

Everything You Always Wanted to Know About Cue Ball Squirt, But Were Afraid to Ask

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Definition: When the cue tip strikes a cue ball with a sideways offset, the ball does not depart from the collision along a direction that is exactly parallel to the cue stick axis. This off-angle cue ball deviation is called *squirt*. The term “squirt” first appeared in Robert Byrne’s *Advanced Technique in Pool and Billiards* (Harvest, San Diego, 1990). Squirt is sometimes called also “cue ball deflection” or “cue ball push.” There have been many suggestions about why this occurs, what stroke and grip techniques affect squirt, and how various equipment characteristics affect squirt. Many of these explanations are incomplete, or have minor errors, or, in some cases, they are outright wrong in their basic premises.

Simple Explanation: Here is a simple physical explanation of squirt. As the tip strikes the cue ball with sidespin, two things occur, the ball is set into linear motion, and the ball acquires spin about its center of mass along the vertical axis. On a normal shot, the cue tip does not slip on the ball, and the tip is in contact with the ball for only a very short time (about 0.001 second). The spinning ball pushes itself away from the tip and sets into sideways motion both the tip and the ball during this short contact period. Conservation of momentum in the sideways direction means that as the ball moves to the side in one direction, the tip must move to the side in the opposite direction. The speed at which the ball and tip move to the side depends on the relative masses of the two objects. If the ball is very heavy compared to the tip, then it will end up moving slowly to the side and the tip will be pushed quickly to the side; if the ball is very light compared to the tip, then the stick will be pushed very little, and the ball will push itself away quickly. The squirt angle is determined by the ratio of the sideways speed to the forward speed of the cue ball.

Importance: Why is squirt important to the player? If there were no squirt, then the player could use any amount of sidespin that he wanted, and he could aim his cue stick along a line that is parallel to the direction that he wants the cue ball to move during the shot. He would still be required to know how to pick the right angle to send the cue ball in order to make the shot succeed, and he would still be required to adjust for the cue ball swerve (the small amount of masse that is caused by stick elevation), but that would be the whole story as far as aiming is concerned. In real life, there is also squirt, which complicates things even more. With squirt, the player must also estimate how much

deviation there will be away from the cue stick direction, and he must compensate for this angle deviation. It does not matter if the player does this consciously, using a definite set of steps, or unconsciously, by shooting in a way that just “looks” right, the player must adjust for squirt. If the player does not adjust correctly, then the cue ball will not go in the correct direction after leaving the tip, and after sliding and rolling on the cloth to the object ball, it will not hit the object ball in the right spot in order for the shot to succeed.

If all cue sticks have squirt, so that players must always adjust for it, what difference does equipment make? The easiest way to understand this is to consider how accurate a typical shot must be in order to succeed. When the object ball is close to a pocket, then there is a relatively large line of contact spots that will result in a successful shot. The spot that results in hitting the far right side of the pocket might be as far as 10mm away from the spot that results in hitting the far left side of the pocket, and hitting any spot in between these extremes on the object ball will also result in shot success. But when the object ball is far from the pocket, then it must be hit very precisely. When a ball is near the center of the table, the spots on the object ball corresponding to the far-right and far left sides of the pocket might be separated by only 1mm. The actual values depend on the table size, the pocket size, the angle into the pockets (particularly for side pockets), and the ball size (small snooker balls require an even more precise hit than the larger pool balls) – the above numbers are examples of typical values.

On the above center-table shot, if there were no squirt, then the player is required to hit a target on the object ball that is about 1mm in size, that will require a certain amount of aiming and stroke accuracy, and that required accuracy will be the same regardless of whether he hits centerball or hits with sidespin. His target should be in the middle of that range, and he is allowed 0.5mm error to one side and 0.5mm error to the other side and the shot will still succeed. If the player has a very high-squirt cue stick and he uses sidespin for this shot, he might be required to aim as much as 100mm to the side of the object ball in order to hit near the desired target spot. The player must aim between 99.5mm and 100.5mm to the side of the actual desired contact point on the object ball in order for the shot to succeed. In other words, his estimation of the squirt must be accurate to within 1% for the shot to succeed. However, if the player has a low-squirt stick, then for this same shot he might be required to aim only 10mm to the side, and the shot will succeed if the cue ball actually hits between 9.5mm and 10.5mm. With the low-squirt cue, the player must judge the amount of squirt accurately to within 10% in order for the shot to succeed. The 10% accuracy required with the low-squirt cue is easier to achieve than the 1% accuracy required with the high-squirt cue. It is much easier to judge small distances to a given accuracy than it is to judge large distances to a given accuracy. As far as aiming adjustment due to squirt is concerned, less squirt is better than more squirt.

If common cue sticks are compared, the real range of squirt is probably more like a factor of 4 or 5 from the high-squirt cues to the low-squirt cues, rather than a factor of

10 as used in the above example, but the same general principle holds in any case. This argument does not mean that one player with a low-squirt cue will be more accurate than another player with a high-squirt cue – that would be comparing apples and oranges. This argument says that a given player, no matter how good he is at estimating the effects of squirt, will more consistently hit within the margin of error when using a low-squirt cue than when using a high-squirt cue. The improved consistency will result for any reduction in squirt, even smaller 10% or 20% improvements are significant, not just the factors of 4 or 5 that correspond to the extreme variations among common sticks.

Detailed Physics: So, if less squirt is better, what needs to be done to get less squirt. One obvious answer is to avoid using sidespin as much as possible, and if sidespin is necessary for the shot, then use as little as possible so that the aiming adjustments are minimized. But sometimes the player needs to use sidespin, so what affects squirt? How does squirt change as the tip offset increases? Are there any stroke mechanics that affect squirt? Is there more squirt for fast shots than for slow shots? What about the type of bridge (open or closed) or the bridge length? What about tip hardness, tip diameter, shaft taper, cue weight, and the cue balance point? How much more do lighter snooker balls squirt than heavier pool balls or even heavier carom balls? In order to attempt to answer some of these questions, we will examine the physics involved when the tip strikes the ball with sidespin.

