**TP B.6**

CB table lengths of travel for different speeds, accounting for rail rebound and drag losses

supporting:
“The Illustrated Principles of Pool and Billiards”
http://billiards.colostate.edu
by David G. Alciatore, PhD, PE ("Dr. Dave")

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Typical speeds for a range of shots:

\[ v_{\text{touch}} := 1.5 \text{ mph} \]
\[ v_{\text{slow}} := 3 \text{ mph} \]
\[ v_{\text{medium,soft}} := 5 \text{ mph} \]
\[ v_{\text{medium}} := 7 \text{ mph} \]
\[ v_{\text{medium,fast}} := 8 \text{ mph} \]
\[ v_{\text{fast}} := 12 \text{ mph} \]
\[ v_{\text{power}} := 20 \text{ mph} \]

Relevant physical properties:

\[ \mu_s := 0.2 \] typical ball-cloth coefficient of sliding friction
\[ \mu_r := 0.01 \] typical ball-cloth coefficient of rolling resistance
\[ R := 2.25 \text{ in} \] ball radius

From TP 4.1, the distance required for a rolling ball to stop is:

\[ d_{\text{roll,stop}}(v) := \frac{v^2}{2 \mu_r g} \]

Table lengths vs. speed, accounting for rail rebound and drag losses:

\[ e_c := 0.7 \] typical ball/rail COR with ball rolling into the rail cushion
(see HSV B.15)

When the CB rebounds off a rail cushion, speed is lost. If we assume the CB rebounds off the rail with stun (see HSV B.15 - a rolling ball usually rebounds with stun), the resulting skid distance and speed change are (from TP 4.1):

\[ d_{\text{skid}}(v) := \frac{12 \cdot v^2}{49 \cdot \mu_s \cdot g} \]

\[ v_{\text{skid}}(v,x) := \sqrt{\frac{2}{\mu_s \cdot g} \cdot x} \]
\[ v_{\text{skid}}(v,d_{\text{skid}}(v)) = \frac{5}{7} \cdot v \]
In the analysis below, to keep things reasonably simple, we assume the CB always rebounds off the rail with stun. HSV B.15 shows that a skidding ball usually rebounds with some roll, but the overall rebound efficiency, taking post-rebound skid into consideration, is fairly consistent for most shots.

While the CB rolls, it slowly loses speed due to rolling resistance over distance $x$:

$$\frac{1}{2} \cdot m \cdot v'^2 = \frac{1}{2} \cdot m \cdot v^2 - \mu_r \cdot m \cdot g \cdot x$$

so the function of speed over distance, during rolling, is:

$$v_{\text{roll}}(v, x) := \sqrt{\frac{v^2 - 2 \cdot \mu_r \cdot g \cdot x}{2}}$$

Determine CB travel distance for a rolling CB with rail collisions:

<table>
<thead>
<tr>
<th>TL := 100 in</th>
<th>TL = 8.333 ft</th>
<th>9' table playing length</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d(v) :=$</td>
<td>$x \leftarrow 0$</td>
<td>$v \leftarrow v$</td>
</tr>
<tr>
<td>$v \leftarrow v$</td>
<td>$n \leftarrow 0$</td>
<td>$\text{roll} \leftarrow 1$</td>
</tr>
<tr>
<td>if $(d_{\text{roll stop}}(v) &lt; TL)$</td>
<td>&quot;less than one table length&quot;</td>
<td>$x \leftarrow d_{\text{roll stop}}(v)$</td>
</tr>
<tr>
<td>otherwise</td>
<td>&quot;ball will roll into a rail&quot;</td>
<td>while $(v &gt; 0)$</td>
</tr>
<tr>
<td>if $(\text{roll} = 1)$</td>
<td>if $d_{\text{roll stop}}(v) &lt; (n + 1)TL - x$</td>
<td>&quot;won't make it to rail again&quot;</td>
</tr>
<tr>
<td>$x \leftarrow x + d_{\text{roll stop}}(v)$</td>
<td>break</td>
<td>otherwise</td>
</tr>
<tr>
<td>otherwise</td>
<td>&quot;roll to the rail&quot;</td>
<td>$n \leftarrow n + 1$</td>
</tr>
<tr>
<td>$v \leftarrow v_{\text{roll}}(v, n \cdot TL - x)$</td>
<td>$x \leftarrow n \cdot TL$</td>
<td>&quot;rebound off the rail&quot;</td>
</tr>
</tbody>
</table>
| $v \leftarrow e_c \cdot v$ | roll $\leftarrow 0$ | }
otherwise

