**TP B.14**

**Draw shot cue ball angle approximations**

supporting:
“The Illustrated Principles of Pool and Billiards”
http://billiards.colostate.edu
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Relevant physical constants and parameters

\[ e := 0.95 \] typical coefficient of restitution between balls

\[ D := \frac{2.25 \text{ in}}{m} \] \[ R := \frac{D}{2} \] ball dimensions, converter to meters

\[ v := \frac{5 \text{ mph}}{m/s} \] typical slow-medium CB speed

\[ \varphi := 0 \text{ deg.} \ldots 90 \text{ deg} \] cut angle range

\[ f := 0, 0.01 \ldots 1 \] ball-hit fraction range

From TP A.23, cut angle (\( \phi \)) and ball-hit fraction (\( f \)) are related according to:

\[ \phi(f) := \sin(1 - f) \]

\[ f_{\phi}(\varphi) := 1 - \sin(\varphi) \]

From TP A.4, the final cue ball deflection angle for any shot is:

\[ \theta_c = \tan^{-1}\left( \frac{5v \sin(\phi) \cos(\phi)}{5v \sin^2(\phi) - 2R \omega} \right) \]

From TP A.20, the amount of backspin for a typical good action draw shot is:

\[ \omega = 0.625 \cdot \frac{v}{R} \]

giving a CB deflection angle of:

\[ \theta_c = \tan^{-1}\left( \frac{\sin(\phi) \cos(\phi)}{\sin^2(\phi) - 1/4} \right) \]

Therefore, the angle between the original CB aiming and the draw line is:

\[ \theta_{\text{actual}}(\varphi) := 180 \text{ deg} - \tan^{-1}\left( \frac{\sin(\varphi)^2 - \frac{1}{4}, \sin(\varphi) \cdot \cos(\varphi)}{\sin(\varphi) \cdot \cos(\varphi)} \right) \]
From TP A.20, here are two approximations that apply to certain ranges of draw shots:

Double-bisect system (double the cut angle and double the total angle again)

$$\theta_{\text{full}}(\varphi) := 4 \cdot \varphi$$

Trisect system (triple the cut angle):

$$\theta_{\text{tri}}(\varphi) := 3 \cdot \varphi$$
The double-bisect system approximates the draw angle very well up to about a 20 degree cut (~5/8-ball hit):

\[ \varphi_{\text{full}} := 0 \text{-deg}, 1 \text{-deg}, 20 \text{-deg} \]

\[ f_{\varphi}(20 \text{-deg}) = 0.658 \]

\[ \frac{5}{8} = 0.625 \]

The trisect system approximates the draw angle fairly well for all shots in the 0-40 degree cut range (i.e., greater than ~3/8-ball hit):

\[ \varphi_{\text{tri}} := 0 \text{-deg}, 1 \text{-deg}, 40 \text{-deg} \]

\[ f_{\varphi}(40 \text{-deg}) = 0.357 \]

\[ \frac{3}{8} = 0.375 \]

The slope of the draw-angle curve can be used to develop additional approximations:

\[ \text{slope}(\varphi, \varphi) := \frac{d}{d\varphi} \left( 180 \text{-deg} - \text{atan} \left( \frac{\sin(\varphi) \cdot \cos(\varphi)}{\sin(\varphi)^2 - \frac{1}{4}} \right) \right) \text{ simplify } \rightarrow \frac{8 \cdot \sin(\varphi)^2 + 4}{8 \cdot \sin(\varphi)^2 + 1} \]

At the important 1/2-ball hit (30 degree cut) benchmark, the slope is:

\[ \text{slope}(30 \text{-deg}) = 2 \]

so a good approximation for an average cut angle (close to 30 degrees) is:

\[ \theta_{\text{avg}}(\varphi) := 2 \cdot \varphi + 30 \text{-deg} \]

This approximation applies fairly well for cut angles in the 10-60 degree (~1/8-7/8 ball-hit fraction) range:

\[ \varphi_{\text{avg}} := 10 \text{-deg}, 11 \text{-deg}, 60 \text{-deg} \]

\[ f_{\varphi}(10 \text{-deg}) = 0.826 \]

\[ \frac{7}{8} = 0.875 \]

\[ f_{\varphi}(60 \text{-deg}) = 0.134 \]

\[ \frac{1}{8} = 0.125 \]

For a 90-degree cut, the slope of the draw-angle curve is:

\[ \text{slope}(90 \text{-deg}) = 1.333 \]

so a good approximation for a thin cut (close to 90 degrees) is:

\[ \theta_{\text{thin}}(\varphi) := \frac{4}{3} \cdot \varphi + 60 \text{-deg} \]

This approximation applies fairly well for cut angles in the 40-90 degree range (i.e., thinner than ~3/8-ball hit):

\[ \varphi_{\text{thin}} := 40 \text{-deg}, 41 \text{-deg}, 90 \text{-deg} \]

\[ f_{\varphi}(40 \text{-deg}) = 0.357 \]

\[ \frac{3}{8} = 0.375 \]
Here's a composite all of the approximations over their good ranges: