technical proof

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TP 7.3
Ball-rail interaction and the effects on vertical plane spin

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
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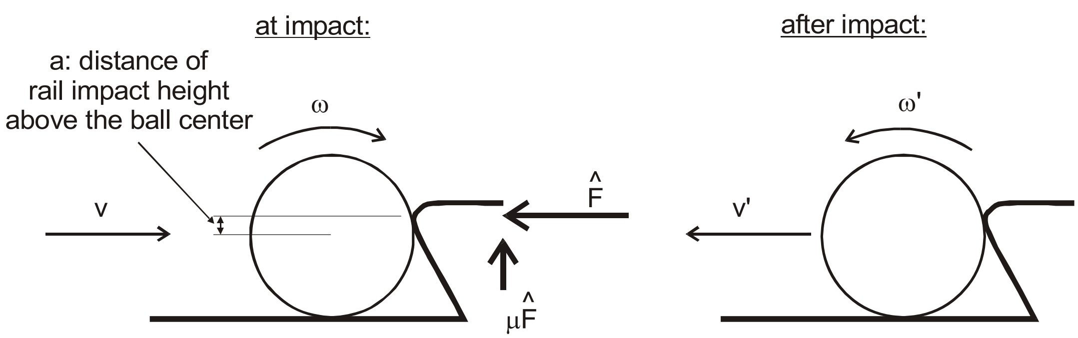

Ball properties:

$$m := 6 \cdot oz \qquad D := 2.25 \cdot in \qquad R := \frac{D}{2} \qquad I_o := \frac{2}{5} \cdot m \cdot R^2$$

From the coefficient of restitution:

$$v' = e \cdot v$$

From linear impulse and momentum:

$$F' = m(v' + v) = m \cdot (1 + e) \cdot v$$

From angular impulse and momentum:

$$\mu \cdot F' \cdot R + F' \cdot a = I_o \cdot (\omega' + \omega)$$

Solving for the ball angular speed after impact gives:

$$\omega' = \frac{F'}{I_{o'}} \cdot (\mu \cdot R + a) - \omega = \frac{m \cdot (1 + e) \cdot v}{\frac{2}{5} \cdot m \cdot R^2} \cdot (\mu \cdot R + a) - \omega = \frac{5 \cdot (1 + e) \cdot v}{2 \cdot R^2} \cdot (\mu \cdot R + a) - \omega$$

Typical values for different initial conditions

$v := 5 \cdot \frac{ft}{sec}$ $\qquad e := 0.7$ $\qquad \mu := 0.17$ $\qquad a := 0.08 \cdot R$

$v' := e \cdot v$ $\qquad v' = 3.5 \frac{ft}{s}$

For rolling without slipping after impact:

$\omega' := \frac{v'}{R}$ $\qquad \omega' = 37.333 \frac{rad}{sec}$

For rolling without slipping at impact, ω = v/R giving:

$\omega := \frac{v}{R}$ $\qquad \omega = 53.333 \frac{rad}{s}$

$\omega' := \frac{5 \cdot (1 + e) \cdot v}{2 \cdot R^2} \cdot (\mu \cdot R + a) - \omega$ $\qquad \omega' = 3.333 \frac{rad}{sec}$ $\qquad$ ω' close to 0

For topspin at impact, ω > v/R giving:

$\omega := 1.5 \cdot \frac{v}{R}$ $\qquad \omega = 80 \frac{rad}{s}$

$\omega' := \frac{5 \cdot (1 + e) \cdot v}{2 \cdot R^2} \cdot (\mu \cdot R + a) - \omega$ $\qquad \omega' = -23.333 \frac{rad}{sec}$ $\qquad 0 < |\omega'| < \frac{v'}{R}$

For stun shot with ω = 0, there is no friction impulse and:

$\omega' := \frac{5 \cdot (1 + e) \cdot v}{2 \cdot R^2} \cdot (a)$ $\qquad \omega' = 18.133 \frac{rad}{sec}$ $\qquad 0 < \omega' < \frac{v'}{R}$

For bottom spin at impact with ω < v/R, the friction impulse is in the opposite direction and:

$\omega := -0.5 \cdot \frac{v}{R}$ $\qquad \omega = -26.667 \frac{rad}{s}$

$\omega' := \frac{5 \cdot (1 + e) \cdot v}{2 \cdot R^2} \cdot (-\mu \cdot R + a) - \omega$ $\qquad \omega' = 6.267 \frac{rad}{sec}$ $\qquad$ ω' close to 0