technical proof

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TP A.24
The effects of follow and draw on throw

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http://billiards.colostate.edu
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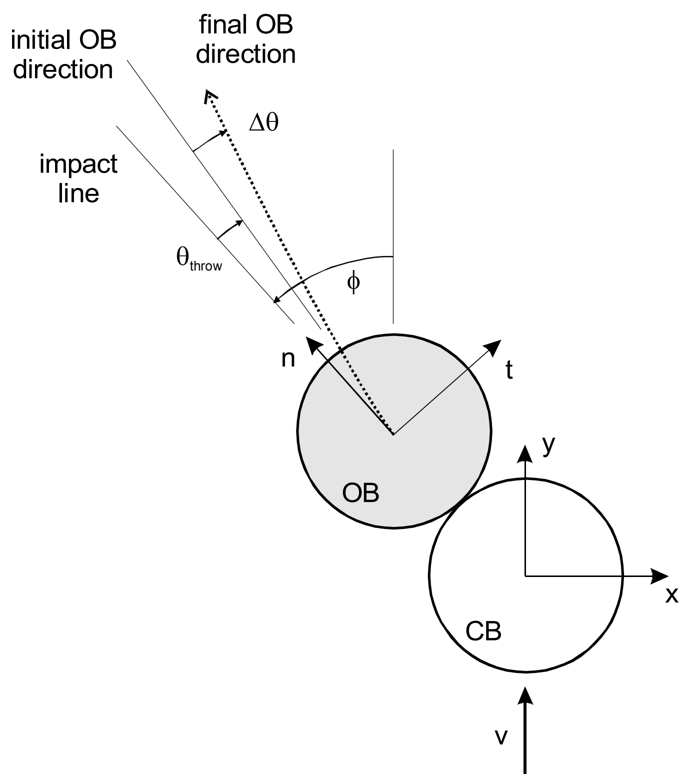

The cue ball (CB) strikes the object ball (OB) with velocity v in the y direction with cut angle ϕ. The OB gets thrown amount θ_{throw}, and spin imparted about the t axis (due to draw or follow) causes the trajectory to curve a small amount $\Delta\theta$.

$$R := \frac{1.125 \cdot in}{m}$$ ball radius converted to meters

$$v := \frac{2 \cdot mph}{\frac{m}{s}}$$ typical cue ball speed converted to meters/sec

$$\omega_{roll} := \frac{v}{R}$$ natural-roll spin rate

From TP A.14, the OB post-impact velocity can be expressed with the following relations:

$$\mu(\mathrm{v}) := 9.951 \times 10^{-3} + 0.108 \cdot e^{-1.088 \cdot \mathrm{v}}$$

$$\mathrm{v_{rel}}(\mathrm{v}, \omega_x, \omega_z, \phi) := \sqrt{(\mathrm{v} \cdot \sin(\phi) - \mathrm{R} \cdot \omega_z)^2 + (\mathrm{R} \cdot \omega_x \cdot \cos(\phi))^2}$$

$$\mathrm{v_{OBt}}(\mathrm{v}, \omega_x, \omega_z, \phi) := \min\begin{pmatrix} \dfrac{\mu(\mathrm{v_{rel}}(\mathrm{v}, \omega_x, \omega_z, \phi)) \cdot \mathrm{v} \cdot \cos(\phi)}{\mathrm{v_{rel}}(\mathrm{v}, \omega_x, \omega_z, \phi)} \\ \dfrac{1}{7} \end{pmatrix} \cdot (\mathrm{v} \cdot \sin(\phi) - \mathrm{R} \cdot \omega_z)$$

$$\mathrm{v_{OBn}}(\mathrm{v}, \omega_x, \omega_z, \phi) := \mathrm{v} \cdot \cos(\phi)$$

The throw angle can then be expressed as:

$$\theta_{\mathrm{throw}}(\mathrm{v}, \omega_x, \omega_z, \phi) := \mathrm{atan}\left(\frac{\mathrm{v_{OBt}}(\mathrm{v}, \omega_x, \omega_z, \phi)}{\mathrm{v_{OBn}}(\mathrm{v}, \omega_x, \omega_z, \phi)}\right)$$

