**TP B.10**

**Draw shot cue elevation effects**

supporting:
“The Illustrated Principles of Pool and Billiards”
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a) cue stick impact

b) CB bounce
The data and equations on this page are from TP B.9.

Relevant physical parameters:

\[ R := 1.125 \text{ in} \quad \text{ball radius} \]
\[ b_{\text{max}} := \frac{R}{2} \quad \text{generally accepted miscue limit} \]
\[ \mu_s := 0.2 \quad \text{typical ball-cloth coefficient of sliding friction} \]
\[ m_r := \frac{6}{19} \quad \text{typical ball-mass-to-cue-mass ratio (m_b/m_s)} \]
\[ \eta := 0.87 \quad \text{typical cue tip efficiency} \]
\[ e_t := 0.5 \quad \text{typical ball-table coefficient of restitution} \]
\[ \text{rps} := \frac{\text{rpm-min}}{\text{sec}} \quad \text{revs per second} \]
\[ v_s := 12 \text{ mph} \quad \text{typical fast cue speed} \]

Speed and spin loss, and spin ratio, for a sliding CB (e.g., with a level-cue draw shot):

\[
\begin{align*}
\tag{1}
\nu_{\text{drag}}(v, x) &:= \sqrt{v^2 - 2 \mu_s g x} & \omega_{\text{drag}}(v, \omega, x) &:= \omega - \frac{5}{2R} \left(v - \sqrt{v^2 - 2 \mu_s g x}\right) \\
\text{drag ratio}(v, \omega, x) &:= \frac{\omega_{\text{drag}}(v, \omega, x) \cdot R}{\nu_{\text{drag}}(v, x)}
\end{align*}
\]

CB speed and spin from cue speed \((v_s)\) and tip offset \((b)\):

\[
\begin{align*}
\nu_{\text{CB}}(v_s, b) &:= v_s \cdot \sqrt{\frac{1 + \eta - \frac{1 - \eta}{m_r} \left[1 + \frac{5}{2} \left(\frac{b}{R}\right)^2\right]}{1 + m_r + \frac{5}{2} \left(\frac{b}{R}\right)^2}} \\
\omega_{\text{CB}}(v_s, b) &:= \frac{5}{2} v_{\text{CB}}(v_s, b) \cdot \frac{b}{R^2}
\end{align*}
\]

Typical fast cue speed with near-maximum tip offset:

\[
\begin{align*}
\nu_s &:= 15 \text{ mph} \\
\frac{b}{b_{\text{max}}} &:= 0.563 \text{ in}
\end{align*}
\]

Cue ball (CB) speed, spin, and angle after tip impact:

\[
\begin{align*}
\nu_0 &:= \nu_{\text{CB}}(v_s, b) = 11.194 \text{ mph} \\
\omega_0 &:= \omega_{\text{CB}}(v_s, b) = 34.841 \text{ rps}
\end{align*}
\]

NOTE - I will assume the cue tip delivers most of its impulse to the CB before any significant force build up between the CB and the table (due to cloth compression and rebound off the slate). This assumption is supported by HSV B.44 for modest cue elevations.
On each bounce of the CB, including the first bounce after tip impact, the ball speed, spin, and angle change according to impact-momentum principles. See the illustrations on the 1st page for the terminology used.

The normal and tangential components of the incoming velocity are:

\[ v_{n_i} = v_i \sin(\theta_i) \quad \quad \quad v_{t_i} = v_i \cos(\theta_i) \]

After the bounce, the normal component is reduced according to the coefficient of restitution between the ball and table:

\[ v_{n_{i+1}} = e_t \cdot v_{n_i} = e_t \cdot v_i \sin(\theta_i) \]

The normal impulse corresponding to this momentum change is:

\[ F'_n = m_b (v_{n_i} + v_{n_{i+1}}) = m_b \left( 1 + e_t \right) \sin(\theta_i) v_i \]

Assuming the ball still has backspin after rebound (i.e., \( \omega_{i+1} > 0 \)), the tangential impulse will be:

\[ F'_t = \mu_s \cdot F'_n = \mu_s \cdot m_b \left( 1 + e_t \right) \sin(\theta_i) v_i \]

This changes the tangential momentum, resulting in a post-rebound tangential speed of:

\[ v_{t_{i+1}} = v_{t_i} - \frac{F'_t}{m_b} = v_i \left[ \cos(\theta_i) - \mu_s \left( 1 + e_t \right) \sin(\theta_i) \right] \]

The total post-rebound speed is:

\[ v_{i+1} = \sqrt{\left( v_{n_{i+1}} \right)^2 + \left( v_{t_{i+1}} \right)^2} = v_i \sqrt{\left( e_t \sin(\theta_i) \right)^2 + \left[ \cos(\theta_i) - \mu_s \left( 1 + e_t \right) \sin(\theta_i) \right]^2} \]

