TP B.18
Pendulum Stroke Cue Tip Trajectory

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
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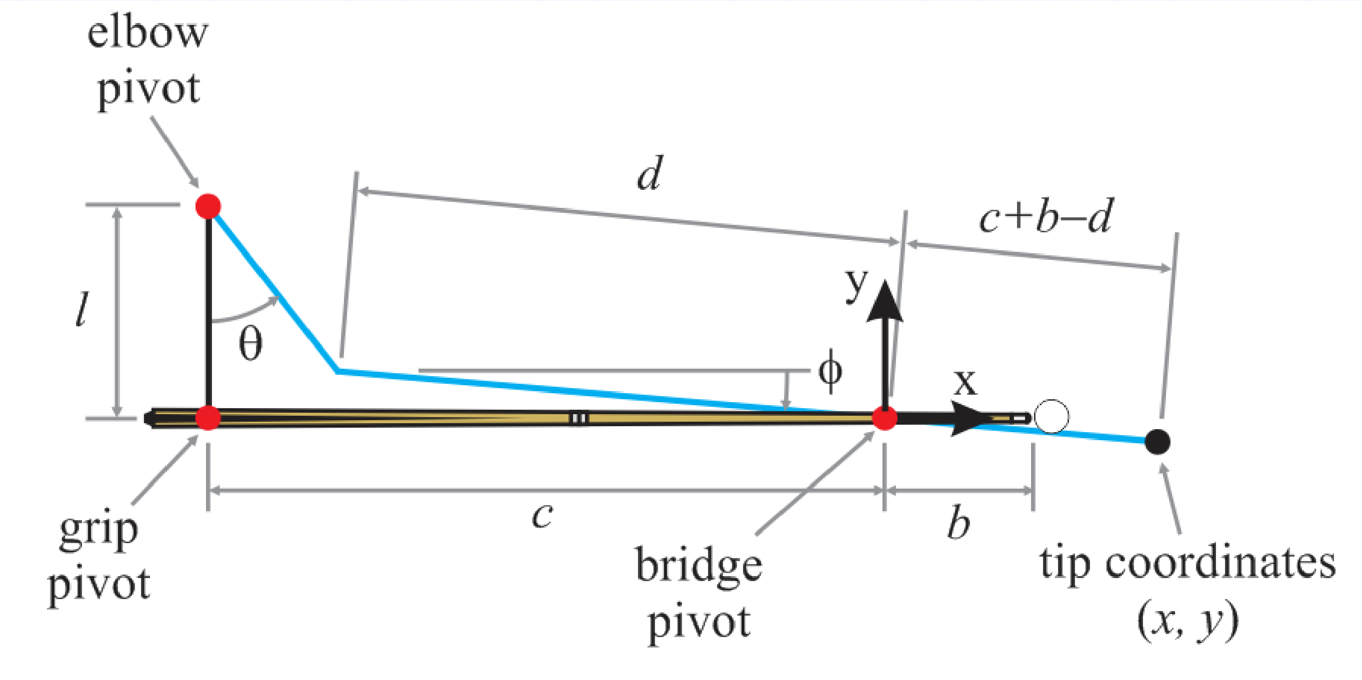


Dr. Dave's shooting dimensions:

forearm length: $l := 14 \cdot in$

length of cue between grip and bridge at set position: $c := 45 \cdot in$

bridge length: $b := 12 \cdot in$

From the geometry in the diagram above:

$$d \cdot cos(\phi) = c - l \cdot sin(\theta) \tag{1}$$

$$d \cdot sin(\phi) = l - l \cdot cos(\theta) \tag{2}$$

Adding the squares of these two equations gives the distance (d) between grip and bridge during the stroke:

$$d(\theta) := \sqrt{c^2 + 2 \cdot l^2 - 2 \cdot c \cdot l \cdot \sin(\theta) - 2 \cdot l^2 \cdot \cos(\theta)}$$

Dividing Equation 2 by Equation 1 gives the cue elevation angle (ϕ) during the stroke:

$$\phi(\theta) := \operatorname{atan}\left(\frac{l \cdot (1 - \cos(\theta))}{c - l \cdot \sin(\theta)}\right)$$