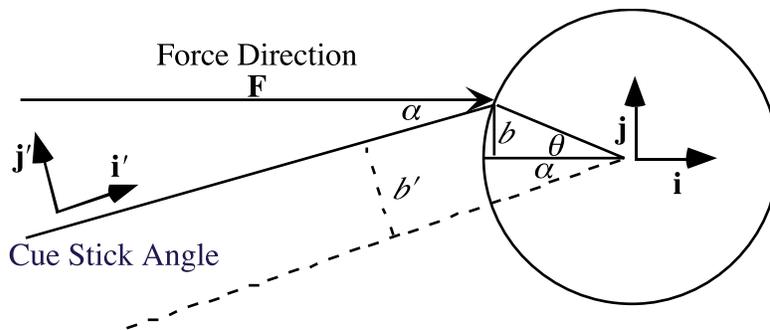


Figure 1. The initial stick angle and the resulting force direction are shown schematically from above. The squirt angle α is greatly exaggerated in this diagram to show the angles clearly.

Figure 1 is a view of the stick striking the cue ball from above, as if an overhead video camera were looking straight down on the shot. The tip is striking the cue ball with left sidespin. The squirt angle α is exaggerated in Figure 1 to make the drawing clear; as will be seen below, actual squirt angles are much smaller. There is a net force on the cue ball as the tip strikes the ball that is directed exactly along the x -axis as the figure is drawn. This force starts off at zero, builds up to some maximum value, and then decreases again to zero as the ball moves away from the tip. This force is applied to the

contact point, and this contact point corresponds to a sideways displacement along the y axis of length $b=R\sin(\theta)$. This force does two things. At any moment in time t during the tip-ball contact period Δ , it accelerates the ball along the x -axis according to the equation

$$\mathbf{F}(t) = M_b \dot{\mathbf{V}}_b$$

and it accelerates the angular motion about the vertical axis according to the equation

$$I\dot{\omega} = \mathbf{r} \times \mathbf{F}(t)$$

In this equation, $I = \frac{2}{5} M_b R^2$ is the moment of inertia of the ball, and

$\mathbf{r} = -R\cos(\theta)\hat{\mathbf{i}} + R\sin(\theta)\hat{\mathbf{j}}$ is the vector from the center of mass of the ball to the tip-ball contact point. R is the ball radius, M_b is the ball mass, and it is assumed that the ball is constructed from a material with a uniform density. The total linear momentum that the ball attains during the tip-ball contact time is given by the equation

$$\int_0^\Delta \mathbf{F}(t) dt = M_b \mathbf{V}_b = M_b V_{bx} \hat{\mathbf{i}}$$

The total spin that the ball attains during this time is given by

$$\begin{aligned} I\omega &= \left(-R\cos(\theta)\hat{\mathbf{i}} + R\sin(\theta)\hat{\mathbf{j}}\right) \times \int_0^\Delta \mathbf{F}(t) dt \\ &= RM_b V_{bx} \left(-\cos(\theta)\hat{\mathbf{i}} + \sin(\theta)\hat{\mathbf{j}}\right) \times \hat{\mathbf{i}} \\ &= -RM_b V_{bx} \sin(\theta) \hat{\mathbf{k}} \\ &= \omega_z \hat{\mathbf{k}} \end{aligned}$$

Using the relative tip offset (b/R), the spin is related to the ball speed according to

$$R\omega = -\frac{5}{2} \left(\frac{b}{R}\right) V_{bx} \hat{\mathbf{k}}$$

It is seen from this last equation that the spin on the cue ball depends linearly on the relative tip offset (b/R), it does not depend on the ball mass M_b , and that it is proportional to the cue ball speed V_{bx} . It is convenient to write the “spin” vector as $R\omega$ because this magnitude is the instantaneous speed of a point on the rotational equator of the ball as the ball spins about the vertical axis. The sign of ω_z is consistent with the right-hand-rule for vector cross products. A positive tip offset ($b>0$) results in a clockwise ball rotation (as viewed from above), which in turn means that the spin vector points down, and the spin vector component is negative ($\omega_z<0$). This is the situation shown in Figure 1. It should also be mentioned that tip offsets are limited approximately to the range $-\frac{1}{2} \leq (b/R) \leq \frac{1}{2}$. Tip offsets outside this range are likely to miscue, or if the offset is large enough, the tip would miss the ball entirely; in these cases, the above equations obviously would not apply.

It is somewhat remarkable that the details of the time-dependent force $\mathbf{F}(t)$ do not matter in this relation between the spin and speed of the cue ball. Presumably, a hard tip would have a shorter contact time Δ than a soft tip, but the magnitude of the force during

this shorter contact time is larger, such that the above time integral, which is called the *impulse*, ends up being the same; this impulse appears in both the equation for the linear velocity and the equation for the angular spin. This also rules out the idea that special stroke techniques (e.g. a tight wrist, or a loose wrist, or a wrist snap) can affect the spin/speed ratio. Such stroke techniques might affect the overall speed, or they might help or hurt other aspects of the shot, such as speed control and consistency, but they must affect both the spin and the speed equally in such a way that the ratio remains constant for a given tip offset.

The tip of the stick is in contact with the ball during the short contact time Δ , and whatever sideways velocity is attained by that point on the ball is also attained by the cue tip during that time. The instantaneous velocity of the contact point on the ball, relative to the ball center of mass, is given by

$$\begin{aligned}\omega \times \mathbf{r} &= \left(-\frac{5}{2}(b/R)V_{bx}\hat{\mathbf{k}}\right) \times \left(-\text{Cos}(\theta)\hat{\mathbf{i}} + \text{Sin}(\theta)\hat{\mathbf{j}}\right) \\ &= \frac{5}{2}(b/R)^2V_{bx}\hat{\mathbf{i}} + \frac{5}{2}(b/R)V_{bx}\sqrt{1-(b/R)^2}\hat{\mathbf{j}}\end{aligned}$$

Notice that the velocity component along the x -axis is always positive, but the velocity component along the y -axis depends on whether the tip strikes with a positive tip offset (left sidespin, as shown in Figure 1) or with a negative tip offset (right sidespin). The total velocity of the tip contact point is the vector sum of the above tangential rotational velocity and the ball center of mass linear velocity.

$$\begin{aligned}\mathbf{V}_{cp} &= \mathbf{V}_b + \omega \times \mathbf{r} \\ &= \left(1 + \frac{5}{2}(b/R)^2\right)V_{bx}\hat{\mathbf{i}} + \frac{5}{2}(b/R)V_{bx}\sqrt{1-(b/R)^2}\hat{\mathbf{j}} \\ &= V_{cpx}\hat{\mathbf{i}} + V_{cpy}\hat{\mathbf{j}}\end{aligned}$$

