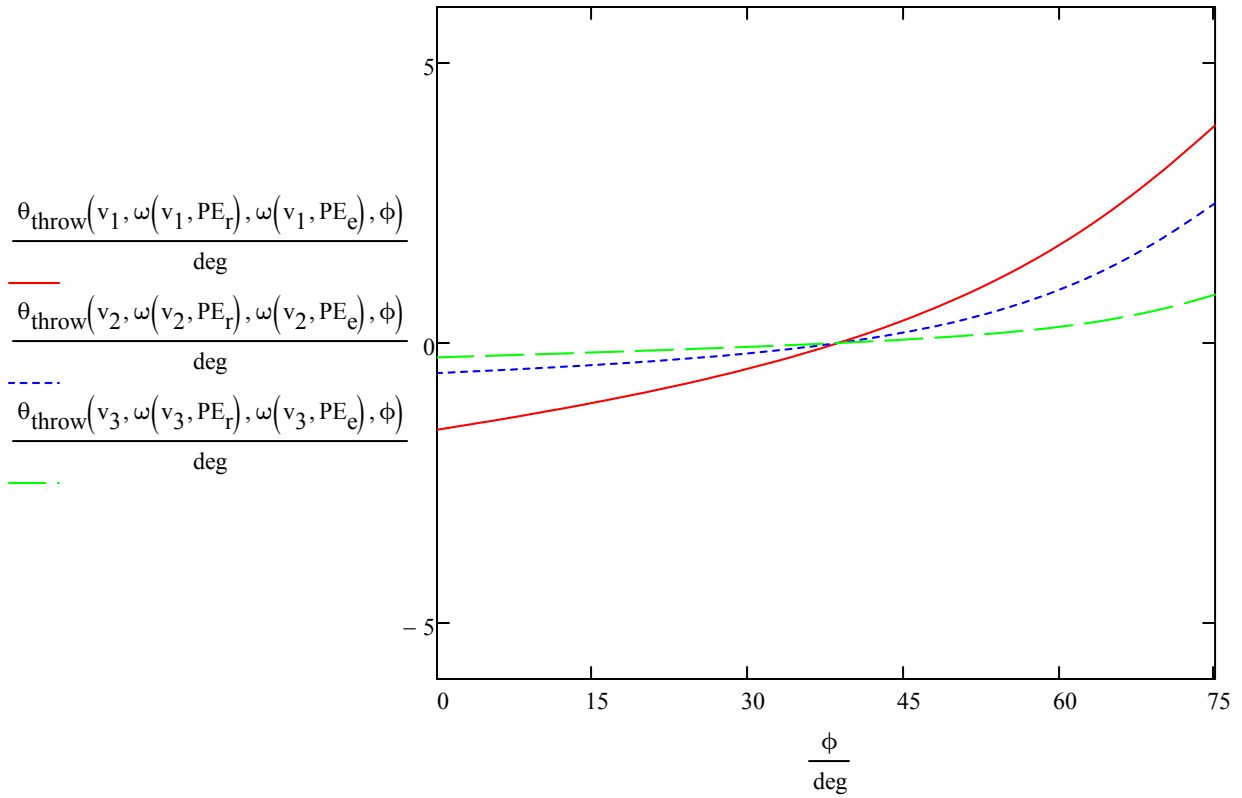
$\underline{PE}_r := 100\%$     $\underline{PE}_e := 0\%$



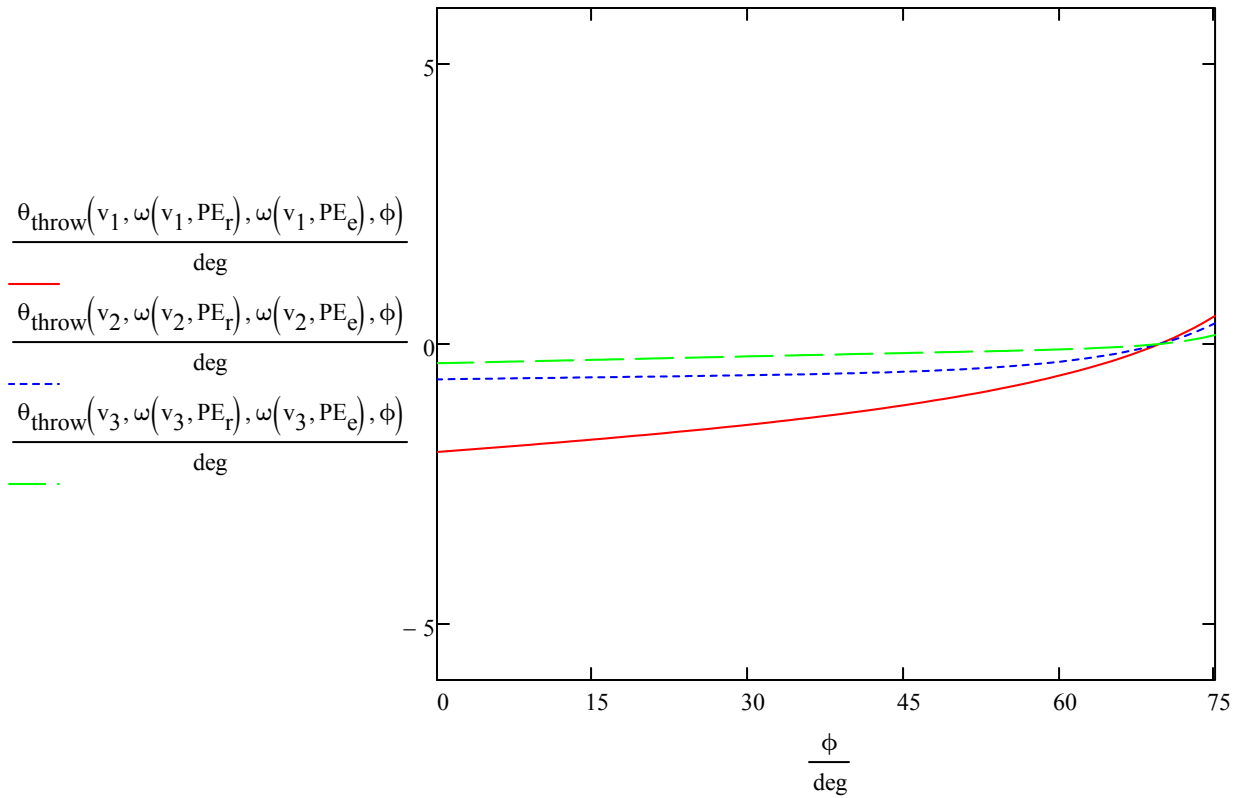
