technical proof

4/7/04

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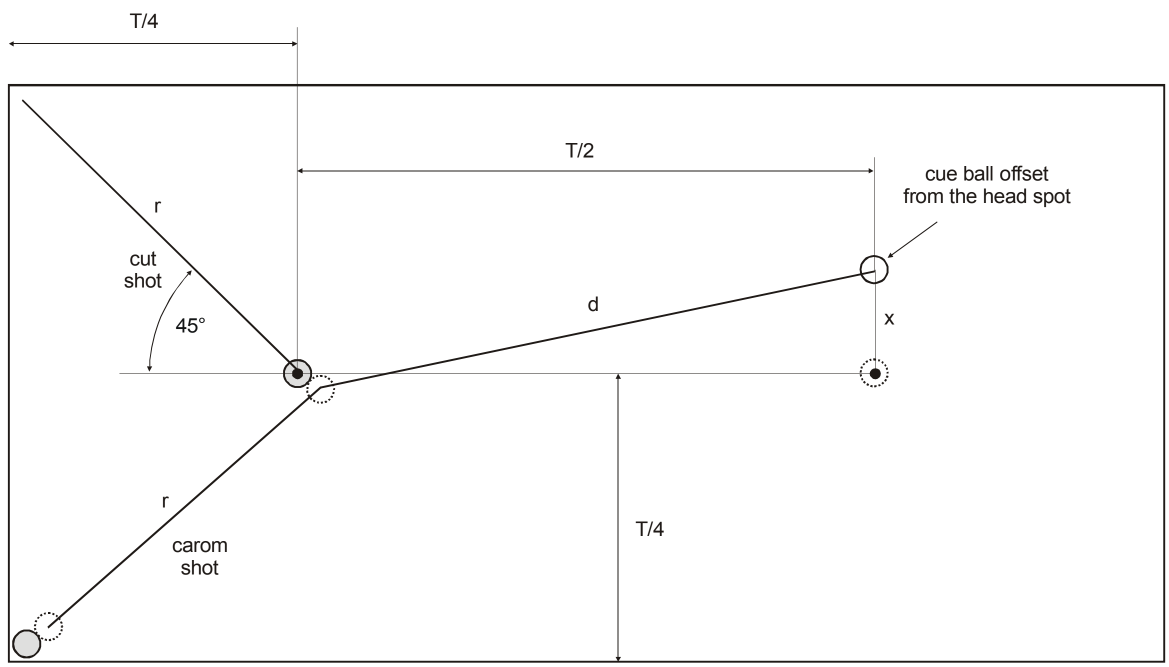
TP A.2
Carom shot vs. cut shot example

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
by David G. Alciatore, PhD, PE ("Dr. Dave")

T: table size (6.5', 7', 8', 9') $T := 8 \cdot ft$

$D := 2.25 \cdot in$ ball diameter

$x := 2.5 \cdot D$ from TP A.1 and the figure below

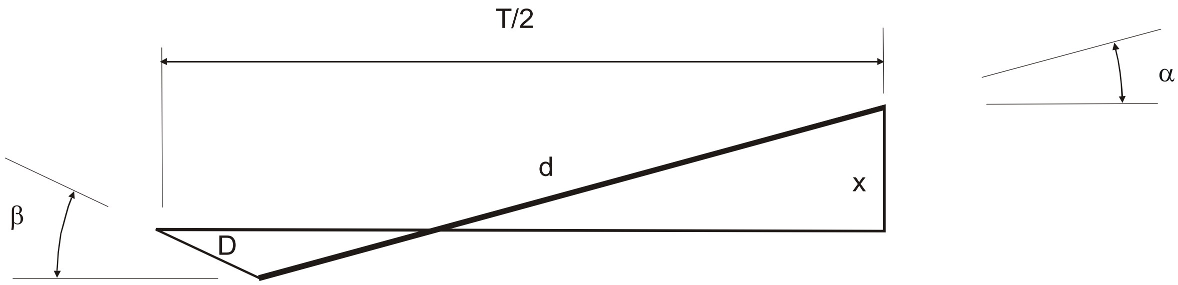


The approximate distance to the target for each shot is:

$$r := \frac{\frac{T}{4}}{\cos(45 \cdot deg)} \qquad r = 2.828\ ft$$

From the figure below, the distance from the cue ball to the impact point is:

$$d(\beta) := \sqrt{\left(\frac{T}{2} - D\cdot\cos(\beta)\right)^2 + (x + D\cdot\sin(\beta))^2}$$



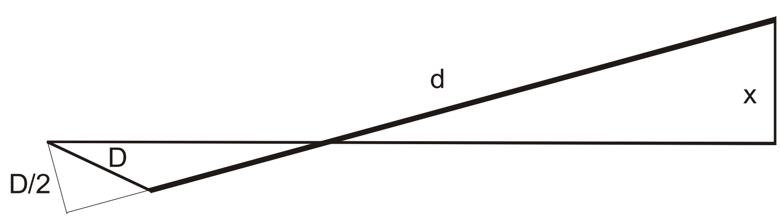
For the cut shot,

$\beta := 45\cdot deg$ $\qquad d_{cut} := d(\beta)$ $\qquad d_{cut} = 3.914\ ft$

$$\alpha_{cut} := \operatorname{atan}\left(\frac{x + D\cdot\sin(\beta)}{\frac{T}{2} - D\cdot\cos(\beta)}\right) \qquad \alpha_{cut} = 8.838\ deg$$