The total momentum of the cue stick and the ball together is conserved during the collision, so the initial momentum, which was aligned along the x' -axis as shown in Figure 1, must be the same as the final momentum, which includes both the stick and the ball motions. In particular, the initial momentum before the collision in the y' -axis is zero, so the final momentum along this axis must also be zero. The equation that results from this momentum conservation is

$$0 = M_{tip}V'_{tip,y} + M_bV'_{by}$$

The parameter M_{ip} is not the total stick mass. Rather, it is the inertial resistance to sideways motion that the tip possesses. This effective mass M_{ip} is sometimes called the *endmass*. The relation between the unit vectors in the (x,y) coordinate system and the (x',y') coordinate system depends on the squirt angle α according to

$$\begin{aligned}\hat{\mathbf{i}} &= \text{Cos}(\alpha)\hat{\mathbf{i}}' - \text{Sin}(\alpha)\hat{\mathbf{j}}' \\ \hat{\mathbf{j}} &= \text{Sin}(\alpha)\hat{\mathbf{i}}' + \text{Cos}(\alpha)\hat{\mathbf{j}}'\end{aligned}$$

Using this transformation, the ball velocity and the tip/contact-point velocity may be rewritten in the (x',y') coordinate axis as

$$\begin{aligned}\mathbf{V}_{tip} &= \mathbf{V}_{cp} \\ &= V_{cpx} \left(\cos(\alpha) \hat{\mathbf{i}}' - \sin(\alpha) \hat{\mathbf{j}}' \right) + V_{cpy} \left(\sin(\alpha) \hat{\mathbf{i}}' + \cos(\alpha) \hat{\mathbf{j}}' \right) \\ &= \left(V_{cpx} \cos(\alpha) + V_{cpy} \sin(\alpha) \right) \hat{\mathbf{i}}' + \left(V_{cpy} \cos(\alpha) - V_{cpx} \sin(\alpha) \right) \hat{\mathbf{j}}' \\ \mathbf{V}_b &= V_{bx} \cos(\alpha) \hat{\mathbf{i}}' - V_{bx} \sin(\alpha) \hat{\mathbf{j}}'\end{aligned}$$

Substitution into the conservation of momentum equation gives the general equation for the squirt angle.

$$\begin{aligned}0 &= M_{tip} \left(V_{cpy} \cos(\alpha) - V_{cpx} \sin(\alpha) \right) - M_b V_{bx} \sin(\alpha) \\ \tan(\alpha) &= \frac{\sin(\alpha)}{\cos(\alpha)} \\ &= \frac{M_{tip} V_{cpy}}{M_b V_{bx} + M_{tip} V_{cpx}} \\ &= \frac{\frac{5}{2} (b/R) \sqrt{1 - (b/R)^2}}{1 + (M_b/M_{tip}) + \frac{5}{2} (b/R)^2}\end{aligned}$$

It is clear from the last expression that the individual endmass and ball mass values are not important, but rather it is only the ball mass to endmass *ratio* that matters. In the limit $(M_b/M_{tip}) \rightarrow \infty$, then $\tan(\alpha) \rightarrow 0$ regardless of the tip offset. That is, there is no squirt, for any tip offset, in the limit of very heavy balls and/or very light endmass. The other limit, $(M_b/M_{tip}) \rightarrow 0$, is also interesting. In this limit of heavy endmass and/or a very light ball, squirt remains a smooth function of the tip offset.

Table I. Squirt Angles[†]

(M_b/M_{tip})	$\tan(\alpha)$	α	Stick Pivot Point
∞	0	0°	∞
200	.0043	0.25°	97.66"
100	.0086	0.49°	49.11"
75	.011	0.65°	36.98"
50	.017	0.97°	24.85"
40	.021	1.20°	20.00"
30	.028	1.59°	15.14"
20	.041	2.33°	10.28"
10	.077	4.38°	5.43"
0	.643	32.7°	0.57"

[†]Squirt angles are shown as a function of the (M_b/M_{tip}) mass ratio. All values are calculated for $(b/R) = \frac{3}{8}$, and the equivalent stick pivot points are calculated for pool balls with $R = \frac{9}{8}$ ".

Table I shows the squirt angle α for a $(b/R)=\frac{3}{8}$ tip offset (comfortably within the miscue limit) for a variety of (M_b/M_{tip}) mass ratios. As will be discussed below, pool cues have (M_b/M_{tip}) ratios that range from 20 (high squirt) to 100 (low squirt).

Figure 2 shows the squirt angle as a function of tip offset (b/R) for (M_b/M_{tip}) mass ratios of 20, 30, 50, and 100, which span the typical range of real cue sticks. Although the entire possible range from 0 to 1 is plotted for (b/R) , the only realistic values are in the $|b/R|\leq\frac{1}{2}$ range due to the miscue limit. It may be seen that the squirt angle is almost a linear function of the tip offset in this practical range for all of the plotted mass ratios; it is only for larger tip offsets that the nonlinearity becomes dominant.

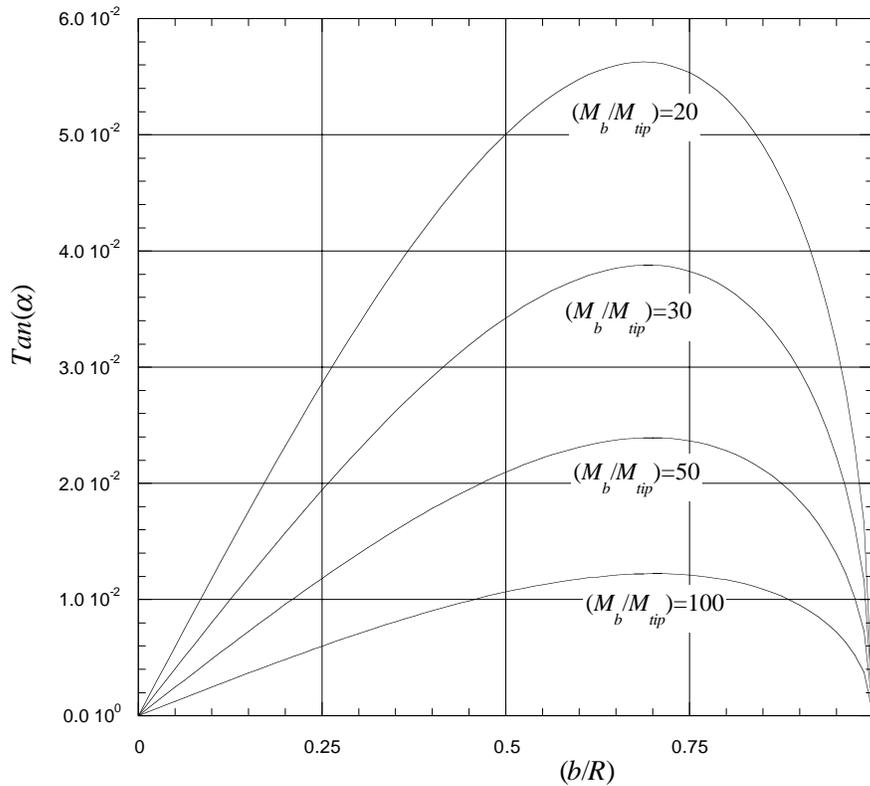


Figure 2. The squirt angle $Tan(\alpha)$ is shown as a function of tip offset (b/R) for several selected (M_b/M_{tip}) mass ratios.