$\underline{PE}_r := 100\%$     $\underline{PE}_e := 25\%$



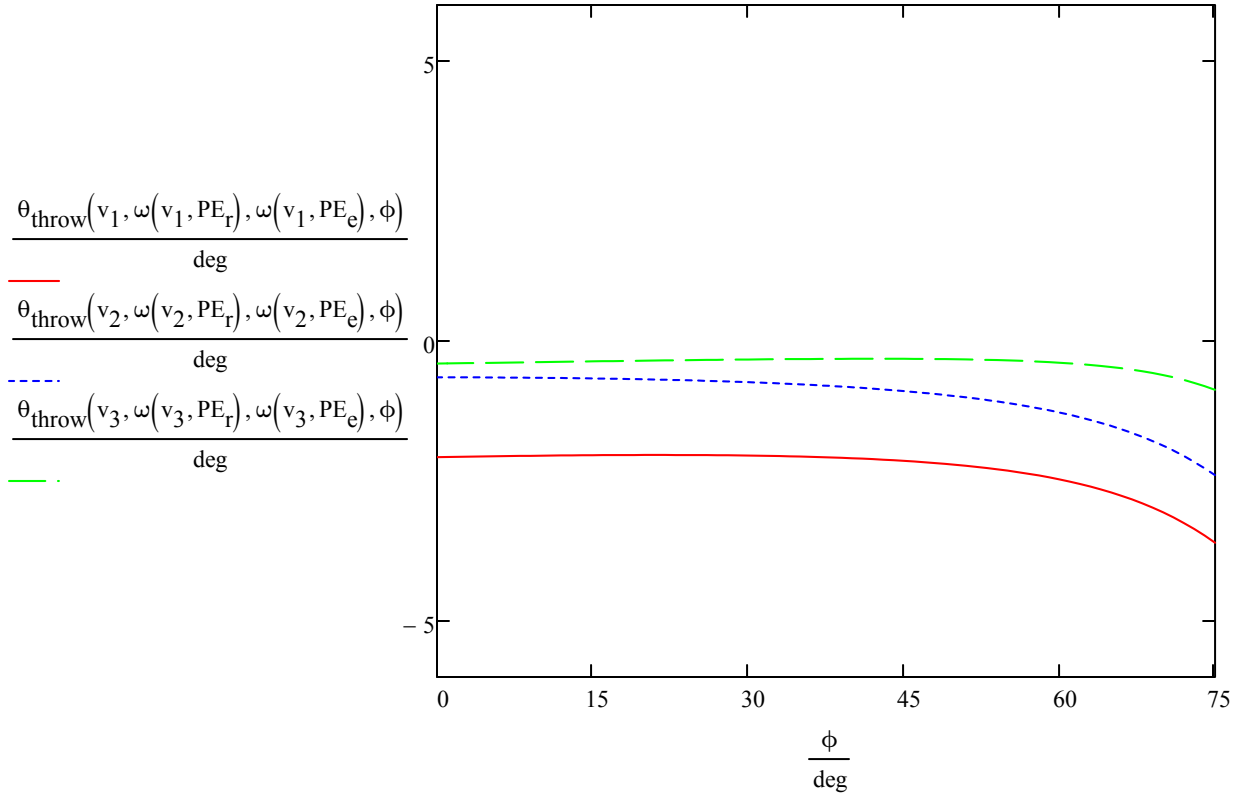
$PE_r := 100\%$     $PE_e := 50\%$