The tangential impulse also changes the angular momentum, resulting in a post-rebound spin of:

\[ \omega_{i+1} = \omega_i - \frac{F'_t \cdot R}{I} = \omega_i - \frac{5}{2 \cdot m_b R^2} = \omega_i - \frac{5}{2} \frac{1}{\mu_s \left( 1 + e_t \right) \sin(\theta_i)} v_i \]

The trajectory angle after the bounce is:

\[ \theta_{i+1} = \arctan \left( \frac{v_{n_{i+1}}}{v_{t_{i+1}}} \right) = \arctan \left[ \frac{e_t \sin(\theta_i)}{\cos(\theta_i) - \mu_s \left( 1 + e_t \right) \sin(\theta_i)} \right] = \arctan \left[ \frac{e_t}{\cot(\theta_i) - \mu_s \left( 1 + e_t \right)} \right] \]

From basic projectile motion, the distance covered in the air after the bounce is:

\[ d_{i+1} = \frac{\left( v_{i+1} \right)^2}{g} \cdot \sin(2 \cdot \theta_{i+1}) \]
Here's a program to calculate the CB spin and spin ratio at OB contact for a given cue elevation and drag distance:

\[
\text{elev}_\text{draw}_\text{shot}(\theta_s, d_{\text{drag}}) := \\
\begin{align*}
\text{d}_{\text{total}} &\leftarrow 0 \\
\theta_0 &\leftarrow \theta_s \\
i &\leftarrow 0 \\
\text{while } (\text{d}_{\text{total}} < d_{\text{drag}}) \\
\begin{cases}
\text{"the CB hasn't reached the OB yet"} \\
\text{if } (\theta_i = 0) \\
\text{"drag with no more bounces"} \\
x &\leftarrow d_{\text{drag}} - d_{\text{total}} \\
d_{i+1} &\leftarrow x \\
v_{i+1} &\leftarrow v_{\text{drag}}(v_i, x) \\
\omega_{i+1} &\leftarrow \omega_{\text{drag}}(v_i, \omega_i, x) \\
\theta_{i+1} &\leftarrow 0 \\
i &\leftarrow i + 1 \\
\text{break}
\end{cases} \\
\text{"otherwise, the CB will bounce"} \\
v_{n_{i+1}} &\leftarrow e_t \cdot v_i \cdot \sin(\theta_i) \\
v_{t_{i+1}} &\leftarrow v_i \cdot \left[ \cos(\theta_i) - \mu_s(1 + e_t) \cdot \sin(\theta_i) \right] \\
v_{i+1} &\leftarrow \sqrt{(v_{n_{i+1}})^2 + (v_{t_{i+1}})^2} \\
\omega_{i+1} &\leftarrow \omega_i - \frac{5}{2} \cdot \mu_s(1 + e_t) \cdot \sin(\theta_i) \cdot v_i \\
\text{if } \left[ (v_{i+1} < 0) \lor (\omega_{i+1} < 0) \right] \\
\text{"CB doesn't make it to the OB with draw"} \\
v_{i+1} &\leftarrow 0 \\
\omega_{i+1} &\leftarrow 0 \\
i &\leftarrow i + 1 \\
\text{break}
\end{cases} \\
\theta_{i+1} &\leftarrow \arctan \left[ \frac{e_t}{\cos(\theta_i) - \mu_s(1 + e_t)} \right] \\
d_{i+1} &\leftarrow \frac{(v_{i+1})^2}{g} \cdot \sin(2 \cdot \theta_{i+1}) \\
d_{\text{total}} &\leftarrow d_{\text{total}} + d_{i+1}
\end{align*}
\]
if \((d_{i+1} < 0.1\text{-in}) \land (\theta_{i+1} < 1\text{-deg})\)

"assume the bouncing ceases"

\[
\theta_{i+1} \leftarrow 0
\]

\[i \leftarrow i + 1\]

\[
\text{spin} \leftarrow \omega_i
\]

\[
\text{ratio} \leftarrow \frac{\omega_i \cdot R}{v_i \cdot \cos(\theta_i)}
\]

"return all calculated data"

\[
\left( i \quad d \quad v \quad \omega \quad \theta \quad \text{spin} \quad \text{ratio} \right)
\]

Functions to extract spin and spin-to-speed ratio from the program results:

\[
\text{spin}(\theta_s, d_{\text{drag}}) := \text{elev\_draw\_shot}(\theta_s, d_{\text{drag}}) \quad \text{ratio}(\theta_s, d_{\text{drag}}) := \text{elev\_draw\_shot}(\theta_s, d_{\text{drag}})
\]