The above discussion results in a relatively simple expression for the squirt angle α , but there are two hidden complications. One is that the squirt depends on the offset of the total force direction b rather than the actual tip offset b' (as observed by the shooter). The relation between these two offsets is

$$\frac{b}{R} = \sin(\theta)$$

$$\frac{b'}{R} = \sin(\theta + \alpha) = \sin(\theta)\cos(\alpha) + \cos(\theta)\sin(\alpha)$$

Because $(\theta + \alpha) < 90^\circ$ in Figure 1, it is clear that $b \leq b'$. For the small squirt angles that occur with real pool cues (see Figure 1), $b \approx b'$, so this minor detail is insignificant for

practical purposes. This small difference between the observed tip offset b' and the smaller effective tip offset b means also that low-squirt cues result in slightly more sidespin, and cue ball speed, than high-squirt cues (all other things being equal). Another way of looking at this is that the high-squirt cue loses more energy during the stick-ball collision by moving the cue stick and cue ball to the side than does a low-squirt cue. However, this spin increase is a very minor effect, probably too small for all but perhaps the most expert players to notice without special equipment or measuring devices.

The other hidden complication is the somewhat uncertain nature of the effective tip mass M_{tip} . In the above discussion M_{tip} plays the role of inertial resistance to a sideways force applied to the tip. Consider two extremes. One extreme is when essentially all of the cue mass is located right at the tip; in this case the effective tip mass will be the same as the stick mass. At the other extreme, consider a very long cue with all of the mass concentrated in the butt far from the tip; in this case, the effective tip mass will be very small and it would be easy for the ball to push the tip to the side with very little force. Actual cue sticks are somewhere in between these two extremes.

Rigid Cue Approximation: Some simplifying approximations are useful to find an estimate for M_{tip} : 1) the cue stick will be considered to be a rigid body, and 2) the interaction between the cue stick and the bridge hand and grip hand during the short tip-ball contact time will be neglected. Real cues are not rigid; in fact, the sideways vibrations can be felt, and sometimes even seen, when a cue ball is struck with sidespin. The neglect of the grip hand is sometimes called the “loose skin” approximation. The general idea is that the tip is in contact with the ball for such a short time, and for such a short distance, that the soft skin and flesh in the hand do not have time to tighten sufficiently in order to apply any additional forces to the stick that may then be transmitted to the ball. However, this assumption will allow the general effects on squirt of the stick length L , the balance point B , and the moment of inertia I_s to be estimated.

When the sideways force acts on the ball during the tip-ball contact time, a sideways force in the opposite direction and with the same magnitude is applied to the tip. The sideways impulse of the ball is opposite to the sideways impulse of the stick. This sideways force accelerates the stick center of mass to the speed

$$M_s \mathbf{V}'_{sy} = \int_0^{\Delta} F'_y(t) dt$$

The tip acquires also an angular velocity ω_s about the stick center of mass B (measured from the butt of the stick)

$$\begin{aligned} I_s \omega_s &= (L - B) \int_0^{\Delta} F'_y(t) dt \\ &= (L - B) M_s V'_{sy} \end{aligned}$$

It is convenient to write the moment of inertia of the stick as

$$I_s = k_s^2 M_s$$

The distance k_s , determined from $k_s^2=(I_s/M_s)$ is called the *radius of gyration*. If all the mass of the stick were located at this distance from the balance point, then the balance point and moment of inertia would remain unchanged. Because of physical constraints, $0 \leq B \leq L$, $0 \leq k_s \leq L$, and $0 \leq I_s \leq M_s L^2$.

The instantaneous tip velocity due to the rotation about the stick center of mass is

$$\begin{aligned}(L-B)\omega_s &= \frac{(L-B)^2 M_s V'_{sy}}{I_s} \\ &= \frac{(L-B)^2 V'_{sy}}{k_s^2}\end{aligned}$$

This rotational velocity component increases when the balance point is farther away from the tip (i.e. as $(L-B)$ increases), a trend that agrees with intuition and the general idea of moving mass from the tip end of the cue to the butt end.

The total sideways tip speed is the sum of the center of mass speed and the rotational speed at the moment after the collision

$$V'_{tip,y} = V'_{sy} + (L-B)\omega_s$$

The effective endmass parameter M_{tip} is defined from the relation $M_{tip}V'_{tip,y} = M_s V'_{sy}$. $M_s V'_{sy}$ is the actual impulse of the stick, and $M_{tip}V'_{tip,y}$ is what the equivalent impulse would have been if a point mass of M_{tip} had been located at the tip. This equivalence results in the relations

$$\begin{aligned}\frac{M_s}{M_{tip}} &= \frac{V'_{tip,y}}{V'_{sy}} \\ &= \frac{V'_{sy} + \frac{(L-B)^2 V'_{sy}}{k_s^2}}{V'_{sy}} \\ &= 1 + \frac{(L-B)^2}{k_s^2}\end{aligned}$$

It is clear from this equation that $(M_s/M_{tip}) \geq 1$ for all possible values of the balance point B , the cue length L , and the radius of gyration k_s . The following limits and bounds also hold for the balance point

$$\begin{aligned}\lim_{B \rightarrow L} \left(\frac{M_s}{M_{tip}} \right) &= 1 \\ \lim_{B \rightarrow 0} \left(\frac{M_s}{M_{tip}} \right) &= 1 + \frac{L^2}{k_s^2} \geq 2\end{aligned}$$

and for the radius of gyration

$$\lim_{k_s \rightarrow L} \left(\frac{M_s}{M_{tip}} \right) = 1 + \frac{(L-B)^2}{L^2} \leq 2$$

$$\lim_{k_s \rightarrow 0} \left(\frac{M_s}{M_{tip}} \right) = \infty$$

The (M_s/M_{tip}) ratio should be as large as possible to minimize squirt, so the rigid cue model suggests that the stick moment of inertia I_s , or equivalently the radius of gyration k_s , should be minimized, and that the balance point B , which is less important to the value of the mass ratio limit, should be as far towards the butt end as practical. A pool ball weighs 6 oz, and a typical pool cue weighs 18 to 20 oz, so the ratio $(M_b/M_{tip}) = (M_b/M_s)(M_s/M_{tip})$, which determines squirt, will be about three times smaller than the above (M_s/M_{tip}) ratio.

