TP B.22
How peak tip contact force and contact patch size vary with shot speed, and drop tests

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
by Dr. Dave Alciatore, PhD, PE ("Dr. Dave")

originally posted: 5/19/2018 last revision: 5/19/2018

Mass of a pool ball and typical cue stick:

$$m_b := 6 \cdot oz \qquad m_s := 19 \cdot oz$$

Typical tip-ball contact times for phenolic and leather tips with fast-speed shots, from the DBKcue link here:
http://billiards.colostate.edu/threads/cue_tip.html#contact

$$\Delta t_{phenolic} := 0.0008 \cdot s \qquad \Delta t_{leather} := 0.0012 \cdot s$$

Typical coefficients of restitution (CORs) for a phenolic tip on a break cue and a typical leather tip on playing cue, from: http://billiards.colostate.edu/threads/cue_tip.html#efficiency

$$e_{phenolic} := 0.85 \qquad e_{leather} := 0.73$$

Typical contact patch sizes for a fast-speed shot with phenolic and leather tips:

$$v_{fast} := 10 \cdot mph \qquad cps_{phenolic} := 3 \cdot mm \qquad cps_{leather} := 4 \cdot mm$$

From TP B.20, the peak force between the cue tip and CB during impact, for a given CB speed v_b and tip contact time Δt is:

$$F_{peak}(v_b, \Delta t) := \frac{2 \cdot m_b \cdot v_b}{\Delta t}$$

Hertz elastic contact-stress equations (e.g., from "Impact Mechanics" by Strong, pp.117-118, 2004) can be used to approximate how contact patch size (cps) varies with peak force (*F*) according to:

$$cps = \left(\frac{3\ F \cdot E}{R}\right)^{\frac{1}{3}} = c \cdot F^{\frac{1}{3}}$$

where *E* depends on tip and CB material properties, *R* depends on the radii of curvature of the tip and CB, and *c* is the resulting constant.

Therefore, the approximate contact patch size can be related to CB speed and tip contact time according to:

$$cps(v_b, \Delta t, c) := c \cdot \left(\frac{2 \cdot m_b \cdot v_b}{\Delta t}\right)^{\frac{1}{3}}$$

And the Hertz constant c can be related to contact patch size according to:

$$c\left(v_b, \Delta t, cps\right) := cps \cdot \left(\frac{\Delta t}{2 \cdot m_b \cdot v_b}\right)^{\frac{1}{3}}$$

We can approximate the Hertz equation constant c for both phenolic and leather tips using the data above:

$$c_{phenolic} := c\left(v_{fast}, \Delta t_{phenolic}, cps_{phenolic}\right) = 0.242 \ \frac{mm}{N^{\frac{1}{3}}}$$

$$c_{leather} := c\left(v_{fast}, \Delta t_{leather}, cps_{leather}\right) = 0.37 \ \frac{mm}{N^{\frac{1}{3}}}$$

As a check to make sure these values are correct, we can see if the cps equation predicts the correct contact patch sizes:

