
technical proof


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TP A.7
The effects of English on the 90° rule

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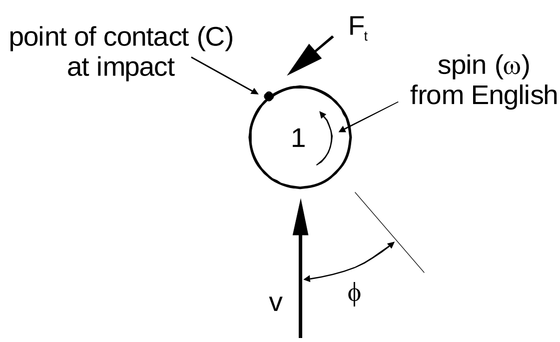
This technical proof is based on the background presented in TP A.5, assuming a perfect coefficient of restitution (e = 1).

From Equations 7 and 9, the normal components of the post-impact velocities are:

$$v_{1n}' = 0 \qquad\qquad v_{2n}' = v_1 \cos(\phi)$$

The direction of the tangential impulse depends on the direction and amount of English, which affect the relative speed of the point of contact at impact (see the figure below). The speed of this point (C) in the tangential direction is given by:

$$v_{Ct} = v_1 \sin(\phi) - \omega R$$



The tangential impulse is in the direction shown in the figure if $v_{Ct} > 0$ (e.g., with inside English or no English), is in the opposite direction if $v_{Ct} < 0$ (e.g., with extreme outside English), and is zero if $v_{Ct} = 0$ (when outside English exactly cancels the tangential sliding motion). Therefore, from Equations 11 and 12, assuming maximum friction is acting (se TP A.5 and TP A.14 for more information), the tangential components of the post-impact velocities are:

$$v_{2t}' = \sigma \mu v_1 \cos(\phi) \qquad\qquad v_{1t}' = v_1\left(\sin(\phi) - \sigma\mu \cos(\phi)\right)$$

where the multiplier σ is defined based on the sign of v_{Ct} according to:

$$\sigma = \begin{cases} +1 & v_{Ct} > 0 \\ 0 & v_{Ct} = 0 \\ -1 & v_{Ct} < 0 \end{cases}$$

From Equations 13 and 14, the ball trajectory angles after impact are:

$$\theta_1 = \tan^{-1}\left(\frac{v_{1n}'}{v_{1t}'}\right) = 0 \qquad \theta_2 = \tan^{-1}\left(\frac{v_{2t}'}{v_{2n}'}\right) = \tan^{-1}(\sigma\mu)$$

so the final angle between the balls, from Equation 15, is:

$$\theta = 90° - \theta_1 - \theta_2 = 90° - \tan^{-1}(\sigma\mu)$$

Here are some example values:

$\mu := 0.06$ (average coefficient of friction between balls)

$\theta_2(\sigma,\mu) := \mathrm{atan}(\sigma\cdot\mu)$ $\qquad$ $\theta(\sigma,\mu) := 90\cdot\mathrm{deg} - \theta_2(\sigma,\mu)$

For a shot with inside English or no English (σ = 1),

$\theta_2(1,\mu) = 3.434\,\mathrm{deg}$ $\qquad$ $\theta(1,\mu) = 86.566\,\mathrm{deg}$

the object ball angle is shortened (i.e., the object ball is thrown forward).

For a shot with just enough outside English to create "gearing" (no sliding) at contact (i.e., σ = 0),

$\theta_2(0,\mu) = 0\,\mathrm{deg}$ $\qquad$ $\theta(0,\mu) = 90\,\mathrm{deg}$

the 90 degree rule applies exactly.

For a shot with an excess of outside English (σ = -1),

$\theta_2(-1,\mu) = -3.434\,\mathrm{deg}$ $\qquad$ $\theta(-1,\mu) = 93.434\,\mathrm{deg}$

the object ball angle is lengthened (i.e., the object ball is thrown backward).