Squirt Measurements: It is possible to determine by experiment what is the effective tip mass for a given cue stick. One approach would be to pick a tip offset, measure the squirt angle α , and then determine the corresponding endmass from the above equations. It is difficult to obtain consistent results with this straightforward experiment for several reasons. First, the actual tip offset is not easy to see from the perspective of the shooter. The shooter can see the shaft offset, but the actual tip offset depends on both the shaft offset and the tip curvature. For a given shaft offset, a flatter tip will have a smaller actual tip offset than a more rounded tip. Secondly, the actual squirt angle as the ball leaves the tip is not easy to see because the cue ball moves too quickly. Instead, the offset of the cue ball direction from the stick angle can be measured at some distance away, for example by noting where the cue ball hits a distant cushion; a longer distance would seem to give more precise measurements of the actual squirt angle, but the problem is that longer distances allow also the cue ball to swerve, the masse effect due to the slightly elevated cue butt on almost all shots, and the measured result is thereby contaminated by this other effect. And finally, there are the small errors introduced in the stick angle because of imperfect stroke technique. Each of these problems can be addressed with the use of special equipment, such as jigs and high-speed video equipment.

An alternative experiment to determine the squirt characteristics of a cue stick is the “aim and pivot” approach proposed by Bob Jewett [*Billiards Digest*, June, 1997]. An object ball is placed about 12 inches (about one diamond on a 9ft table) from the cue ball. The precise distance is not important, but it should not be too large because of the potential for swerve contamination. The shooter lines up the shot for a centerball hit for an exactly straight-on shot. The shooter picks a trial “pivot point” a given distance away from the cue ball, and uses that distance for the bridge length. The shooter then pivots the butt of the cue about this pivot point, moving the tip to achieve a sideways tip offset

of about $(b/R)^{3/8}$, and then the shooter strokes the cue stick along this new line. A firm level stroke should be used, enough to send the object ball at least two table lengths, in order to minimize the swerve contamination. If the trial pivot point is too close to the cue ball, then the stick angle change induced by the pivot will be larger than the squirt angle; in this case the cue ball will not hit the object ball straight on, it will instead hit on the same side as the tip offset. If the trial pivot point is too far from the cue ball, then the stick angle change induced by the pivot will be smaller than the squirt angle; in this case the cue ball will not hit the object ball straight on, it will instead hit on the opposite side as the tip offset. By adjusting the trial pivot point forward and backward appropriately, a pivot point can be found for which the induced stick angle is exactly the same as the squirt angle; in this case the cue ball will hit the object ball straight on, and the cue ball will sit spinning in place. This special pivot location is called the *stick pivot point*. All three of these possible situations are shown in Figure 3. The only purpose of the object ball in this procedure is to provide a sensitive measure of where the cue ball hits: left, right, or straight on. A selection of stick pivot points is given in Table I along with the corresponding (M_b/M_{ip}) ratios.

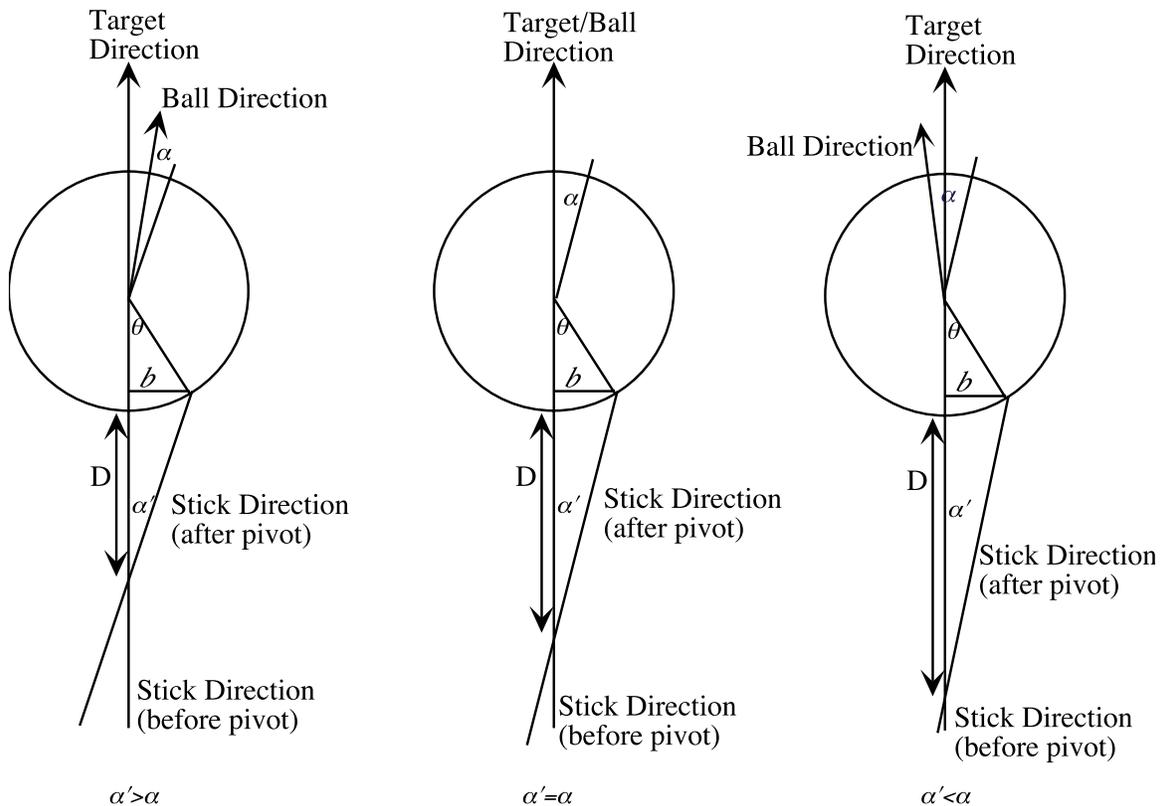


Figure 3. The aim-and-pivot approach is shown schematically for three different trial pivot points. The trial pivot point is too short in the left diagram, too long in the right diagram, and is at the correct distance in the center diagram.