The coordinates of the tip (x, y) during the stroke are given by:

$$x(\theta) := (c + b - d(\theta)) \cdot \cos(\phi(\theta))$$

$$y(\theta) := -(c + b - d(\theta)) \cdot \sin(\phi(\theta))$$

The minimum forearm angle (θ_{min}) possible at the end of the backstroke, with the tip at the bridge (x=0) is:

$$\theta := -45 \cdot deg$$

$$\theta_{min} := \mathbf{root}(x(\theta), \theta) = -56.369\ deg$$

Assuming the forward stroke is the same length as the backstroke results in the same angle forward:

$$\theta_{max} := -\theta_{min}$$

$$\theta := -80 \cdot deg, -79 \cdot deg .. 80 \cdot deg$$

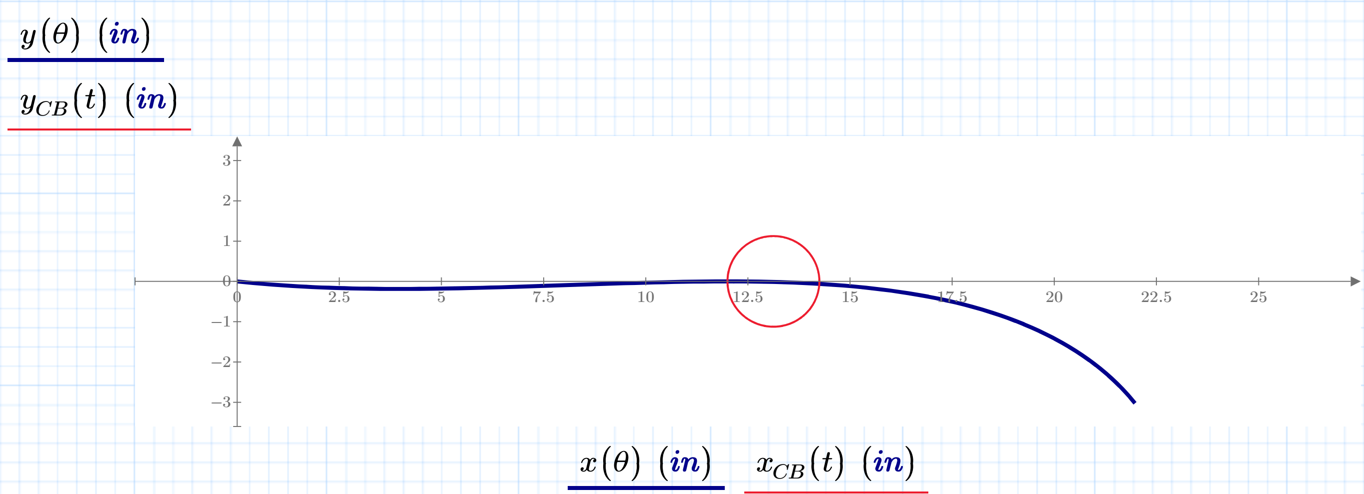
Cue ball geometry (added to the tip trajectory plots below for scale):

$$t := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg \qquad R := \frac{2.25}{2} \cdot in$$

