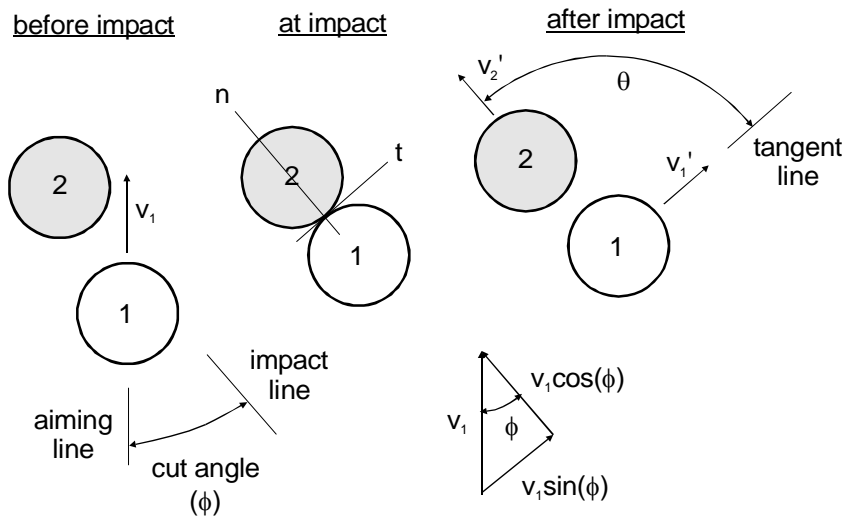




TP 3.1 **90° rule**

supporting:
 “The Illustrated Principles of Pool and Billiards”
<http://billiards.colostate.edu>
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Assumptions:

- the coefficient of friction between the balls is negligible
- the collision is perfectly elastic (the coefficient of restitution is 1)
- both balls have the same mass
- ball 2 is stationary initially

Because there are no forces in the t direction, conservation of linear momentum gives:

$$v'_{1t} = v_{1t} = v_1 \cdot \sin(\phi) \quad \text{the t component of ball 1's velocity remains constant}$$

$$v'_{2t} = v_{2t} = 0 \quad \text{ball 2 has no t component of velocity before or after impact}$$

Conservation of momentum in the n direction gives:

$$v'_{1n} + v'_{2n} = v_{1n} = v_1 \cdot \cos(\phi) \quad (1)$$

The coefficient of restitution relation gives:

$$v'_{2n} - v'_{1n} = v_{1n} \quad (2)$$

the separation speed is equal to the approach speed

Solving Equation 1 and 2 gives:

$$v'_{1n} = 0 \quad \text{ball 1 loses all of its speed in the n direction}$$

$$v'_{2n} = v_{1n} \quad \text{ball 1 transfers all speed in the n direction to ball 2}$$

After impact, ball 1 has a t component only and ball 2 has an n component only. Therefore,

v'_1 and v'_2 are perpendicular

$$\theta = 90\text{-deg}$$

Here is a more elegant proof:

Conservation of momentum gives:

$$V_1 = V'_1 + V'_2 \quad (3) \quad \text{where a capital letter denotes a vector}$$

Conservation of energy gives:

$$v_1^2 = v'^2_1 + v'^2_2 \quad (4) \quad \text{where a lower case letter denotes magnitude}$$

Taking the vector dot product of Equation 3 with itself gives:

$$v_1^2 = v'^2_1 + 2 \cdot v'_1 \cdot v'_2 \cdot \cos(\theta) + v'^2_2 \quad (5)$$

Subtracting Equation 4 from Equation 5 gives:

$$2 \cdot v'_1 \cdot v'_2 \cdot \cos(\theta) = 0$$

This can be true only if:

$$\theta = 90\text{-deg}$$

or if

$$v'_1 = 0 \quad \text{for a head-on stop shot}$$