A high-squirt cue might have a pivot point of 10 inches ($M_b/M_{tip} \approx 20$), an average cue will have a pivot point in the 16 to 18 inch range ($M_b/M_{tip} \approx 30$), a good cue will have a pivot point around 30 inches (close to the joint on a two-piece cue, $M_b/M_{tip} \approx 50$), and a low-squirt cue will have a pivot point of 40 inches or longer ($M_b/M_{tip} \approx 100$). It is difficult to stroke accurately with bridge lengths exceeding ~ 18 inches, and the above pivot procedure must be modified in order to allow for a reasonable bridge length in these cases. As a practical matter, it is recommended also to use a mixture of both left and right offsets when executing the shots to determine the pivot point; this tends to eliminate the bias due to imperfect, but consistent, swoops in the player's stroke.

The advantage of the pivot point method for determining the squirt characteristic of a cue stick, compared to an attempted direct measurement based on the squirt angle, may be seen by examining the dependence of the measured pivot point on the tip offset. Consider the center diagram in Figure 3. The correct stick pivot point has been found for a particular tip offset b such that the stick angle α' is exactly the same as the corresponding squirt angle α . The stick angle after the pivot is given by

$$\tan(\alpha') = \frac{(b/R)}{(D/R) + 1 - \sqrt{1 - (b/R)^2}}$$

in which D is the pivot point distance as shown in Figure 3. If the correct pivot point has been found, then the stick angle α' is equal to the squirt angle α for this offset. This results in the relation

$$\tan(\alpha') = \tan(\alpha)$$

$$\frac{(b/R)}{(D/R) + 1 - \sqrt{1 - (b/R)^2}} = \frac{\frac{5}{2}(b/R)\sqrt{1 - (b/R)^2}}{1 + (M_b/M_{tip}) + \frac{5}{2}(b/R)^2}$$

This equation can be solved for (D/R) as a function of the mass ratio and the tip offset.

$$\left(\frac{D}{R}\right) = \frac{2(M_b/M_{tip}) + 7 - 5\sqrt{1 - (b/R)^2}}{5\sqrt{1 - (b/R)^2}}$$

$$\approx \frac{2(1 + (M_b/M_{tip}))}{5} + \frac{(7 + 2(M_b/M_{tip}))}{10}(b/R)^2 + O((b/R)^4)$$

It may be verified that (D/R) is a slowly varying function of the tip offset (b/R) in the practical range $0 \leq (b/R) \leq 1/2$. It is only as (b/R) approaches 1 that the measured stick pivot point would begin to change significantly. For example, for a (M_b/M_{tip}) ratio of 40, the stick pivot point for an offset of $(b/R) = 1/4$ has the value of $D = 19.0''$, the offset of $(b/R) = 3/8$ has the value of $D = 20.0''$, and for the maximum practical tip offset of $(b/R) = 1/2$, the pivot point has the value of $D = 21.5''$. Even the $(b/R) \rightarrow 0$ limit results in $D = 82''/5 = 16.4''$, which is still only a modest change in the pivot point distance. Therefore, the stick pivot point is, to a reasonable approximation, essentially independent of the tip offset in the range

$\frac{1}{4} < (b/R) < \frac{1}{2}$, greatly reducing the effects of an imperfect stroke on the squirt measurement. Because the stroke errors are reduced, due to the design of the experiment, the aim-and-pivot method is a reliable way to measure and to specify the amount of squirt for a given stick. Due to the way the (M_b/M_{tip}) ratio appears in the numerator of the (D/R) expression, the variation of the pivot point with tip offset is somewhat less for a low-squirt cue than for a high-squirt cue.

Notice that for a given mass ratio and relative tip offset, the stick pivot point D depends on the ball radius. This is because the “natural unit” for the pivot point is in terms of the ball radius or ball diameter. If all other things were equal, a given stick would have a shorter stick pivot point D for smaller snooker balls, and it will have a longer stick pivot point for larger carom balls. Of course, all other things aren’t equal in this case, and, in particular, the ball mass M_b increases with the ball radius, and this also tends to make the stick pivot point D shorter for lighter snooker balls and longer for heavier carom balls.

The amount of squirt changes with the tip offset, so how does the stick pivot point end up being (almost) independent of the tip offset? It is because of a cancellation of errors. When pivoted about the correct pivot point distance, a given stick angle results in a particular tip offset, and that tip offset results in an amount of squirt that compensates almost exactly for the stick angle; the result is that the cue ball goes in the intended direction after being struck by the tip. If the stick angle is increased by a small amount, then the tip offset increases by a small amount, and that increased tip offset results in an increased squirt angle; the result is that the increased squirt angle compensates for the increased stick angle, and the cue ball still goes in the intended direction after being struck by the tip.

In addition to using the aim-and-pivot shot as a way to measure the squirt inherent in a given cue stick, it may also be used by the player to aim shots with sidespin. In this context, this technique is sometimes called “backhand English.” The player aligns the shot as if he were going to shoot with no sidespin. He then pivots about the bridge hand, by moving his back hand holding the butt of the cue, in order to align the cue stick along a new angle, and strokes along this new angle to execute the shot. If his bridge length is close to the stick pivot point, then the cue ball will go along the correct line after being struck with the tip. Backhand English is sometimes taught to beginners when they first begin to learn how to use sidespin. Unfortunately, this beginner instruction usually does not include an explanation of squirt, so the student is left with an incomplete knowledge of why the technique works, and, perhaps more importantly, why it fails for some cue sticks but appears to work well with other cue sticks. Indeed, a low-squirt cue will have a pivot point near the rear grip hand, and the appropriate “fronthand English” technique in this case is to pivot with the rear hand held fixed by sliding the bridge hand to the side, rather than holding the bridge fixed and moving the rear hand.

Rigid Cone Model: It is instructive to consider the pool cue to be shaped like a cone and constructed of wood with a constant density throughout. This model allows the balance point and moment of inertia to be estimated and predicted squirt can be compared to actual squirt measurements. Although the butt and part of the cue shaft are usually conically shaped, the end of the shaft near the tip is shaped like a cylinder, with constant diameter. Different types of woods are usually spliced together to form the butt, and these different types of woods have various densities, usually with the heavier woods in the butt and the lighter woods in the shaft. Even one-piece cues are usually constructed with heavy wood spliced in the butt. Some cue makers are designing shafts that are purposely lighter near the tip end than usual; the Predator shaft, for example, is drilled to form a hollow tube for a few inches near the tip in order to reduce the endmass. There are materials other than wood that are used in the tip, ferrule, joint, and in various places in the butt, and these materials all have various densities. Having pointed out all of these shortcomings, we now proceed to characterize this *rigid cone* cue model.

