David Alciatore ("Dr. Dave") **ILLUSTRATED PRINCIPLES**

"Throw – Part II: results"

***Note**: Supporting narrated video (NV) demonstrations, high-speed video (HSV) clips, and technical proofs (TP) can be accessed and viewed online at **billiards.colostate.edu**. The reference numbers used in the article help you locate the resources on the website. If you have a slow or inconvenient Internet connection, you might want to view the resources from a CD-ROM or DVD. See the website for details.*

I had a tough time deciding how to start this month's article. How about ...

$$\theta_{throw} = \tan^{-1}\left\{ \left[\min\left(\frac{\left(a_\mu + b_\mu e^{-c_\mu \sqrt{(v\sin(\phi) - R\omega_z)^2 + (R\omega_x \cos(\phi))^2}} \right) v\cos(\phi)}{\sqrt{(v\sin(\phi) - R\omega_z)^2 + (R\omega_x \cos(\phi))^2}},\ \ 1/7 \right) \left(v\sin(\phi) - R\omega_z\right) \right] \Big/ v\cos(\phi) \right\}$$

Isn't it beautiful! Actually, I know that it probably isn't very pretty to most of you, but take my word for it ... the equation is sweet. This might be the first time an equation like this has ever appeared in Billiards Digest, and I bet most of you are thinking: "I hope it never happens again." Well, before you go and write a nasty letter to the editor, please bear with me for a while.

I know the equation probably just "looks like Greek" to you (as it should, because it has many Greek letters in it). I also know many of you might be thinking: "What in the heck does math and physics have to do with playing pool?" Don't worry ... I don't plan to go through the physics and math derivation of the equation (although, you can find it in **TP A.14** if you are interested and/or sadistic). Instead, I just want to look at all of the marvelous results and insights the equation can provide. Amazingly, this single equation explains practically every effect and principle related to throw. The equation provides insight on the effects of speed, cut angle, English, and ball friction, and the results are in agreement with numerous observations and experiments.


technical proof

TP A.14 – The effects of cut angle, speed, and spin on object ball throw

This is the second of a series of articles concerning "throw." Last month, I started with some basic terminology and some examples of where throw can help or hurt you in game situations. To refresh your memory, **throw** is the change in object ball (OB) direction due to sliding friction forces between the cue ball (CB) and OB during impact. **NV 4.15, 4.16, 7.5, and 7.6** show various examples of both **cut-induced throw** (**CIT**) and **spin-induced throw** (**SIT**). See the video demos and last month's article for more information.


normal video

NV 4.15 – Using throw to make a partially blocked shot
NV 4.16 – Over-cutting a cut shot to compensate for throw
NV 7.5 – Frozen ball throw
NV 7.6 – Frozen cue-ball throw

Diagram 1 shows an example plot that can be created with the throw equation. This particular plot shows how the amount of throw changes with cut angle for a stun shot (i.e., sliding CB) at various speeds (slow, medium, and fast). **Diagram 2** illustrates how the terminology "cut angle" and "throw angle" are defined. The 2-ball is the OB and the 1-ball is placed at the ghost-ball (GB) target. Useful conclusions one can observe from **Diagram 1** include:

- for small cut angle shots (i.e., fuller hits), the amount of throw does not vary with shot speed, but increases with cut angle.
- for larger cut angle shots (i.e., thinner hits), the amount of throw is significantly larger for slower speed shots as compared to faster speed shots.
- the amount of throw decreases a little at large cut angles, but not by much (especially for slower speed shots).
- the maximum throw occurs at close to a half-ball hit (30º cut angle).

