technical proof

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TP B.6
CB table lengths of travel for different speeds, accounting for rail rebound and drag losses

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
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originally posted: 1/30/2009   last revision: 2/5/2009

Typical speeds for a range of shots:

$v_{touch} := 1.5 \cdot mph$ $v_{slow} := 3 \cdot mph$ $v_{medium\_soft} := 5 \cdot mph$ $v_{medium} := 7 \cdot mph$

$v_{medium\_fast} := 8 \cdot mph$ $v_{fast} := 12 \cdot mph$ $v_{power} := 20 \cdot mph$

Relevant physical properties:

$\mu_s := 0.2$ typical ball-cloth coefficient of sliding friction

$\mu_r := 0.01$ typical ball-cloth coefficient of rolling resistance

$R := 2.25 \cdot in$ ball radius

From TP 4.1, the distance required for a rolling ball to stop is:

$$d_{roll\_stop}(v) := \frac{v^2}{2 \cdot \mu_r \cdot g}$$

Table lengths vs. speed, accounting for rail rebound and drag losses:

$e_c := 0.7$ typical ball/rail COR with ball rolling into the rail cushion (see HSV B.15)

When the CB rebounds off a rail cushion, speed is lost. If we assume the CB rebounds off the rail with stun (see HSV B.15 - a rolling ball usually rebounds with stun), the resulting skid distance and speed change are (from TP 4.1):

$$d_{skid}(v) := \frac{12 \cdot v^2}{49 \cdot \mu_s \cdot g}$$

$$v_{skid}(v, x) := \sqrt{v^2 - 2 \cdot \mu_s \cdot g \cdot x} \qquad v_{skid}(v, d_{skid}(v)) = \frac{5}{7} \cdot v$$

In the analysis below, to keep things reasonably simple, we assume the CB always rebounds off the rail with stun. HSV B.15 shows that a skidding ball usually rebounds with some roll, but the overall rebound efficiency, taking post-rebound skid into consideration, is fairly consistent for most shots.

While the CB rolls, it slowly loses speed due to rolling resistance over distance x:

$$\frac{1}{2}\cdot m\cdot v'^2 = \frac{1}{2}\cdot m\cdot v^2 - \mu_r\cdot m\cdot g\cdot x$$

so the function of speed over distance, during rolling, is:

$$v_{roll}(v, x) := \sqrt{v^2 - 2\cdot\mu_r\cdot g\cdot x}$$

Determine CB travel distance for a rolling CB with rail collisions:

$TL := 100\cdot in$ $\quad TL = 8.333\cdot ft$ $\quad$ 9' table playing length

```
d(v) := | x ← 0
        | v ← v
        | n ← 0
        | roll ← 1
        | if (d_roll_stop(v) < TL)
        |   | "less than one table length"
        |   | x ← d_roll_stop(v)
        | otherwise
        |   | "ball will roll into a rail"
        |   | while (v > 0)
        |   |   | if (roll = 1)
        |   |   |   | if d_roll_stop(v) < (n + 1)TL − x
        |   |   |   |   | "won't make it to rail again"
        |   |   |   |   | x ← x + d_roll_stop(v)
        |   |   |   |   | break
        |   |   |   | otherwise
        |   |   |   |   | "roll to the rail"
        |   |   |   |   | n ← n + 1
        |   |   |   |   | v ← v_roll(v, n·TL − x)
        |   |   |   |   | x ← n·TL
        |   |   |   |   | "rebound off the rail"
        |   |   |   |   | v ← e_c·v
        |   |   |   |   | roll ← 0
```

otherwise

"(roll = 0) : ball skidding"

if $\left(d_{skid}(v) < TL\right)$

"ball will roll before next rail"

$x \leftarrow x + d_{skid}(v)$

$v \leftarrow \frac{5}{7} \cdot v$

$roll \leftarrow 1$

otherwise

"ball will skid into next rail"