$$cps\left(v_{fast}, \Delta t_{phenolic}, c_{phenolic}\right) = 3 \ mm$$

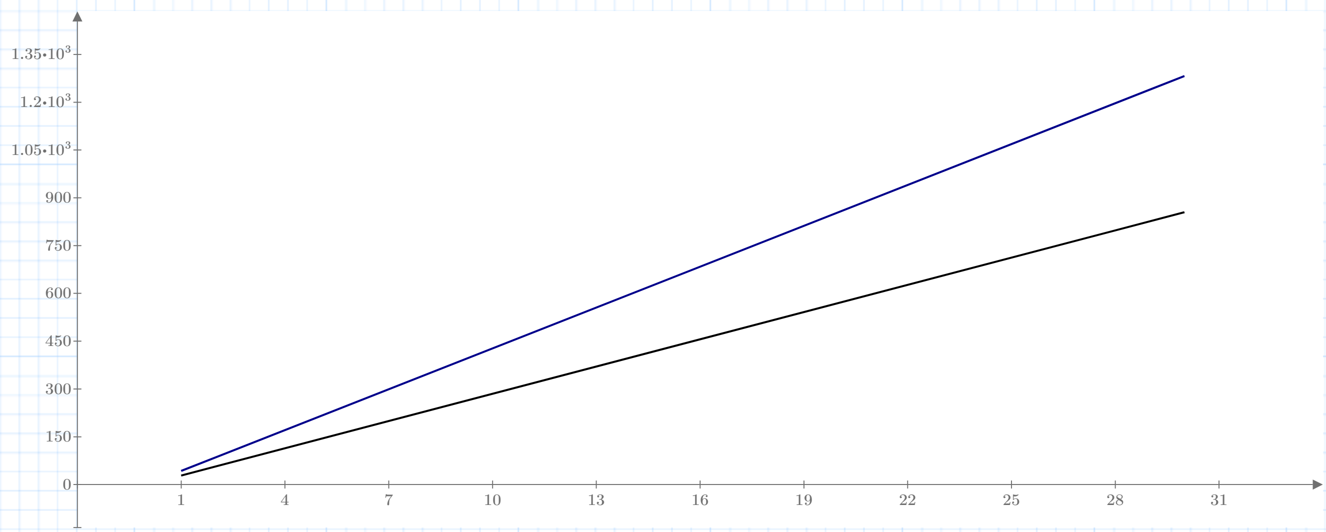
$$cps\left(v_{fast}, \Delta t_{leather}, c_{leather}\right) = 4 \ mm$$

Now we can look at how both peak contact force (in pounds) and contact patch size (in mm) vary with shot speed for both phenolic and leather tips:

$$v_b := 1 \cdot mph, 2 \cdot mph .. 30 \ mph$$

$$F_{peak}\left(v_b, \Delta t_{phenolic}\right) \ (lbf)$$

$$F_{peak}\left(v_b, \Delta t_{leather}\right) \ (lbf)$$



$$v_b \ (mph)$$

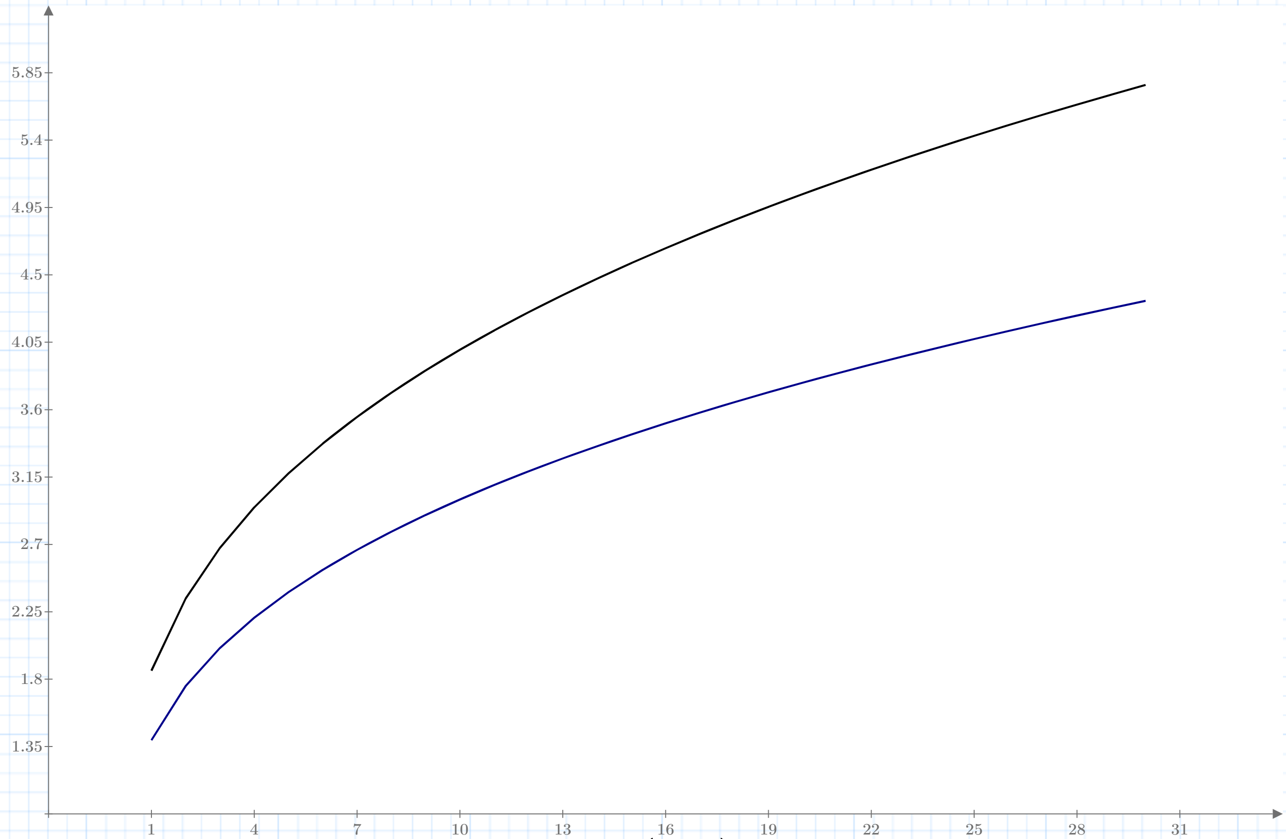
As expected, the peak contact force increases with CB speed, and is greater for a phenolic tip as compared to a leather tip. With a powerful break (25 mph), the peak forces on both phenolic and leather tips are:

$$F_{peak}\left(25 \cdot mph, \Delta t_{phenolic}\right) = 1068 \ lbf \qquad F_{peak}\left(25 \cdot mph, \Delta t_{phenolic}\right) = 4753 \ N$$

$$F_{peak}\left(25 \cdot mph, \Delta t_{leather}\right) = 712 \ lbf \qquad F_{peak}\left(25 \cdot mph, \Delta t_{leather}\right) = 3168 \ N$$

$cps(v_b, \Delta t_{phenolic}, c_{phenolic})\ (mm)$

$cps(v_b, \Delta t_{leather}, c_{leather})\ (mm)$



$v_b\ (mph)$

As expected, the contact patch size increases with CB speed, and is larger for a leather tip as compared to a phenolic tip. With a powerful break (25 mph), the contact patch sizes for phenolic and leather tips are approximated to be:

$$cps(25 \cdot mph, \Delta t_{phenolic}, c_{phenolic}) = 4.1\ mm$$

$$cps(25 \cdot mph, \Delta t_{leather}, c_{leather}) = 5.4\ mm$$

One way to simulate cue-tip-CB impact is to drop a cue from different heights onto a heavy/solid/hard/flat/smooth surface (e.g., a big steel block). From conservation of energy, the cue speed *v* after falling height *h* is:

$$v = \sqrt{2 \cdot g \cdot h}$$

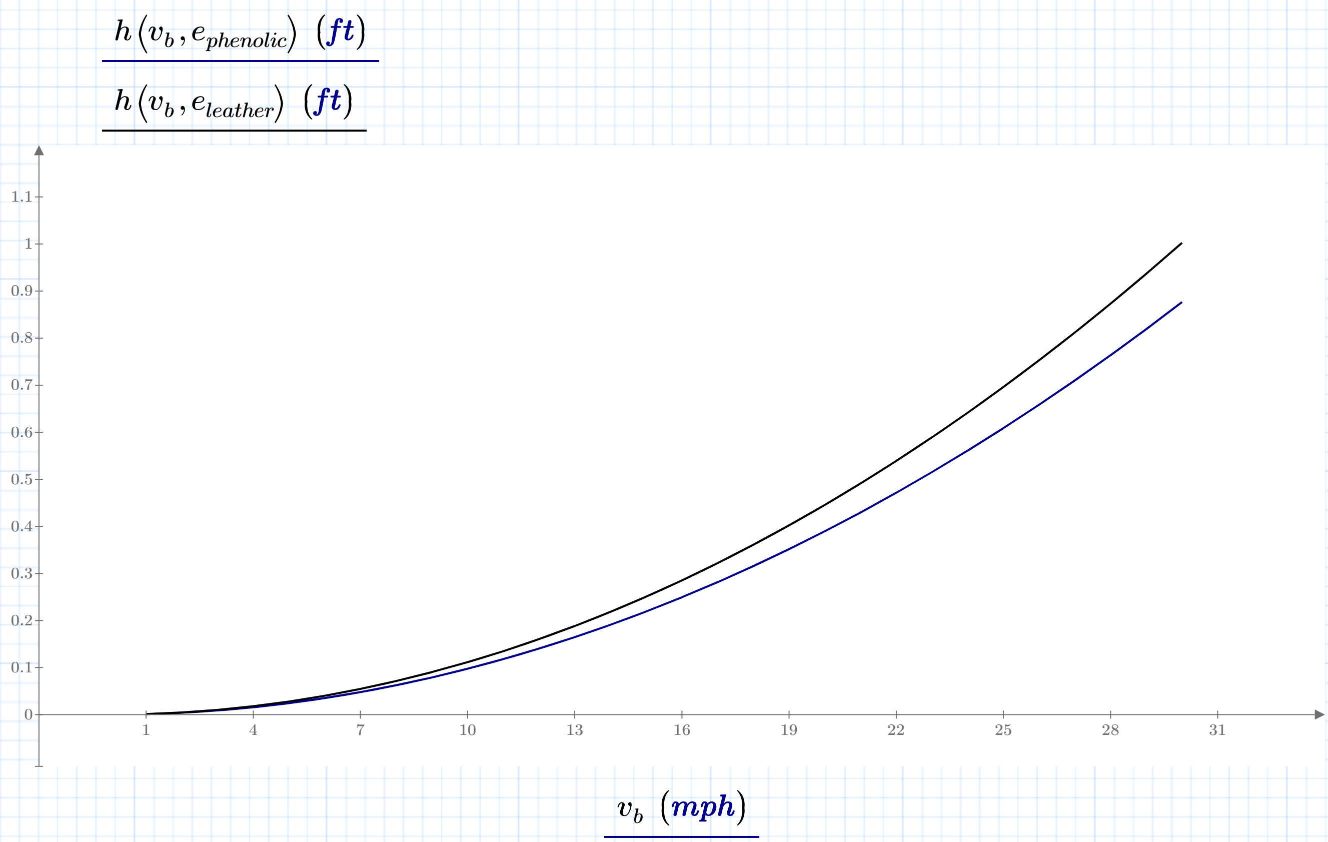
From impulse-mementum principles, if we want the impulse (and peak force) with a drop test to match the impulse (and peak force) of a CB hit, we can relate drop height (*h*) to CB speed (v_b) and drop rebound COR (*e*) with:

$$m_b \cdot v_b = m_s \cdot (v + e \cdot v) = m_s \cdot \sqrt{2 \cdot g \cdot h}\ (1 + e)$$

Solving for h gives us the required drop height to simulate different CB speeds:

$$h(v_b, e) := \frac{1}{2\ g} \left(\frac{m_b \cdot v_b}{m_s \cdot (1 + e)} \right)^2$$

Here's a plot of how required drop height varies with simulated CB speed for both phenolic and leather tips:



As expected, a larger drop height is required to simulate faster CB speeds, and the drop height for a leather tip needs to be a little higher compared to a phenolic tip. With a powerful break (25 mph), the required drop heights for both phenolic and leather tips are approximately:

$$h(25 \cdot mph, e_{phenolic}) = 0.61\ ft \qquad h(25 \cdot mph, e_{phenolic}) = 18.6\ cm$$

$$h(25 \cdot mph, e_{leather}) = 0.7\ ft \qquad h(25 \cdot mph, e_{leather}) = 21.2\ cm$$