"(roll = 0) : ball skidding"
if \((d_{skid}(v) < TL)\)

"ball will roll before next rail"
x \(\leftarrow x + d_{skid}(v)\)
\(v \leftarrow \frac{5}{7}v\)
roll \(\leftarrow 1\)

otherwise

"ball will skid into next rail"
n \(\leftarrow n + 1\)
\(v \leftarrow v_{skid}(v, TL)\)
x \(\leftarrow n \cdot TL\)
"rebound off the rail"
v \(\leftarrow e_{c} \cdot v\)
roll \(\leftarrow 0\)

\[
\begin{align*}
\text{d(v)} &= \text{touch}, \quad \text{d(v)} = 7.522 \text{ ft} \\
\text{d(v)} &= \text{slow}, \quad \text{d(v)} = 14.033 \text{ ft} \\
\text{d(v)} &= \text{medium}, \quad \text{d(v)} = 25.041 \text{ ft} \\
\text{d(v)} &= \text{fast}, \quad \text{d(v)} = 30.752 \text{ ft} \\
\text{d(v)} &= \text{power}, \quad \text{d(v)} = 39.291 \text{ ft} \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>table lengths of travel:</th>
</tr>
</thead>
</table>
| \[
\begin{align*}
\frac{\text{d(v)}}{TL} &= \text{touch}, \quad \frac{\text{d(v)}}{TL} = 0.903 \\
\frac{\text{d(v)}}{TL} &= \text{slow}, \quad \frac{\text{d(v)}}{TL} = 1.684 \\
\frac{\text{d(v)}}{TL} &= \text{medium}, \quad \frac{\text{d(v)}}{TL} = 3.005 \\
\frac{\text{d(v)}}{TL} &= \text{fast}, \quad \frac{\text{d(v)}}{TL} = 3.69 \\
\frac{\text{d(v)}}{TL} &= \text{power}, \quad \frac{\text{d(v)}}{TL} = 4.715 \\
\end{align*}
\]

rolling lag shot: \(v := v_{\text{slow}} = 3 \text{ mph} \quad \text{(estimate)}\)
given
\[
\begin{align*}
d(v) &= 2 \cdot TL \\
v_{\text{lag}} &= \text{find}(v) \quad v_{\text{lag}} = 3.465 \text{ mph} \\
\frac{d(v_{\text{lag}})}{TL} &= 2 \\
\end{align*}
\]
\[ v := 0\text{-mph}, 0.5\text{-mph} \rightarrow v_{\text{power}} \quad v_{\text{power}} = 20\text{-mph} \]

\[ \frac{v_{\text{fast}}}{v_{\text{medium}}} = 1.714 \quad \frac{d(v_{\text{fast}})}{d(v_{\text{medium}})} = 1.228 \]

\[ \frac{v_{\text{fast}}}{v_{\text{slow}}} = 4 \quad \frac{d(v_{\text{fast}})}{d(v_{\text{slow}})} = 2.191 \]

\[ \frac{v_{\text{power}}}{v_{\text{touch}}} = 13.333 \quad \frac{d(v_{\text{power}})}{d(v_{\text{touch}})} = 5.224 \]

So the speed must be increased by a much larger percentage to create a given percentage of distance increase, and this effect is even stronger at faster speeds and longer distances. In other words, it takes a lot more speed to create more distance, especially at higher speeds and longer distances.