David Alciatore, PhD ("Dr. Dave") **ILLUSTRATED PRINCIPLES**

"90° and 30° Rule Follow-up – *Part III: inelasticity and friction*"

***Note**: Supporting narrated video (NV) demonstrations, high-speed video (HSV) clips, and technical proofs (TP) can be accessed and viewed online at **billiards.colostate.edu**. The reference numbers used in the article (e.g., **NV 3.4**) help you locate the resources on the website. You might also want to view the resources from a CD-ROM. See the website for more details.*

In my previous two articles, I answered a few basic questions that have come up concerning my series of articles on the 90° and 30° rules (see my January '04 through July '04 articles at ***billiards.colostate.edu***). In last month's article, we looked at the effects of follow, draw, and shot speed on both rules. In this article, we will look at the effects of ball inelasticity and friction. In next month's article, I'll conclude the series by looking at the effects of side English.

If you don't remember what the 90° and 30° rules are and when they apply, see **NV 3.4-3.5** and **NV 3.7-3.10**. Readers with engineering or physics backgrounds might also find **TP 3.1** and **TP 3.3** interesting. Remember, the 90° rule states that for a stun shot, where the cue ball is sliding at object ball impact, the cue ball and object ball paths separate at 90° (i.e., the separating paths are perpendicular). The 30° rule states that when the cue ball is rolling at object ball impact, and when the cut angle is between a 1/4-ball and 3/4-ball hit fraction, the cue ball's path will be deflected by approximately 30°. If these previous two sentences are not clear, you might want to look at the online videos and articles mentioned above.


normal video

NV 3.4 – 90° rule with various cut angles
NV 3.5 – Using your hand to visualize the 90° rule impact and tangent lines
NV 3.7 – Using the 90° rule to check for and prevent a scratch
NV 3.8 – Using your hand to visualize the 30° rule
NV 3.9 – 30° rule example
NV 3.10 – Using the 30° rule to check for and prevent a scratch


technical proof

TP 3.1 – 90° rule
TP 3.3 – 30° rule

Ball inelasticity is described by a technical factor called the **coefficient of restitution**. It describes how well the cue ball delivers energy to the object ball during impact. In other words, it quantifies how much energy is lost during impact. If the balls were perfect, and if they were colliding in a vacuum (i.e., with no surrounding air), there would be no loss in energy and the coefficient of restitution would be 1.00 or 100%. However, actual pool balls are not perfectly elastic, and when they collide in air some energy is lost to sound. That's right ... if you can hear a sound when the balls collide, that means energy of motion is being lost to acoustic (sound) energy. Most experimental numbers I have seen for the coefficient of restitution for pool balls have been in the range 0.90 to 0.96.

Friction between the cue ball and the object ball creates a sideways force between the balls during impact. It is caused by relative sliding motion between the balls, due to cut angle or cue ball spin. Ball friction is what causes throw effects often referred to as English throw, cut-induced throw (CIT), cut throw, spin-induced throw (SIT), drag, collision-induced throw, cling, skid, kick, drag, or push (for more information, see "throw" under the "Discussion Threads" area of my website). Friction is described by a technical factor called the **coefficient of friction**. This coefficient quantifies the amount of friction force as a percentage of the amount of normal (impact line) force between the balls. Most reported values for pool balls are close to 0.06, meaning that the friction force can be only about 6% as large as the perpendicular impact force between the balls.

Diagram 1 illustrates the results of a technical analysis that shows the effects of both inelasticity and friction on the 90° rule (see **TP A.5** if you want to see the physics and math details). Ball friction causes throw, which shortens the object ball angle, and ball inelasticity shortens the cue ball angle. **Principle 18** summarizes these effects. Both effects reduce the angle between the final object ball and cue ball paths, so the 90° rule is usually a few degrees off (e.g., the 87° degree rule), depending on ball conditions. The greater the friction and inelasticity, the more the 90° angle is reduced. Old, poor quality, and/or dirty balls will show the most variance from 90°. But remember, if there is draw or follow on the cue ball at object ball impact, the angle can be significantly more or less than 90° (see my March ‘05 article).