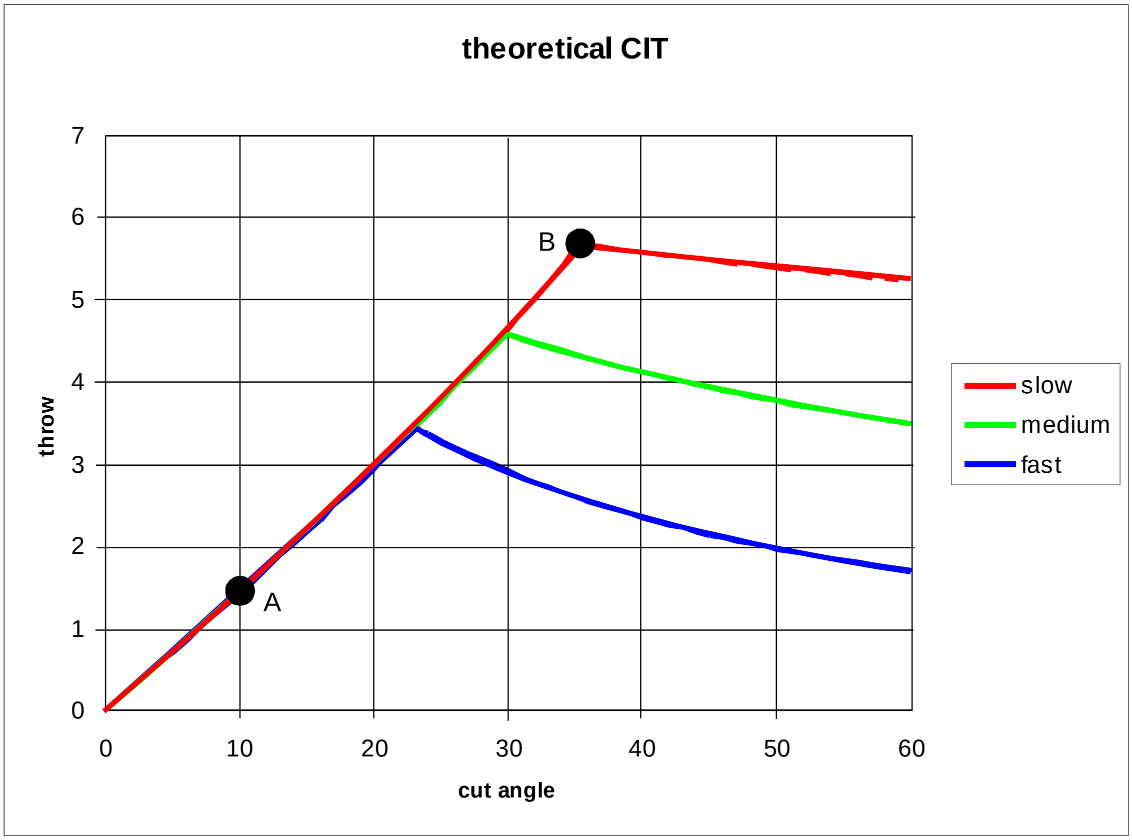


Diagram 1 **Predicted (theoretical) stun shot cut-induced throw (CIT)**

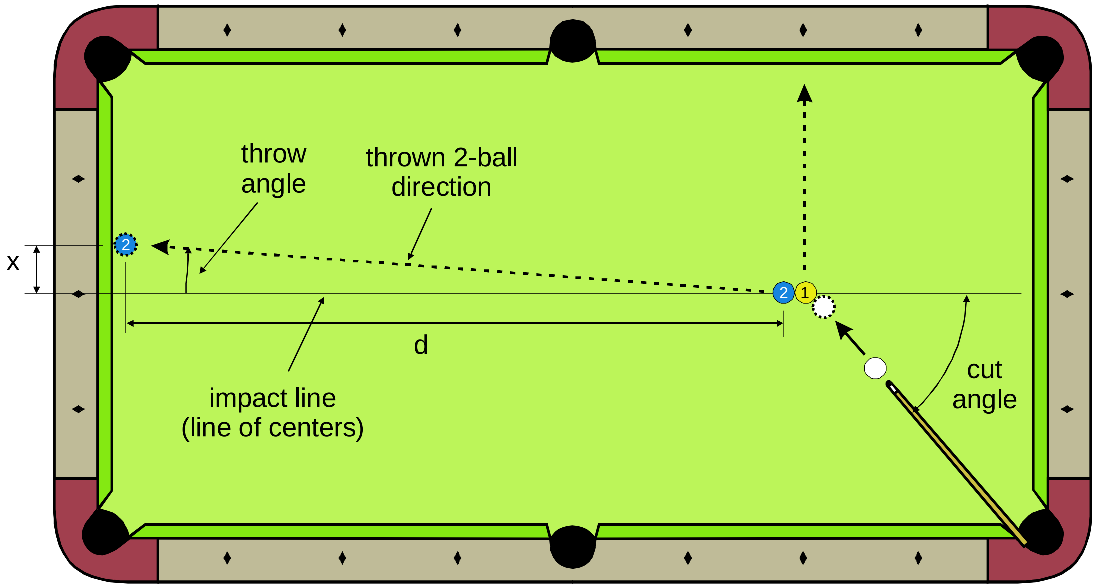


Diagram 2 Cut-induced throw (CIT) terminology and experiment

Many more plots like **Diagram 1**, which apply to various types of shots, can be found in **TP A.14**. Here's a summary of some of the other useful conclusions that can be drawn from the other plots:

- Both CIT and SIT are larger at slower speeds.
- CIT is larger for stun shots.
- SIT is larger, and most sensitive to sidespin, with stun shots.
- Inside English increases CIT, especially at small cut angles.
- Outside English creates maximum SIT at small cut angles.
- "Gearing" outside English results in absolutely no throw. (I'll look at this in more detail in a future article).
- both follow and draw reduce throw, and they do so by the same amount (I'll look at this in detail much more next month. Bob Jewett also looked at this effect in his May '06 article.)
- "cling" resulting from chalk smudges on the CB or OB can result in significantly higher friction, which can increase throw dramatically.

