**TP 3.3**

**30° rule**

supporting:
“The Illustrated Principles of Pool and Billiards”
[http://billiards.colostate.edu](http://billiards.colostate.edu)
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An complete derivation, arrived at by solving the general equations of motion, can be found in TP A.4.

Ball-hit fraction:

\[ f := 0, 0.01, 1 \]

Cut angle:

\[ \varphi(f) := \arcsin(1 - f) \]

Cue ball final deflected angle:

\[ \theta(\varphi) := \arctan\left(\frac{\sin(\varphi) \cos(\varphi)}{\sin(\varphi)^2 + \frac{2}{5}}\right) \]

For a half-ball hit:

\[ \varphi\left(\frac{1}{2}\right) = 30 \text{ deg} \]
\[ \theta\left(\varphi\left(\frac{1}{2}\right)\right) = 33.67 \text{ deg} \]

For a 1/4-ball hit:

\[ \varphi\left(\frac{1}{4}\right) = 48.59 \text{ deg} \]
\[ \theta\left(\varphi\left(\frac{1}{4}\right)\right) = 27.267 \text{ deg} \]

For a 3/4-ball hit:

\[ \varphi\left(\frac{3}{4}\right) = 14.478 \text{ deg} \]
\[ \theta\left(\varphi\left(\frac{3}{4}\right)\right) = 27.626 \text{ deg} \]
**deflected cue ball angle vs. ball-hit fraction:**

\[ \theta(\phi(f)) \mapsto \frac{\text{deg}}{\text{deg}} \]

**deflected cue ball angle vs. cut angle:**

\[ \theta(\phi) \mapsto \frac{\text{deg}}{\text{deg}} \]

\[ \phi \mapsto 0\deg, 1\deg, \ldots, 89\deg \]
Maximum cue ball deflected angle:

Deflected angle as a function of cut angle is:

\[
\text{atan} \left( \frac{\sin(\phi) \cdot \cos(\phi)}{\sin(\phi)^2 + \frac{2}{5}} \right)
\]

The derivative of this with respect to \( \phi \) (using MathCAD) is:

\[
\frac{45 \cdot \sin(\phi)^2 - 10}{45 \cdot \sin(\phi)^2 + 4}
\]

At the maximum, the numerator of this expression must be 0, so:

\[
\sin(\phi) = \frac{\sqrt{10}}{\sqrt{45}} = \frac{\sqrt{2}}{3}
\]

Therefore, the maximum cue ball deflection occurs at a cut angle of:

\[
\phi := \arcsin \left( \frac{\sqrt{2}}{3} \right) = 28.126 \text{ deg}
\]

which corresponds to a ball-hit fraction of:

\[
f(\phi) := 1 - \sin(\phi) \quad \Rightarrow \quad f(\phi) = 0.529
\]

and the maximum deflected angle is:

\[
\theta(\phi) = 33.749 \text{ deg}
\]