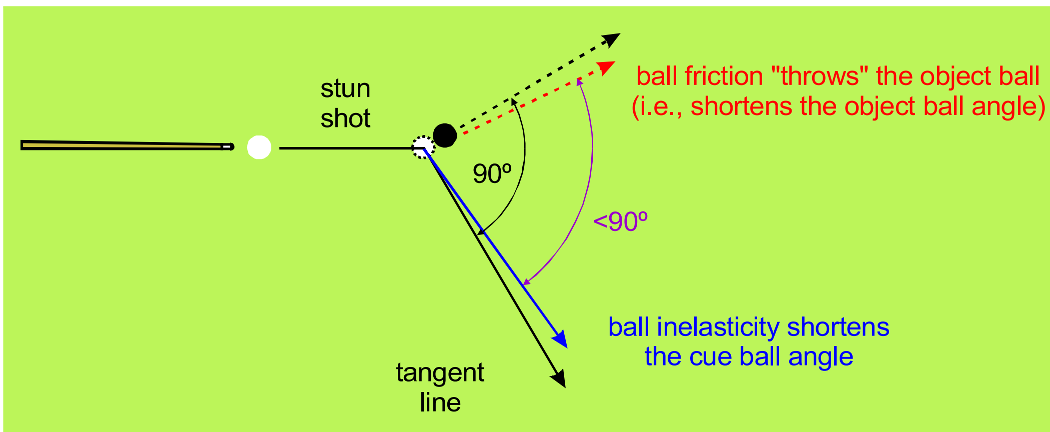

Diagram 1 The effects of ball inelasticity and friction on the 90° rule

<u>Principle 18</u> 90° rule inelasticity and friction effects

Ball inelasticity and friction reduce the angle predicted by the 90° rule (see Diagram 1).

- Ball friction shortens the object ball angle, and ball inelasticity shortens the cue ball angle. Both effects make the total angle between the cue ball and object ball paths smaller.

technical proof

TP A.5 –The effects of ball inelasticity and friction on the 90° rule

Diagram 2 shows the effects of ball imperfections on the 30° rule (see **TP A.6** if you want to see the physics and math details of the technical derivation). As with the 90° rule, ball inelasticity shortens the deflected cue ball angle. However, the effect of ball friction is different. The friction force between the cue ball and object ball affects the spin of the cue ball, which changes the cue ball trajectory. Remember that the 30° rule applies when the cue ball is rolling with forward spin. This spin is reduced by friction, and the friction force also adds side spin (see **TP A.6** for details). The result is that friction lengthens the final cue ball deflected angle. This is actually good news, because the friction effect tends to offset the inelasticity effect. **Principle 19** summarize this result.