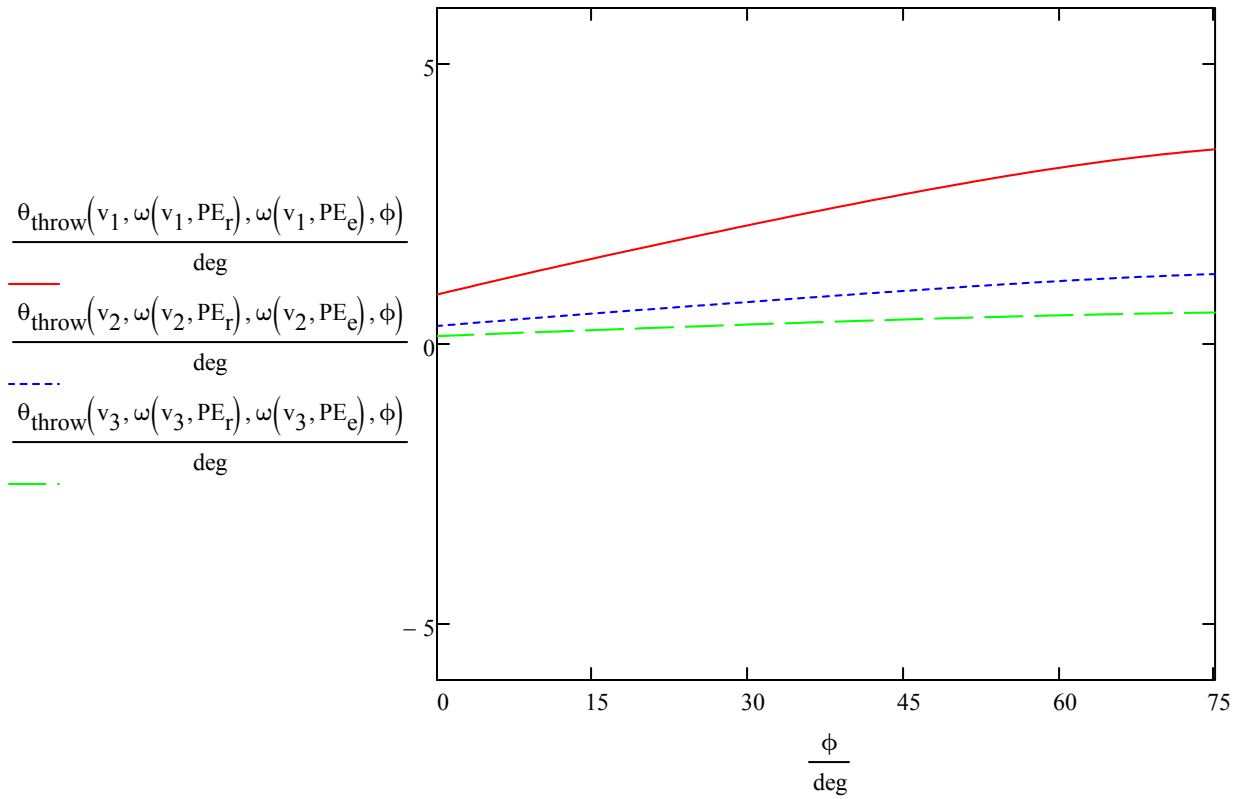
$PE_r := 100\%$     $PE_e := 75\%$



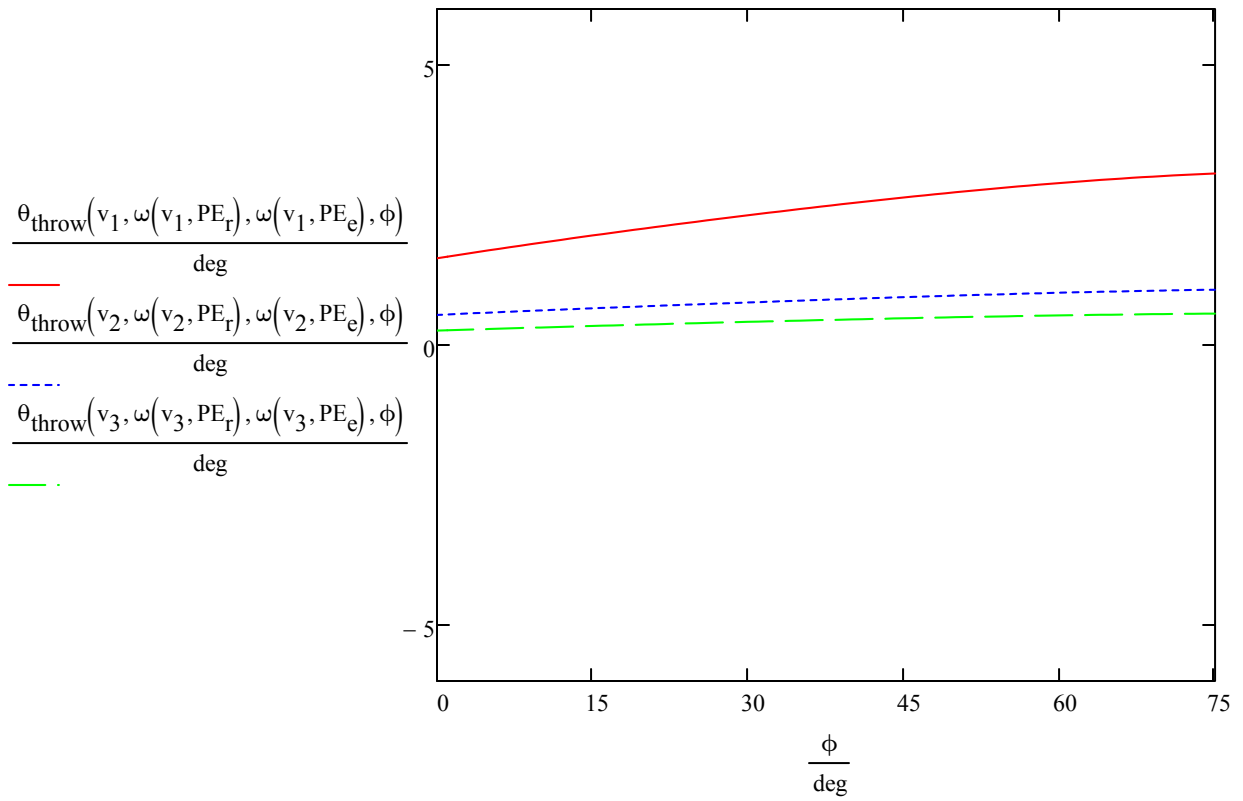
$PE_r := 100\%$     $PE_e := 100\%$