Over the next few months, we will look at some of these conclusions in detail and look at some examples where the knowledge can be applied at the table. **Diagram 3** illustrates examples of how you interpret the numbers in the plots. The two shots shown correspond to the points labeled "A" and "B" in **Diagram 1**. For shot "A," the cut angle is 10º (about a 7/8-ball-hit per **TP A.23**). At any speed (slow, medium, or fast), the amount of throw is the same (about 1.5º) for this cut angle. For shot "B," where the cut angle is about 35º (about a 1/2-ball-hit per **TP A.23**) and the shot speed is slow (the red curve), the amount of throw is the largest possible (almost 6º). The exact throw values will vary for different balls under different conditions (e.g., new, clean, and polished balls don't throw as much as typical used and dirty pool hall and bar balls), but the trends exhibited by the curves and the conclusions listed above will still apply. Remember, the plot in **Diagram 1** applies only for a stun shot. That's why slight bottom spin is shown for shot "B" in **Diagram 3**. At the slow speed, the bottom spin wears off and becomes stun at OB impact (e.g., see **HSV 3.1**).

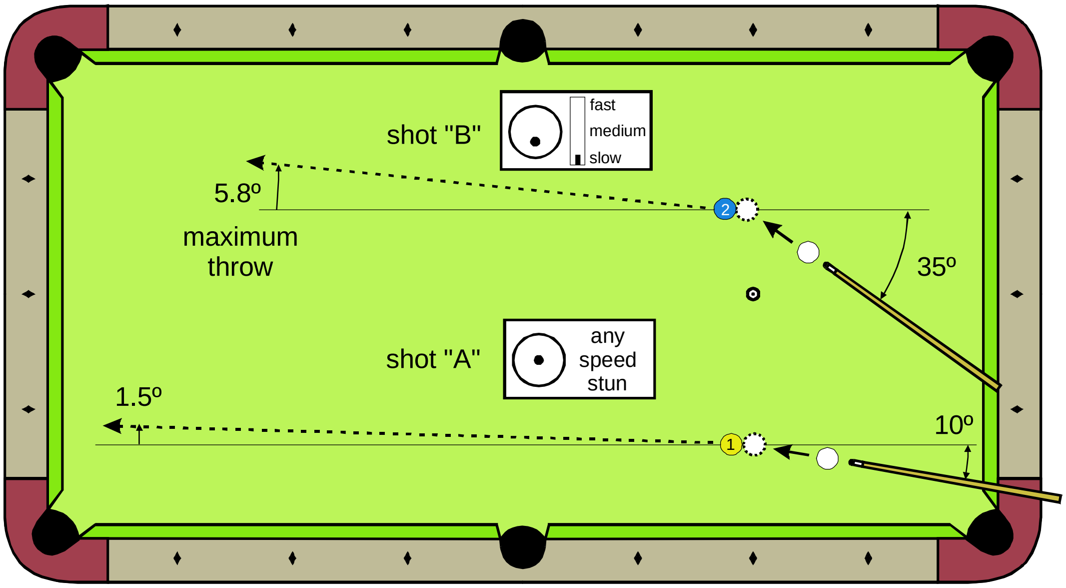
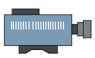