For a level cue, the program results are the same as for a drag shot:

\[
\theta_s := 0\text{-deg} \quad d_{\text{drag}} := 2\text{-ft} \quad \text{spin}(\theta_s, d_{\text{drag}}) = 31.43 \quad \text{ratio}(\theta_s, d_{\text{drag}}) = 1.186
\]

\[
\omega_{\text{drag}}(v_0, \omega_0, d_{\text{drag}}) = 31.43\text{-rps} \quad \text{drag\_ratio}(v_0, \omega_0, d_{\text{drag}}) = 1.186
\]

Here are some example data at a typical cue elevation and medium drag distance:

\[
\theta := 5\text{-deg} \quad d := 2\text{-ft} \quad \text{results} := \text{elev\_draw\_shot}(\theta, d)
\]

\[
N := \text{results}_0 = 9 \quad \text{number of bounces}
\]

\[
db := \text{results}_1 \quad \text{distances between bounces (in feet)}
\]

\[
\text{db}_1 = 0.708 \quad \text{db}_2 = 0.349 \quad \text{db}_3 = 0.174 \quad \text{db}_4 = 0.086 \quad \text{db}_5 = 0.043
\]

\[
v_b := \text{results}_2 \quad \text{ball speed after each bounce (in mph)}
\]

\[
\text{vb}_1 = 10.87 \quad \text{vb}_2 = 10.715 \quad \text{vb}_3 = 10.64 \quad \text{vb}_4 = 10.603 \quad \text{vb}_5 = 10.585
\]

\[
\omega_b := \text{results}_3 \quad \text{ball spin after each bounce (in rps)}
\]

\[
\text{wb}_1 = 33.019 \quad \text{wb}_2 = 32.108 \quad \text{wb}_3 = 31.652 \quad \text{wb}_4 = 31.424 \quad \text{wb}_5 = 31.311
\]

\[
\theta_b := \text{results}_4 \quad \text{ball bounce angle (in deg)}
\]

\[
\theta_b_1 = 2.572 \quad \theta_b_2 = 1.304 \quad \theta_b_3 = 0.657 \quad \theta_b_4 = 0.33 \quad \theta_b_5 = 0.165
\]

\[
\text{results}_5 = 30.143 \quad \text{spin}(\theta_s, d_{\text{drag}}) = 30.143
\]

\[
\text{results}_6 = 1.164 \quad \text{ratio}(\theta_s, d_{\text{drag}}) = 1.164
\]
Example data at higher cue elevation:

\[ \begin{align*}
\theta_s &= 15\text{deg} & d_{\text{drag}} &= 2\text{ft} & \text{results} &= \text{elev\_draw\_shot}(\theta_s, d_{\text{drag}}) \\
N &= \text{results}_0 = 2 & \text{number of bounces} \\
db &= \text{results}_1 & \text{distances between bounces (in feet)} \\
db_1 &= 1.926 & db_2 &= 0.921 \\
v_b &= \text{results}_2 & \text{ball speed after each bounce (in mph)} \\
v_b_1 &= 10.049 & v_b_2 &= 9.537 \\
\omega_b &= \text{results}_3 & \text{ball spin after each bounce (in rps)} \\
\omega_b_1 &= 29.43 & \omega_b_2 &= 26.725 \\
\theta_b &= \text{results}_4 & \text{ball bounce angle (in deg)} \\
\theta_b_1 &= 8.289 & \theta_b_2 &= 4.356 \\
\text{results}_5 &= 26.725 & \text{spin}(\theta_s, d_{\text{drag}}) &= 26.725 \\
\text{results}_6 &= 1.129 & \text{ratio}(\theta_s, d_{\text{drag}}) &= 1.129
\end{align*} \]

Comparing elevated before-bounce and after-bounce spin and ratio to a level-cue drag shot:

original CB speed, spin, and spin ratio:

\[ \begin{align*}
v_0 &= 11.194\text{-mph} & \omega_0 &= 34.841\text{-rps} & \frac{\omega_0}{v_0} &= 1.25
\end{align*} \]

just before the second bounce:

\[ \begin{align*}
d_{\text{drag}} &= 1.925\text{-ft} \\
\text{level cue: } \omega_{\text{drag}}(v_0, \omega_0, d_{\text{drag}}) &= 31.561\text{-rps} \quad \frac{\omega_{\text{drag}}(v_0, \omega_0, d_{\text{drag}})}{v_0} &= 1.188 \\
\text{elevated cue: } \text{spin}(\theta_s, d_{\text{drag}}) &= 29.43 \quad \text{ratio}(\theta_s, d_{\text{drag}}) &= 1.189
\end{align*} \]