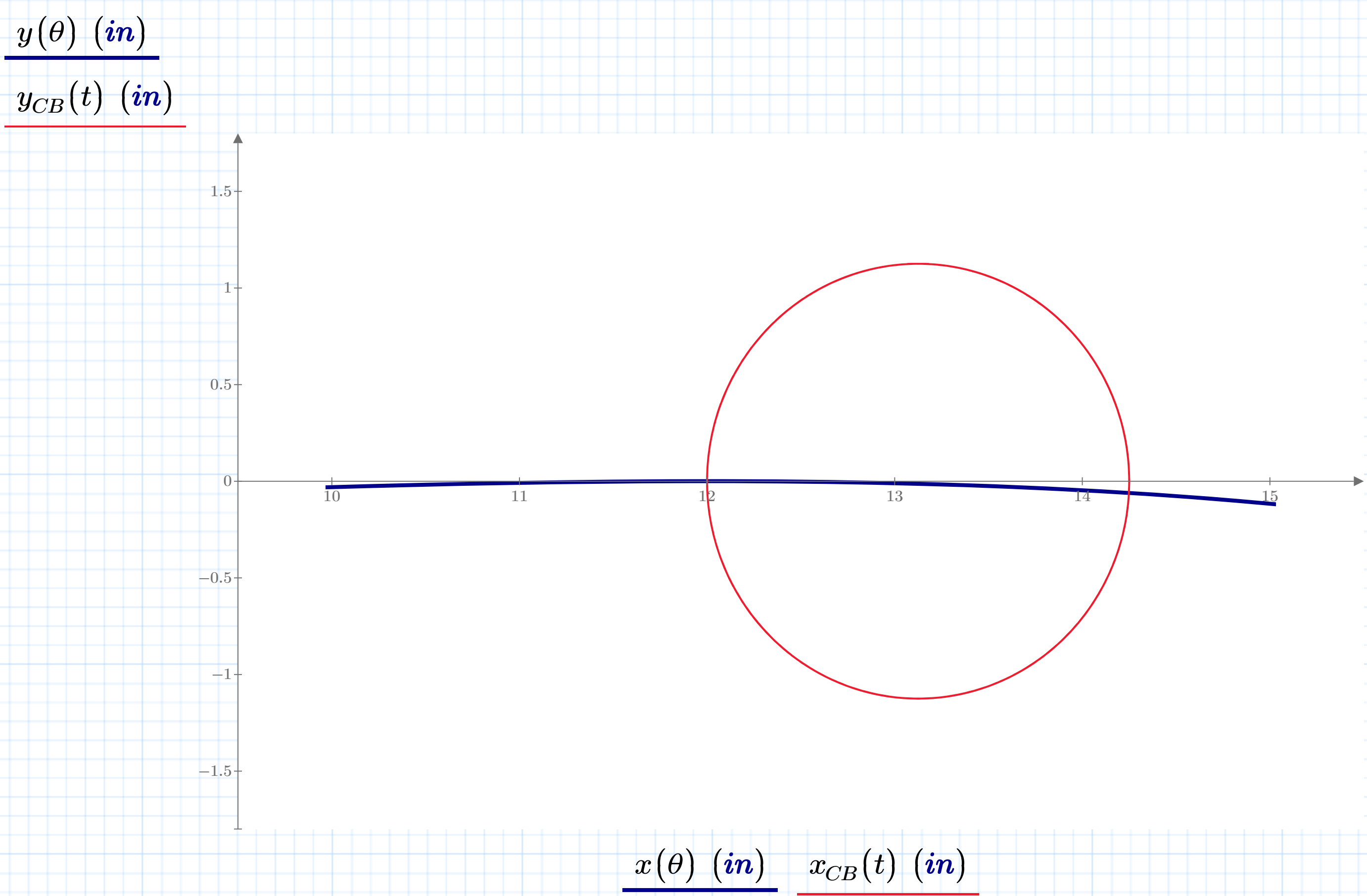
$$x_{CB}(t) := (b + R) + R \cdot \cos(t) \qquad y_{CB}(t) := R \cdot \sin(t)$$

Plot of the cue tip trajectory during the entire stroke:

$$\theta := \theta_{min}, \theta_{min} + 1 \cdot deg .. \theta_{max}$$



Close-up of the cue tip trajectory close to the CB contact point:



Total tip height variance (Δy) over a given distance (Δx) around the tip contact point:

$\Delta x := 4 \cdot in$ $\quad x_{start} := b - \frac{\Delta x}{2} = 10\ in$ $\quad x_{end} := b + \frac{\Delta x}{2} = 14\ in$

$\theta := 0$ $\quad \theta_{start} := \mathbf{root}\left(x(\theta) - x_{start}, \theta\right) = -8.212\ deg$ $\quad x\left(\theta_{start}\right) = 10\ in$

$\theta := 0$ $\quad \theta_{end} := -\theta_{start}$ $\quad x\left(\theta_{end}\right) = 13.999\ in$

$\Delta y := y\left(\theta_{start}\right) - y\left(\theta_{end}\right) = 0.016\ in$ $\quad \Delta y = 0.411\ mm$

Obviously, from the plots and numbers above, a pure pendulum stroke results in the tip moving very straight into and through the CB position, with an accurate tip contact point.