Diagram 3 **Example shots from Diagram 1**

TP A.23 – Ball-hit fraction vs. cut angle



HSV 3.1 – Stop-shot showing loss of bottom spin over distance

Diagram 2 shows the setup for an experiment that can be used to experimentally verify the results of the theoretical analysis (**Diagram 1**). The 2-ball is the OB, and it is positioned on the foot spot. The 2-ball will get thrown different amounts for different cut angles and speeds. The 1-ball, which is frozen to the 2-ball, serves several purposes in the experiment. It

- ensures an accurate impact line.
- makes it easy to aim; because for all cut angles, the CB is aimed at the center of the 1-ball.
- minimizes the effects of any unintentional English on the CB since very little can transfer through two balls.
- helps ensure a stun shot (i.e., the 1-ball will not have vertical-plane spin during impact). Even if the CB has a little unintentional draw or follow, very little will transfer to the 1-ball.

Diagram 4 shows the results of the experiment I performed. For each cut angle (13 total), and for each speed (3 total), I took three attempts. If I thought my aim, stroke, or speed was off for any attempt, I would redo it. The data points in the plot represent the average of the three attempts for each shot. If you want to perform the experiment yourself (which I highly recommend ... it is very good practice and helps builds throw intuition), I have a couple of files you can print from my website to help. They are under “templates” in the “Instructor and Student Resources” section. The first is a large graphic ruler you can tape to the head rail (adjacent to the cushion edge) to accurately measure the amount of throw (“x” in **Diagram 2**). The second is a GB and cut-angle graphic you can tape down on the table to help you aim along the different cut angle lines. You can cut or punch holes in the template to help position the balls accurately for each attempt. It also helps to “tap” the ball locations into place by striking down with an extra ball to create small dents in the cloth. This helps ensure accurate and repeatable ball placements. I used a digital video camera above the end rail so I could accurately measure the rail distance "x" for each shot (with later slow-motion playback), but a friend with a good eye and clipboard should be acceptable. (Unfortunately, I don't have many friends willing or patient enough to share my enthusiasm.) I think it is best if the shooter not see or hear the results as the attempts are being made; otherwise, the shooter might start subconsciously trying to compensate or adjust (i.e., cheat). Because the camera was looking at the rail for me, I didn't have to. Instead, I was able to concentrate on my aim, stroke, and shot speed.