Consider the butt end of the cue stick to have a radius R_1 , and the tip end of the cue to have a, presumably smaller, radius of βR_1 , and a total length L . The radius of the conical cue stick at a given distance z along the length is

$$R(z) = R_1 + \frac{R_1(\beta - 1)}{L} z$$

The total mass of such a cue is

$$M_s = \int_V \rho(\mathbf{r}) dV = \rho \int_0^L \int_0^{2\pi} \int_0^{R(z)} r dr dz d\theta = \frac{1}{3} \pi \rho L R_1^2 (1 + \beta + \beta^2)$$

Cylindrical coordinates are convenient for the evaluation of the volume integrals. The center of mass (i.e. the balance point) B measured from the butt end is

$$B = \frac{\rho}{M} \int_0^L \int_0^{2\pi} \int_0^{R(z)} r z dr dz d\theta = \frac{(1 + 2\beta + 3\beta^2)}{4(1 + \beta + \beta^2)} L$$

The moment of inertia about this balance point around the vertical axis is

$$\begin{aligned} I_s &= \int_V \rho (y^2 + (z - B)^2) dV = \rho \int_0^L \int_0^{2\pi} \int_0^{R(z)} (r^3 \sin^2 \theta + r(z - B)^2) dr dz d\theta \\ &= \frac{3((1 + \beta)^4 + 4\beta^2)}{80(1 + \beta + \beta^2)^2} M_s L^2 + \frac{3(1 + \beta + \beta^2 + \beta^3 + \beta^4)}{20(1 + \beta + \beta^2)} M_s R_1^2 \end{aligned}$$

Pool, billiard, and snooker cues have length $55'' \leq L \leq 60''$, have a diameter at the butt $R_1 \approx 1.25''$, and a tip diameter of about $0.5''$, which corresponds to $\beta = \frac{2}{5}$. Consequently, the second term in the above expression for I_s is much smaller than the first term, and in the remainder of this discussion it will be ignored. The balance point of a $58''$ conical $\beta = \frac{2}{5}$ cue would be about 21 inches from the end of the butt. Actual pool cues have a balance

point 15 to 19 inches from the butt. This gives an idea of the accuracy of the conical model and also the kind of deviations to expect when compared to actual cue sticks.

These expressions for the balance point and moment of inertia may be used to predict (M_s/M_{tip}) ratios.

$$\frac{M_s}{M_{tip}} = \frac{8(6 + 9\beta + 10\beta^2 + 4\beta^3 + \beta^4)}{3((1 + \beta)^4 + 4\beta^2)} \quad (\text{rigid cone approximation})$$

The endmass is a small fraction of the stick mass for this rigid cone model. It is useful to consider a couple of examples to see exactly what is the range of values. If the stick were a cylinder with constant radius, corresponding to $\beta=1$, then the rigid cone model would predict $(M_s/M_{tip})=4$ and if the tip goes all the way down to a point, corresponding to $\beta=0$, then $(M_s/M_{tip})=16$. The typical pool cue value of $\beta=2/5$ results in an intermediate value of $(M_s/M_{tip})=6.83$ for a rigid cone. Although a rigid cone is a coarse approximation of an actual cue stick, this simple model gives an idea of what might be expected for the endmass and how that mass might depend on the cue stick weight, the stick balance point, and the tip diameter. A pool ball weighs 6 oz, and a typical pool cue weighs 18 to 20 oz, suggesting a maximum value of $(M_b/M_{tip})\approx 5$. As discussed above, even this maximum estimate is a factor of 4 to 20 too small compared to actual pool cues. As already mentioned, the balance point is different for the conical cue model than for actual pool cues, and this accounts for some of this discrepancy, but this alone is not sufficient to account for a factor of 4, much less a factor of 20. However, the general trends predicted for the rigid cone model regarding the endmass are expected to be valid, despite the lack of quantitative agreement with actual pool cues.

The major discrepancies between these predictions and the actual observations are probably due to failure of the rigid cue approximation. Indeed, if the mass in the cue is far enough away from the tip, it would be irrelevant to the cue ball because the speed of sound in the stick limits the amount of information that can be reflected back from the butt end of the stick during the short contact time. The balance point and moment of inertia are static quantities, and do not depend on the dynamics of the tip-ball collision or on the internal vibrations of the stick during this short collision time. This implies that there should be some kind of, as yet uncharacterized, relationship between the effective endmass and the mass distribution as a function of the distance from the tip other than that predicted by the rigid cue approximation. At present, the endmass is best considered simply an empirical quantity that characterizes a given cue stick. In contrast, the loose skin approximation appears to be valid, or at least it is not inconsistent with the observed discrepancies between the measured squirt and the predicted squirt based on the rigid cone approximation.

Other Squirt Experiments: In addition to numerous stick pivot point measurements that have been reported by several people, a few other squirt experiments have been performed that should be mentioned. All of these have been reported and discussed in the newsgroup `rec.sport.billiard`.

- Bob Jewett has reported the squirt results from gluing a cue tip onto a solid metal rod; he observed large squirt for this “cue.”
- Mike Page has reported the results of clamping a small lead weight onto the ferrule of a cue; large squirt was observed, and the amount of squirt decreased as the weight was moved farther from the tip until the weight was about 8” away at which distance the normal amount of squirt was observed.
- Jim Buss has inserted a brass rod into the end of a normal pool cue shaft and then attached a normal ferrule and tip in the normal way; a large amount of squirt was observed with this cue.
- The author has reported the squirt results of using a normal pool cue along with a very light Styrofoam ball (found in hobby and craft shops); a large amount of squirt was observed in this experiment.
- The Predator web site contains the reports of several experiments. A jig (called *Iron Willie*) is used to eliminate stroke imperfections and to allow for precise shot repetition. The cue ball squirt angle is measured as a function of shaft offset for a variety of cue shafts including a very light one made of synthetic composite materials. Also, some data is presented that suggests a dependence on the tip curvature.