For the carom shot with a half-ball hit (see the figures above and below and TP A.1),

$$\frac{T}{2} = \frac{x}{\tan(\alpha)} + \frac{\frac{D}{2}}{\sin(\alpha)}$$



Solving for α numerically,

$$\alpha := 10\cdot deg$$

Given

$$\frac{T}{2} = \frac{x}{\tan(\alpha)} + \frac{\frac{D}{2}}{\sin(\alpha)}$$

$$\alpha_{carom} := \mathrm{Find}(\alpha) \qquad \alpha_{carom} = 8.018\ deg$$

$$\beta := 30\cdot deg - \alpha_{carom} \qquad d_{carom} := d(\beta) \qquad d_{carom} = 3.864\ ft$$

Error analysis for the cut shot:

the cut angle is:

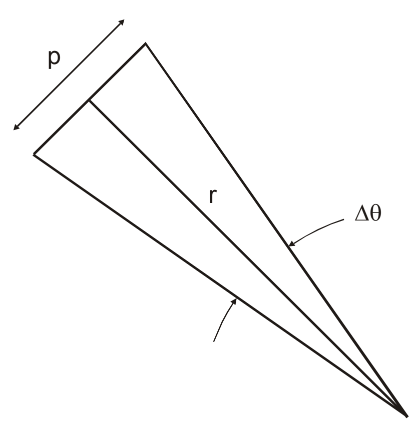
$$\phi := 45\cdot deg + \alpha_{cut} \qquad \phi = 53.838\ deg$$

From TP 3.6, the effective pocket size for a slow, 0 degree entry angle shot is:

$$p := 3.052\cdot in$$

The allowable angle error from the object ball to the pocket is therefore (see figure below):

$$\Delta\theta_{cut} := 2\cdot \mathrm{atan}\left(\frac{\frac{p}{2}}{r}\right) \qquad \Delta\theta_{cut} = 5.149\ deg$$



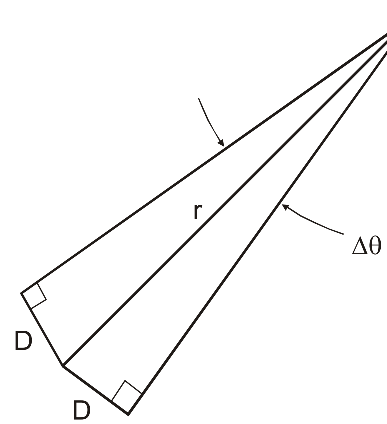
From TP 3.4, the allowable angle error in the aiming line direction is:

$$\Delta\phi_{cut} := 0.155\cdot deg$$

Error analysis for the carom shot:

Assuming the object ball will be pocketed with just the slightest hit by the deflected cue ball, on either side (and neglecting possible interaction with the rail), the allowable angle error in the deflected cue ball path is (see figure below):

$$\Delta\theta_{carom} := 2\cdot\operatorname{asin}\left(\frac{D}{r}\right) \qquad \Delta\theta_{carom} = 7.602\ \text{deg}$$



From TP 3.3:

The cut angle ϕ, as a function of ball-hit fraction f, is given by:

$$\phi(f) := \operatorname{asin}(1 - f)$$

and the cue ball deflection angle θ, as a function of cut angle ϕ, is given by:

$$\theta(\phi) := \operatorname{atan}\left[\frac{\sin(\phi)\cdot\cos(\phi)}{\left(\sin(\phi)^2 + \frac{2}{5}\right)}\right]$$

The maximum cue ball deflection angle occurs for a half-ball hit where f = 0.5:

$$\theta_{max} := \theta(\phi(.5)) \qquad \theta_{max} = 33.67\ \text{deg}$$

Therefore, the least cue ball deflection allowed is:

$$\theta_{min} := \theta_{max} - \Delta\theta_{carom} \qquad \theta_{min} = 26.069\ \text{deg}$$

The ball-hit fraction limits that result in θ_{min} are:

$f_{min} := .22477$ $\qquad$ $f_{max} := .77325$

as shown by:

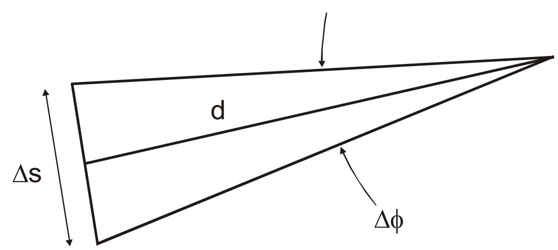
$\theta\left(\phi\left(f_{min}\right)\right) = 26.069\,\mathrm{deg}$ $\qquad$ $\theta\left(\phi\left(f_{max}\right)\right) = 26.069\,\mathrm{deg}$

Therefore, the effective target size for the cue ball at the foot spot object ball is (see figure below):

$\Delta s := \left(f_{max} - f_{min}\right)\cdot D$ $\qquad$ $\Delta s = 0.031\,\mathrm{m}$

and the resulting allowable angle error in the cue ball aiming line is:

$$\Delta\phi_{carom} := 2\cdot \mathrm{atan}\left(\frac{\frac{\Delta s}{2}}{d_{carom}}\right) \qquad \Delta\phi_{carom} = 1.525\,\mathrm{deg}$$



Finally, the ratio of the carom shot margin of error to the cut shot margin of error is:

$$\frac{\Delta\phi_{carom}}{\Delta\phi_{cut}} = 9.838$$

Therefore, the carom shot is almost 10-times easier to execute!!!