





## <u>TP A.17</u> The effect of the amount of friction on throw

supporting: "The Illustrated Principles of Pool and Billiards" <u>http://billiards.colostate.edu</u> by David G. Alciatore, PhD, PE ("Dr. Dave")

## See TP A.14 for background information and fundamental results.

Typical values for the parameters used in the equations:

$\mu := 0.06$	average coefficient of friction between the balls	
$\underset{m}{R} \coloneqq \frac{1.125 \cdot \text{in}}{\text{m}}$	ball radius converted to meters	R = 0.029
$\mathbf{v} := \frac{3 \cdot \mathbf{mph}}{\frac{\mathbf{m}}{\mathbf{s}}}$	average cue ball speed converted to meters/sec	v = 1.341
$\omega_{\text{roll}} \coloneqq \frac{v}{R}$	natural-roll spin rate	

Relationship between friction and sliding contact speed (from TP A.14):

$$\mu(\mathbf{v}_{rel}) \coloneqq \begin{pmatrix} -0.77 \cdot \mathbf{v}_{rel} \\ 0.009951 + 0.108 \cdot e \end{pmatrix}$$

Relative speed and throw equations from TP A.14:

$$\begin{aligned} \mathbf{v}_{rel}(\mathbf{v}, \boldsymbol{\omega}_{x}, \boldsymbol{\omega}_{z}, \boldsymbol{\phi}) &\coloneqq \sqrt{\left(\mathbf{v} \cdot \sin(\boldsymbol{\phi}) - \mathbf{R} \cdot \boldsymbol{\omega}_{z}\right)^{2} + \left(\mathbf{R} \cdot \boldsymbol{\omega}_{x} \cdot \cos(\boldsymbol{\phi})\right)^{2}} \\ \theta_{throw}(\mathbf{k}, \mathbf{v}, \boldsymbol{\omega}_{x}, \boldsymbol{\omega}_{z}, \boldsymbol{\phi}) &\coloneqq \left[ \frac{\min\left(\frac{\mathbf{k} \cdot \mu\left(\mathbf{v}_{rel}\left(\mathbf{v}, \boldsymbol{\omega}_{x}, \boldsymbol{\omega}_{z}, \boldsymbol{\phi}\right)\right) \cdot \mathbf{v} \cdot \cos(\boldsymbol{\phi})}{\mathbf{v}_{rel}\left(\mathbf{v}, \boldsymbol{\omega}_{x}, \boldsymbol{\omega}_{z}, \boldsymbol{\phi}\right)} \right] \\ & \left[ \cdot \left(\mathbf{v} \cdot \sin(\boldsymbol{\phi}) - \mathbf{R} \cdot \boldsymbol{\omega}_{z}\right) \right] \\ & \frac{1}{7} \\ & \mathbf{v} \cdot \cos(\boldsymbol{\phi}) \end{aligned} \right] \end{aligned}$$

Note: A scale factor (k) is added to the friction term to see how changes in friction would affect results.

$k_{less} \coloneqq 0.5$	for half the amount of typical friction
k <sub>avg</sub> := 1.0	for an average amount of friction
$k_{more} := 1.5$	for 50% more than the typical amount of friction

## collision-induced throw vs. cut angle for natural-roll shots with different amounts of friction:



 $\phi := 0 \cdot \deg_{1} \cdot \deg_{2} \cdot 90 \cdot \deg_{2}$ 

So, for natural roll shots, there is more collisions-induced throw when there is more friction. No surprise here.

## collision-induced throw vs. cut angle for stun shots with different amounts of friction



So for small cut angles, the amount of friction does not affect the amount of throw. However, there is more throw with more friction for larger cut angles.



So for small amounts of sidespin, the amount of friction does not affect the amount of spin-induced throw. However, with larger amounts of sidespin, there is more throw with more spin (up to a point) and with more friction.