$n \leftarrow n + 1$

$v \leftarrow v_{skid}(v, TL)$

$x \leftarrow n \cdot TL$

"rebound off the rail"

$v \leftarrow e_c \cdot v$

$roll \leftarrow 0$

x

$d\left(v_{touch}\right) = 7.522 \cdot ft$ $d\left(v_{slow}\right) = 14.033 \cdot ft$ $d\left(v_{medium}\right) = 25.041 \cdot ft$

$d\left(v_{fast}\right) = 30.752 \cdot ft$ $d\left(v_{power}\right) = 39.291 \cdot ft$

table lengths of travel:

$\frac{d\left(v_{touch}\right)}{TL} = 0.903$ $\frac{d\left(v_{slow}\right)}{TL} = 1.684$ $\frac{d\left(v_{medium}\right)}{TL} = 3.005$

$\frac{d\left(v_{fast}\right)}{TL} = 3.69$ $\frac{d\left(v_{power}\right)}{TL} = 4.715$

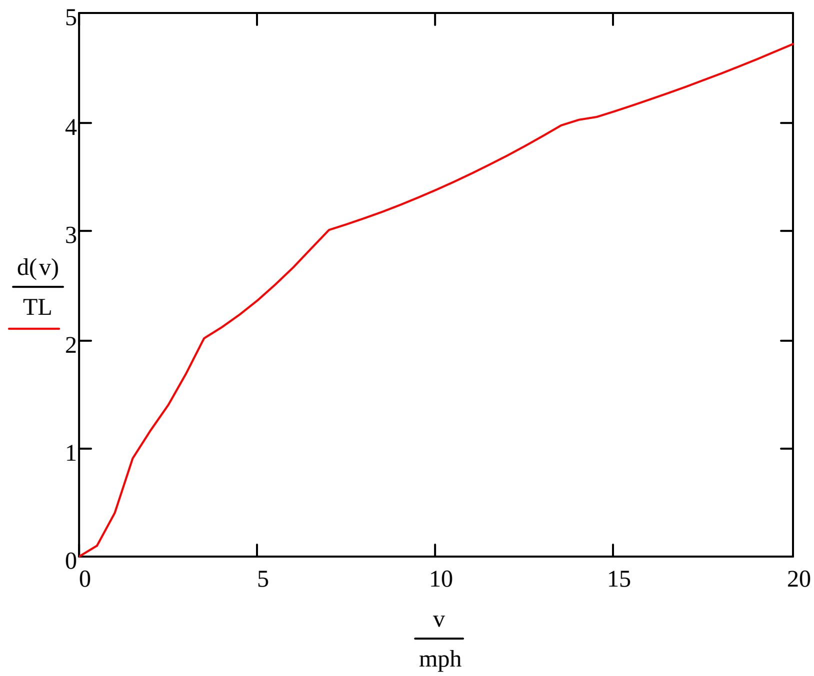
rolling lag shot: $v := v_{slow} = 3\, mph$ (estimate)

given

$d(v) = 2 \cdot TL$

$v_{lag} := find(v)$ $v_{lag} = 3.465\, mph$ $\frac{d\left(v_{lag}\right)}{TL} = 2$

$v := 0 \cdot mph, 0.5 \cdot mph .. v_{power}$ $\qquad$ $v_{power} = 20 \cdot mph$



$$\frac{v_{fast}}{v_{medium}} = 1.714 \qquad \frac{d(v_{fast})}{d(v_{medium})} = 1.228$$

$$\frac{v_{fast}}{v_{slow}} = 4 \qquad \frac{d(v_{fast})}{d(v_{slow})} = 2.191$$

$$\frac{v_{power}}{v_{touch}} = 13.333 \qquad \frac{d(v_{power})}{d(v_{touch})} = 5.224$$

So the speed must be increased by a much larger percentage to create a given percentage of distance increase, and this effect is even stronger at faster speeds and longer distances. In other words, it takes a lot more speed to create more distance, especially at higher speeds and longer distances.