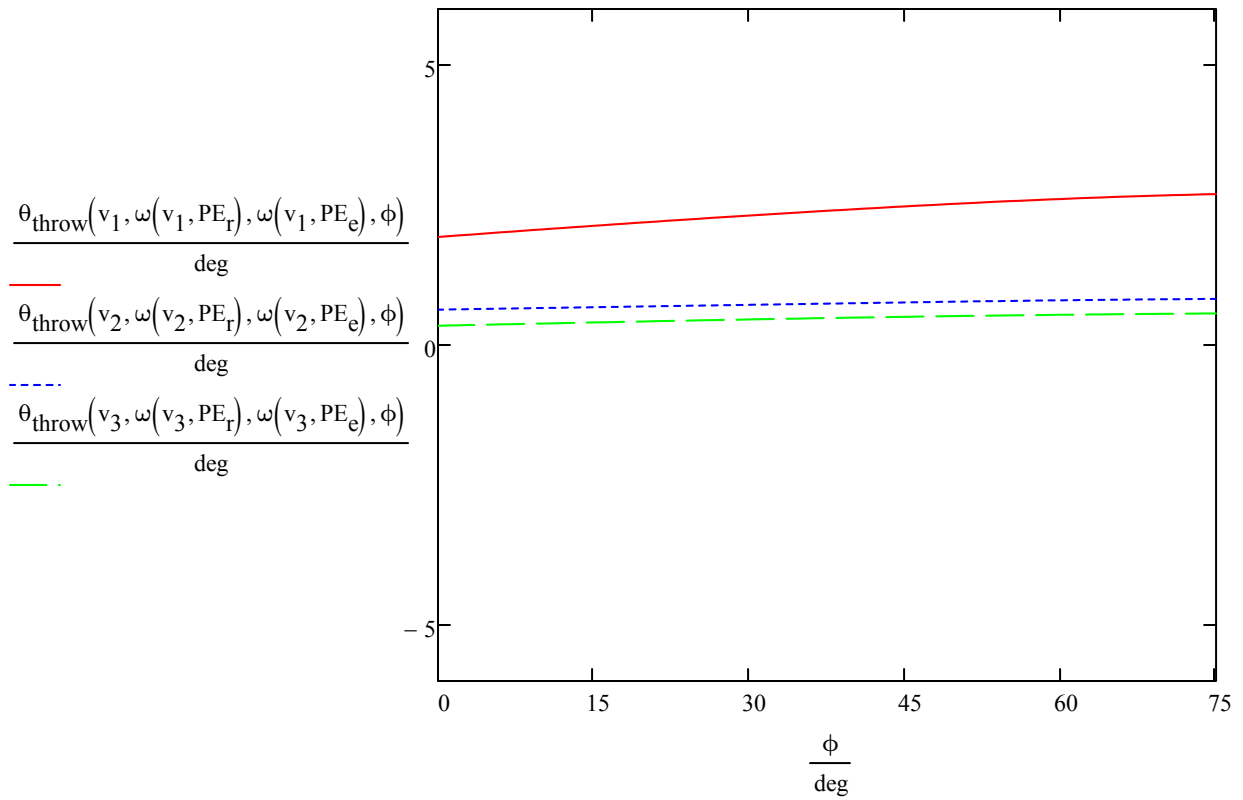
$PE_r := 100\%$     $PE_e := -25\%$



$\underline{\underline{PE_r}} := 100\%$     $\underline{\underline{PE_e}} := -50\%$



$\underline{\underline{PE_r}} := 100\%$     $\underline{\underline{PE_e}} := -75\%$



$\overline{PE}_r := 100\%$     $\overline{PE}_e := -100\%$