From Equation 10 in TP A.14, the friction impulse component in the tangential direction is:

$$\widetilde{F}_t = m v_{OB_t}$$

From Equation 5, the total frictional impulse is then:

$$\widetilde{F}_\mu = \frac{\widetilde{F}_t}{e_{\mu_t}}$$

and the friction impulse component in the vertical direction, using Equation 3, is:

$$\widetilde{F}_z = \widetilde{F}_\mu e_{\mu_z} = \widetilde{F}_t \frac{e_{\mu_z}}{e_{\mu_t}} = \widetilde{F}_t \frac{v_{B_z}}{v_{B_t}} = m v_{OB_t} \frac{R\omega_x \cos(\phi)}{v\sin(\phi) - R\omega_z}$$

This frictional impulse imparts spin to the OB about the t axis, which can be found from the following angular impulse-momentum equation:

$$\omega_{OB_t} = -\frac{1}{I}\left(\widetilde{F}_z R\right) = -\frac{5}{2Rm}\widetilde{F}_z = \frac{-5 v_{OB_t} \omega_x \cos(\phi)}{2(v\sin(\phi) - R\omega_z)}$$

$$\omega_{\mathrm{OBt}}(\mathrm{v}, \omega_x, \omega_z, \phi) := \frac{-5 \cdot \mathrm{v_{OBt}}(\mathrm{v}, \omega_x, \omega_z, \phi) \cdot \omega_x \cdot \cos(\phi)}{2 \cdot (\mathrm{v} \cdot \sin(\phi) - \mathrm{R} \cdot \omega_z)}$$

This induced spin curves the trajectory of the OB slightly with a masse type action.

From TP A.4, applying Equations 21 and 22 to the OB motion, the final (post curve) OB motion will have the following velocity components:

$$v_{OBtf}(v, \omega_x, \omega_z, \phi) := \frac{5}{7} \cdot v_{OBt}(v, \omega_x, \omega_z, \phi)$$

$$v_{OBnf}(v, \omega_x, \omega_z, \phi) := \frac{1}{7} \cdot \left(5 \cdot v_{OBn}(v, \omega_x, \omega_z, \phi) - 2 \cdot R \cdot \omega_{OBt}(v, \omega_x, \omega_z, \phi)\right)$$

Therefore, the change in OB angle due to curve caused by follow or draw is:

$$\Delta\theta(v, \omega_x, \omega_z, \phi) := \operatorname{atan}\left(\frac{v_{OBtf}(v, \omega_x, \omega_z, \phi)}{v_{OBnf}(v, \omega_x, \omega_z, \phi)}\right) - \theta_{throw}(v, \omega_x, \omega_z, \phi)$$

Example values for a 1/2-ball hit: $\phi := 30 \cdot deg$

stun shot: $\omega_x := 0$ $\omega_z := 0$

$$\theta_{throw}(v, \omega_x, \omega_z, \phi) = 4.366\ deg$$

$$\Delta\theta(v, \omega_x, \omega_z, \phi) = 0$$

A stun shot has the most throw, and there is no OB curving.

draw shot: $\omega_x := \omega_{roll}$ $\omega_z := 0$

$$\theta_{throw}(v, \omega_x, \omega_z, \phi) = 1.454\ deg$$

$$\Delta\theta(v, \omega_x, \omega_z, \phi) = -0.061\ deg$$

A draw shot has less throw than a stun shot, and the curve effect tends to decrease the throw, but only by an extremely small amount.

follow shot: $\omega_x := -\omega_{roll}$ $\omega_z := 0$

$$\theta_{throw}(v, \omega_x, \omega_z, \phi) = 1.454\ deg$$

$$\Delta\theta(v, \omega_x, \omega_z, \phi) = 0.067\ deg$$

A follow shot has the same throw as a draw shot, and the curve effect tends to increase the throw, but only by an extremely small amount.