just after the second bounce:

\[ \begin{align*}
d_{\text{drag}} &= 1.927\text{-ft} \\
\text{level cue: } \omega_{\text{drag}}(v_0, \omega_0, d_{\text{drag}}) &= 31.558\text{-rps} \quad \frac{\omega_{\text{drag}}(v_0, \omega_0, d_{\text{drag}})}{v_0} &= 1.188 \\
\text{elevated cue: } \text{spin}(\theta_s, d_{\text{drag}}) &= 26.725 \quad \text{ratio}(\theta_s, d_{\text{drag}}) &= 1.129
Now let's look at how spin and spin ratio vary with drag distance for a typical cue elevation, as compared to a level-cue drag shot:

\[ \theta_s := 5\text{-deg} \]
\[ d_{drag} := 1\text{-ft, 1.1-ft.. 4-ft} \]

With a near level cue, there is little effect from the CB bouncing, and the CB doesn't bounce for very long (see the small spikes in the curves on the left side of the graphs).
Now let's look at how spin and spin ratio vary with drag distance for an elevated cue, as compared to a level-cue drag shot:

\[ \theta_s := 15\text{-deg} \]

\[ d_{\text{drag}} := 1\text{-ft}, 1.1\text{-ft} \ldots 4\text{-ft} \]

Notice how both the spin and the spin ratio take "hits" after each bounce, but don't change while the CB is in airborne between the bounces.
Now let's look at how spin and spin ratio vary with cue elevation, as compared to a level-cue drag shot:

\[ \theta_s := 0 \text{ deg}, 1 \text{ deg} \ldots 30 \text{ deg} \]

\[ d_{\text{drag}} := 2 \text{ ft} \]
The reasons for the spikes in the graphs above are related to the distances at which bounces occur for different cue elevations. The following data explain the two spikes in the curves:

\[ \theta_s := 9\text{-deg} \quad \text{results} := \text{elev\_draw\_shot}(\theta_s, d_{\text{drag}}) \quad \text{spin}(\theta_s, d_{\text{drag}}) = 29.118 \]
\[ N := \text{results}_0 = 3 \quad d := \text{results}_1 \quad d_1 + d_2 = 1.834 \quad \text{ratio}(\theta_s, d_{\text{drag}}) = 1.154 \]

\[ \theta_s := 10\text{-deg} \quad \text{results} := \text{elev\_draw\_shot}(\theta_s, d_{\text{drag}}) \quad \text{spin}(\theta_s, d_{\text{drag}}) = 29.396 \]
\[ N := \text{results}_0 = 2 \quad d := \text{results}_1 \quad d_1 + d_2 = 2.016 \quad \text{ratio}(\theta_s, d_{\text{drag}}) = 1.163 \]

\[ \theta_s := 15\text{-deg} \quad \text{results} := \text{elev\_draw\_shot}(\theta_s, d_{\text{drag}}) \quad \text{spin}(\theta_s, d_{\text{drag}}) = 26.725 \]
\[ N := \text{results}_0 = 2 \quad d := \text{results}_1 \quad d_1 = 1.926 \quad \text{ratio}(\theta_s, d_{\text{drag}}) = 1.129 \]

\[ \theta_s := 16\text{-deg} \quad \text{results} := \text{elev\_draw\_shot}(\theta_s, d_{\text{drag}}) \quad \text{spin}(\theta_s, d_{\text{drag}}) = 29.079 \]
\[ N := \text{results}_0 = 1 \quad d := \text{results}_1 \quad d_1 = 2.029 \quad \text{ratio}(\theta_s, d_{\text{drag}}) = 1.187 \]

**Conclusions from all of the analysis and graphs above:**

1. Elevating the cue reduces the amount of CB spin at OB contact, resulting in less draw distance (see TP B.8 for more info). The loss in spin is small for small cue elevations, but increases with more elevation (for a given cue speed and tip offset).

2. Modest cue elevations (about 0-15 degrees) reduce the spin-to-speed ratio of the CB at OB contact, resulting in “slower” draw (see TP B.9 for more info).

3. As you increase cue elevation above about 20 degrees, the spin-to-forward-speed ratio increases, allowing for “quicker” draw (see TP B.9 for more info). An extreme example is a highly elevated masse draw (pique) shot, where you create lots of backspin with very little forward speed.

Sometimes cue elevation is required to clear over an obstacle ball, or to prevent a double hit when there is a small gap between the CB and OB. And as noted above, with larger cue elevations, better "quick draw" action can result. However, **for maximum draw distance, a level cue (or as close to level is possible) appears to be best.**