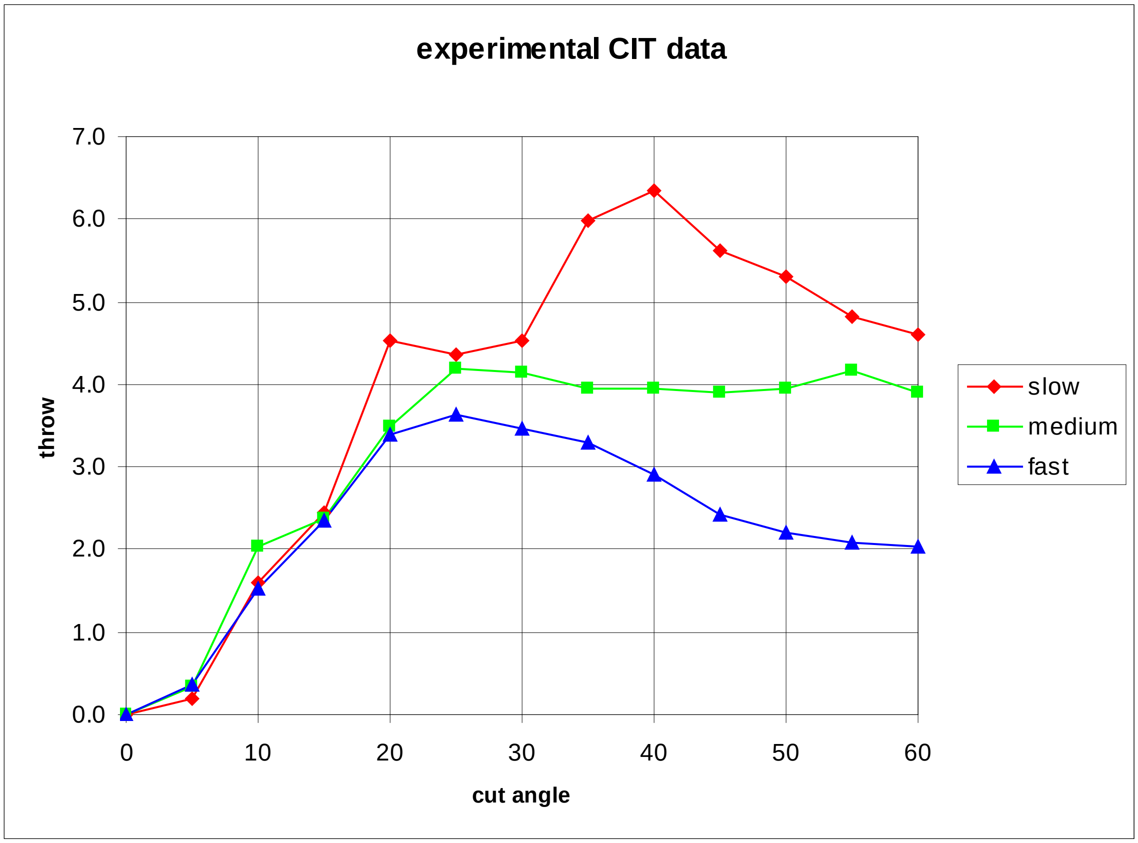


Diagram 4 Example experimental data for cut-induced throw (CIT)

If you want to calculate throw angles, you need to also measure distance "d" between the foot spot and the head rail (see **Diagram 2**). The throw angle can be calculated using a scientific calculator, a spreadsheet program (e.g., Excel), or even Google (www.google.com) with the following equation: $\tan^{-1}(x/d)$. For example, if you measure a rail throw distance (x) of 6 inches and the table distance (d) is 65 inches, the throw angle would be $\tan^{-1}(6/65) = 5.3°$. In Google, you just need to type "ATAN (6/65) =" in the search box and the answer magically appears. I learned this trick from Bob Jewett. Isn't Google amazing?

Bob Jewett has also performed the throw experiment described above (see his June, 1995 article). I didn't perform the experiment because I don't trust Bob, I just wanted to experience the results myself. Actually, Bob's plots (see www.sfbilliards.com/Misc/throw.gif) look much better (smoother) than mine, but our results match up very closely. Isn't it nice when independently performed experiments and theory can all happily agree? I was very pleased (and honestly, a little surprised) when I saw how well the experimental results and theory matched up. Science actually does work sometimes.

Well, that's enough for now. Next month, we'll take a closer look at the effects of follow, draw, cut angle, and speed on the amount of throw. I hope this series of articles will help you improve your understanding and intuition of throw effects. Although nothing beats tons of practice, knowledge can sometimes be helpful in your game. I hope you agree.

Good luck with your game,
Dr. Dave

PS:

- If you want to refer back to any of my previous articles and resources, you can access them online at ***billiards.colostate.edu***.

Dr. Dave is a mechanical engineering professor at Colorado State University in Fort Collins, CO. He is also author of the book: "The Illustrated Principles of Pool and Billiards."