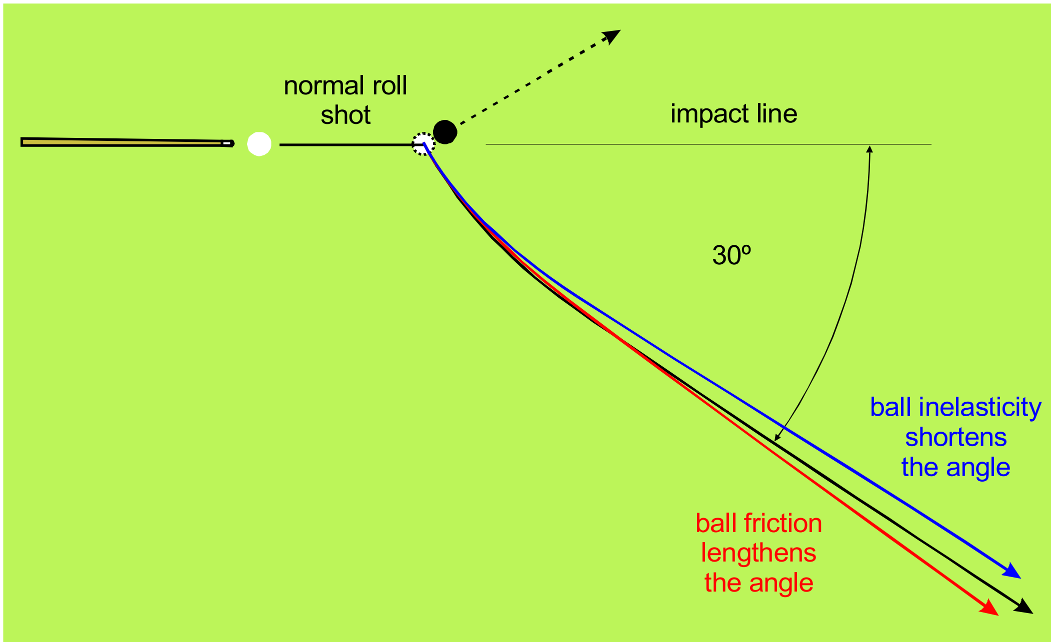

Diagram 2 The effects of ball inelasticity and friction on the 30° rule

<u>Principle 19</u> 30° rule inelasticity and friction effects

Ball inelasticity and friction have opposite effects on the 30° rule (see Diagram 2).

- Ball friction lengthens the cue ball angle and ball inelasticity shortens the cue ball angle. With both effects combined, the deflected cue ball angle is still very close to 30°.


technical proof

TP A.5 –The effects of ball inelasticity and friction on the 30° rule

I almost didn't want to write this article, because the information is probably not useful for most people. I know at least the physics nerds out there will appreciate it. Probably the only time the information would be useful in practice is if you come across playing conditions where the balls are old, of poor quality, and/or very dirty. Then you might notice the difference in the actual vs. expected cue ball paths based on the 90° and 30° rules. Then, you could correct for the errors by modifying your predicted ball paths slightly.

Object ball throw is very important and definitely needs to be accounted for, especially at sow speeds and with dirty balls. However, knowing exactly where the cue ball will go (within a few degrees) is usually not so critical (e.g., with position play). Also, as pointed out in **TP A.5**, the coefficient of restitution and the coefficient of friction can vary significantly with shot speed, cut angle, and ball conditions, so it is not very useful to try to quantify things too exactly. And remember, the 30° rule isn't perfect anyway (see my February '04 article). As the cut angle changes from a 1/4-ball hit to a 3/4-ball hit, the final deflected angle of the cue ball varies over the range 27° to 34° (see **TP 3.3** for exact numbers and more details).

Bottom line: The 90° and 30° rules are precise enough for most people and practically all situations. Although, it is useful to know that for dirty balls, the 90° rule angle will be less than 90° and the 30° rule angle will be a little larger than 30°.

I hope you are enjoying my series of follow-up articles on the 90° and 30° rules. Next month, I'll conclude the series by looking at the effects of side English on both rules.

Good luck with your game, and practice hard,
Dr. Dave

PS:

- If you want to refer back to any of my previous articles and resources, you can access them online at ***billiards.colostate.edu***.
- FYI, I have a new section on my website entitled "Online Discussion Threads." There you can find links to interesting online discussions from the Billiards Digest Cue Chalk Board (CCB) relating to "Illustrated Principles" topics. Feel free to participate by posting questions or responses.
- FYI, over the next three years I will be presenting a multimedia seminar across the country, sponsored by the *American Society of Mechanical Engineers*. The title is "The Illustrated Principles of Pool and Billiards." The talks are usually open to the public, so periodically check out the dates and locations on my website. It would be fun to have some BD readers (and not just engineers) in the audience. The seminar is geared toward a general audience (even non pool players), but it is usually well received by both engineers and seasoned pool players.

Dr. Dave is a mechanical engineering professor at Colorado State University in Fort Collins, CO. He is also author of the book: "The Illustrated Principles of Pool and Billiards" (2004, Sterling Publishing).