Desirability: Is squirt ever desirable? If a player has a physical handicap that prevents him from stroking straight, then it might be advantageous for that player to use a cue that has a pivot point near his bridge length. If the player has the ability to line up correctly for the shot in the first place, then such a cue stick will tend to compensate for his unusually large stroke errors. For such a player, the stroke errors are larger than the judgment errors required to compensate for squirt, and he might be better off eliminating the large errors and accepting the smaller judgment errors.

Another possible exception is when a player uses a separate cue for the break shot for open-break games such as 9-ball or 8-ball. In this case, the player wants to hit the cue ball with no sidespin, his stroke involves more body movement and upper arm movement than the usual stroke, his bridge length and stroke distance are longer than usual, and his arm applies more force than usual during the forward stroke. All these things contribute to larger than normal errors in the placement of the tip on the ball and in stick alignment. The sideways errors in tip placement are equivalent to a pivot about the bridge (see Figure 3). With a low squirt cue (with the bridge length shorter than the stick pivot

point), any unintentional pivot results in a different cue ball direction and eventually in an off-center hit on the head ball. If the stick pivot point is at the bridge length however, then the unintentional sidespin is accompanied by squirt that compensates for the change in the stick angle, and the cue ball goes along the original intended target direction. The cue ball does have sidespin on it, so some energy is lost in the break shot due to this unwanted rotation, but the cue ball will still hit the head ball straight on as intended. In other words, a high-squirt cue does not solve all the problems associated with imperfect technique on the break shot, but it addresses the most important problem, hitting the head ball straight. Most players bridge longer on their break shots than for other normal shots, so the pivot point of the break cue should be at the appropriate distance for the individual player. With such a break cue, the player should take extra care when setting up initially for the break shot in order to take full advantage of the compensating squirt characteristics of the stick.

Summary: With the detailed discussion given above for squirt, several of the questions posed in the beginning can be answered.

- How can squirt be minimized? By reducing the tip offset or by using a cue stick with a small endmass.
- Does shaft flexibility affect squirt? No, not directly. Shaft flexibility may affect the endmass and thereby affect squirt indirectly, but this is probably a relatively minor effect. This means that the player is able to choose a cue with a desired amount of shaft flexibility for other reasons, without having to compromise the squirt characteristics.
- Does tip curvature affect squirt? No, not according to the above analysis, except for the fact that the tip curvature affects the actual tip offset for a given shaft offset. However, there are data at the Predator web site that suggests that a rounder tip (i.e. a dime radius compared to the larger nickel radius) reduces squirt.
- Does squirt depend on shot speed? No, not directly. Shot speed might affect the endmass, but this is probably a relatively minor effect. The observation that different aiming is required for different shot speeds, particularly on longer shots, is probably due to cue ball swerve, not to squirt.
- Does squirt depend on the cue stick weight? Yes, but only when it affects the endmass. Weight added more than about 10" away from the tip apparently has little effect. This means that the player is free to choose stick weight for other reasons without compromising squirt characteristics.
- Does squirt depend on the stick balance point? Yes, according to the rigid cone approximation, but only because it has a small effect on the endmass.
- Do ivory or brass ferrules have more squirt than synthetic materials? If so, it is primarily because of the density of the material and the resulting effect on the

endmass. It appears unlikely that the hardness or other physical characteristics affect directly the squirt. However, a thick brass ferrule will probably have more squirt than a thin brass ferrule. Also, some ferrules are attached to the shaft with a metal screw or stud; these cues probably have larger squirt than an otherwise equivalent cue with a standard wood tenon.

- Does the tip diameter affect squirt? Yes, because smaller tips will have smaller endmass, all other things being equal. This trend is predicted by the rigid cone model. However, there are other ways to reduce endmass than by using a small tip diameter, such as the approach used by the Predator design.
- Does tip hardness affect squirt? No, not according to the above analysis. This also suggests that the tip-ball contact time, which is related to tip hardness, does not directly affect squirt, but this has not been proven independently, and it is possible that contact time does play at least a minor role in determining the effective endmass through speed-of-sound mechanisms. To the extent that tip hardness is independent of squirt, this means that the player is free to choose tip hardness based on personal preference without compromising the squirt characteristics.
- Is squirt caused by the tip slipping on the ball? No, squirt occurs even when the tip does not slip. The tip does not slip on normal shots.
- Are there any stroke techniques that can be used to minimize squirt? No, not unless the technique somehow reduces the tip offset, in which case the same shot could have been achieved simply by stroking normally and using the smaller tip offset in the first place.
- Is it better to use an open bridge than a closed bridge? It is possible that a tight closed bridge might increase the effective endmass, and thereby increase squirt, but this is probably a very minor effect.
- For a given cue stick, will snooker balls squirt more than pool balls, and will carom balls squirt less? There are two separate effects, mass and ball diameter. Due to the ball/tip mass ratio, the lighter snooker balls will tend to squirt more than pool balls, and the heavier carom balls will tend to squirt less. Furthermore, the stick pivot point depends on the ball radius, so, for a given relative tip offset and ball/tip mass ratio, the larger balls will tend to have longer stick pivot points.
- If I use a low-squirt cue, must I suffer from excessive or sensitive sidespin? All other things being equal, high-squirt cues will appear to get slightly less spin (and speed) than a low-squirt cue for a given tip offset, but this is a very minor effect, smaller, for example, than the differences in tip curvature between two tips might affect the amount of sidespin.
- Does the amount of shaft bend, buckle, or vibration affect squirt? No, the shaft is set into motion as the tip strikes the ball, but the actual bending and/or buckling occurs after the short tip-ball contact time. If the ball has already separated from

the tip before the bending occurs, then the bending and buckling can have no effect on the ball.

- Must I trust the advertising of the cue makers to tell me how much squirt a stick will have? No, the aim-and-pivot squirt test is a reliable way to determine squirt, it requires no special equipment, and with a little practice, it can be performed by anyone with a reasonably straight stroke.
- What is the optimal squirt characteristics for a break cue? The stick pivot point should be at the bridge length. For most players, this will be typically in the 14" to 16" range.

With these answers in mind, there are still many details that are not understood at present regarding squirt. A more detailed understanding of endmass appears to be the most important. How exactly does the mass distribution along the shaft contribute to the observed effective endmass? And even if a complete and correct understanding of endmass and squirt were known, there would be still the subjective challenge of choosing a cue stick with the optimal amount of squirt for each player.

Postscript: This document is a first step at collecting together information and ideas related to squirt, it is not the final answer to this interesting and practically important topic. This document will evolve in the future in order to report and describe new developments, new explanations, and new squirt data. Comments and corrections should be sent to the author at email : //shepard@tcg.anl.gov.