technical proof

technical proof

TP B.5
Rolling CB, direct-hit hop and ball travel distances

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
by David G. Alciatore, PhD, PE ("Dr. Dave")

originally posted: 1/30/2009 last revision: 1/30/2009

Typical speeds for a range of shots:

$v_{touch} := 1.5 \cdot mph$ $v_{slow} := 3 \cdot mph$ $v_{medium\_soft} := 5 \cdot mph$ $v_{medium} := 7 \cdot mph$

$v_{medium\_fast} := 8 \cdot mph$ $v_{fast} := 12 \cdot mph$ $v_{power} := 20 \cdot mph$

Rolling CB with square hit on OB (see details in TP A.5):

Normal impulse between the balls:

$$F'_n = \frac{(1 + e_b)}{2} \cdot m \cdot v$$

Tangential impulse between the balls during impact (assuming slip):

$$F'_t = \mu_b \cdot F'_n$$

Vertical speed of CB after impact, due to tangential friction force:

$$v_z = \frac{1}{m} \cdot F'_t = \frac{\mu_b \cdot (1 + e_b) \cdot v}{2}$$

Loss of CB spin, which is the same as the gain in OB spin, due to the tangential friction force is:

$$\Delta\omega = \frac{F'_t \cdot R}{I} = \frac{\mu_b \cdot \left[\frac{(1 + e_b)}{2} \cdot m \cdot v\right] \cdot R}{\frac{2}{5} \cdot m \cdot R^2} = \frac{5}{4 \cdot R} \cdot \mu_b \cdot (1 + e_b) \cdot v$$

where:

$\mu_b := 0.06$ typical COF between the balls

$e_b := 0.94$ typical COR between the balls

CB hop height and time:

Energy is conserved during the CB hop of height h, so:

$$m \cdot g \cdot h = \frac{1}{2} \cdot m \cdot v_z^2 \qquad h(v) := \frac{\mu_b^2 \cdot \left(1 + e_b\right)^2}{8g} \cdot v^2$$

$$h\left(v_{medium}\right) = 0.067 \cdot in \qquad h\left(v_{fast}\right) = 0.196 \cdot in \qquad h\left(v_{power}\right) = 0.544 \cdot in$$

The time required for the entire hop (up and down) is given by:

$$v_z = 2g \cdot \Delta t \qquad \Delta t(v) := \frac{\mu_b \cdot \left(1 + e_b\right) \cdot v}{g}$$

$$\Delta t\left(v_{fast}\right) = 0.064\,s \qquad \Delta t\left(v_{power}\right) = 0.106\,s$$

Ball travel distances after impact:

$\mu_s := 0.2$ typical ball-cloth coefficient of sliding friction

$\mu_r := 0.01$ typical ball-cloth coefficient of rolling resistance

$R := 2.25 \cdot in$ ball radius

From TP 4.1, the skid distance and final speed are given by:

$$d_{skid}(v, \omega) := \operatorname{sign}\left(\frac{v}{R} - \omega\right) \cdot \frac{2}{49 \cdot \mu_s \cdot g} \cdot \left[6 \cdot v^2 - 5v \cdot R \cdot \omega - (R \cdot \omega)^2\right]$$

$$v_{skid}(v, \omega) := \frac{5}{7} \cdot v + \frac{2}{7} \cdot R \cdot \omega$$

The distance required for rolling to stop is:

$$d_{roll\_stop}(v) := \frac{v^2}{2 \cdot \mu_r \cdot g}$$

After impact, the initial CB and OB speeds and spins are (from TP A.5 and above):

$$v_{CB}(v) := \frac{\left(1 - e_b\right)}{2} \cdot v \qquad v_{OB}(v) := \frac{\left(1 + e_b\right)}{2} \cdot v$$

$$\Delta\omega(v) := \frac{5}{4 \cdot R} \cdot \mu_b \cdot \left(1 + e_b\right) \cdot v \qquad \omega_{CB}(v) := \frac{v}{R} - \Delta\omega(v) \qquad \omega_{OB}(v) := -\Delta\omega(v)$$

Here, we will ignore ball hop effects (i.e., the skid is assumed to take place smoothly, without hop interruption).

Therefore, the distances the CB and OB travel after a rolling direct hit are:

$$d_{CB}(v) := d_{skid}\left(v_{CB}(v), \omega_{CB}(v)\right) + d_{roll\_stop}\left(v_{skid}\left(v_{CB}(v), \omega_{CB}(v)\right)\right)$$

$$d_{OB}(v) := d_{skid}\left(v_{OB}(v), \omega_{OB}(v)\right) + d_{roll\_stop}\left(v_{skid}\left(v_{OB}(v), \omega_{OB}(v)\right)\right)$$

$$\frac{d_{OB}\left(v_{slow}\right)}{d_{CB}\left(v_{slow}\right)} = 6.08 \qquad \frac{d_{OB}\left(v_{medium}\right)}{d_{CB}\left(v_{medium}\right)} = 6.08$$

These numbers are similar to the results from the analyses in TP A.16.