TP B.20
Peak forces and tip contact distance during a break shot

technical proof

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
by Dr. Dave Alciatore, PhD, PE ("Dr. Dave")

originally posted: 7/24/2016 last revision: 1/8/2018

$m_b := 6 \cdot oz$ ball mass

$m_s := 19 \cdot oz$ typical cue mass

$m_{rack} := \frac{3}{2} \cdot m_b$ effective mass of a rack of balls
from: http://billiards.colostate.edu/high_speed_videos/new/HSVB-45.htm

$v_b := 25 \cdot mph$ ball speed after impact (for a fast-speed break shot)

$\Delta t_{tip} := 0.0005 \cdot s$ typical tip-ball contact time (with a hard tip at fast speed)
from: http://billiards.colostate.edu/threads/cue_tip.html#contact

$\Delta t_{ball} := 0.00025 \cdot s$ typical ball-ball contact time
from: http://billiards.colostate.edu/threads/balls.html#contact

The average CB speed during tip contact is:

$$v_{b\_avg} := \frac{v_b}{2} = 12.5\ mph$$

Therefore, the distance the CB (and tip) travels while the tip is in contact is:

$$\Delta x := v_{b\_avg} \cdot \Delta t_{tip} = 0.11\ in \qquad \Delta x = 2.794\ mm$$

The momentum of the CB is created by the impulse between the tip and CB.
Assuming a triangular and symmetric impulse-force profile (at peak force F_{peak}):

$$\frac{1}{2} \cdot F_{peak} \cdot \Delta t_{tip} = m_b \cdot v_b$$

So the peak force between the tip and CB is:

$$F_{peak} := \frac{2 \cdot m_b \cdot v_b}{\Delta t_{tip}} = 1709\ lbf$$

This corresponds to a peak acceleration (in units of g):

$$a_{peak} := \frac{F_{peak}}{m_b} = 4559\ g$$

When the CB hits the rack of balls and bounces back (with speed $v_{b'}$), both momentum and energy are conserved (assuming a perfect collision):

$$m_b \cdot v_b = m_{rack} \cdot v_{rack} - m_b \cdot v_{b'}$$

$$\frac{1}{2} \cdot m_b \cdot {v_b}^2 = \frac{1}{2} \cdot m_b \cdot {v_{b'}}^2 + \frac{1}{2} \cdot m_{rack} \cdot {v_{rack}}^2$$

Re-writing and solving these equations for $v_{b'}$ gives:

$$v_b = \frac{3}{2} \cdot v_{rack} - v_{b'}$$

$$ {v_b}^2 = {v_{b'}}^2 + \frac{3}{2} \cdot {v_{rack}}^2$$

$$\left({v_b}^2 = {v_{b'}}^2 + \frac{2}{3} \cdot \left(v_b + v_{b'}\right)^2 \right) \xrightarrow{solve\,, v_{b'}} \begin{bmatrix} 5 \cdot mph \\ -25 \cdot mph \end{bmatrix}$$

$$v_{b'} := 5 \cdot mph$$

The impulse between the CB and the rack is equal to the change in momentum of the CB:

$$\frac{1}{2} \cdot F_{peak} \cdot \Delta t_{ball} = m_b \cdot \left(v_b + v_{b'}\right)$$

So the peak force between the CB and rack is:

$$F_{peak} := \frac{2 \cdot m_b \cdot \left(v_b + v_{b'}\right)}{\Delta t_{ball}} = 4103 \ lbf$$

This corresponds to a peak acceleration (in units of g) of:

$$a_{peak} := \frac{F_{peak}}{m_b} = 10940 \ g$$