**TP B.18**

Pendulum Stroke Cue Tip Trajectory

supporting:
“The Illustrated Principles of Pool and Billiards”
http://billiards.colostate.edu
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Dr. Dave’s shooting dimensions:
- forearm length: \( l := 14 \cdot \text{in} \)
- length of cue between grip and bridge at set position: \( c := 45 \cdot \text{in} \)
- bridge length: \( b := 12 \cdot \text{in} \)

From the geometry in the diagram above:

\[
\begin{align*}
  d \cdot \cos(\phi) &= c - l \cdot \sin(\theta) \\
  d \cdot \sin(\phi) &= l - l \cdot \cos(\theta)
\end{align*}
\]

Adding the squares of these two equations gives the distance (d) between grip and bridge during the stroke:

\[
d(\theta) := \sqrt{c^2 + 2 \cdot l^2 - 2 \cdot c \cdot l \cdot \sin(\theta) - 2 \cdot l^2 \cdot \cos(\theta)}
\]
Dividing Equation 2 by Equation 1 gives the cue elevation angle ($\phi$) during the stroke:

$$\phi(\theta) := \tan \left( \frac{l \cdot (1 - \cos(\theta))}{c - l \cdot \sin(\theta)} \right)$$

The coordinates of the tip ($x, y$) during the stroke are given by:

$$x(\theta) := (c - d(\theta)) \cdot \cos(\phi(\theta)) + b$$
$$y(\theta) := -(c - d(\theta)) \cdot \sin(\phi(\theta))$$

The minimum forearm angle ($\theta_{min}$) possible at the end of the backstroke, with the tip at the bridge ($x=0$) is:

$$\theta := 0$$
$$\theta_{min} := \text{root}(x(\theta), \theta) = -56.848 \ \text{deg}$$

Assuming the forward stroke is the same length as the backstroke results in the same angle forward:

$$\theta_{max} := -\theta_{min}$$

Cue ball geometry (added to the tip trajectory plots below for scale):

$$t := 0 \cdot \text{deg.} 1 \cdot \text{deg} \ldots 360 \cdot \text{deg}$$
$$R := \frac{2.25 \cdot \text{in}}{2}$$
$$x_{CB}(t) := (b + R) + R \cdot \cos(t)$$
$$y_{CB}(t) := R \cdot \sin(t)$$

Plot of the cue tip trajectory during the entire stroke:

$$\theta := \theta_{min}, \theta_{min} + 1 \cdot \text{deg} \ldots \theta_{max}$$

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Close-up of the cue tip trajectory close to the CB contact point:

\[ y(\theta) \ (\text{in}) \]
\[ y_{CB}(t) \ (\text{in}) \]

Total tip height variance \((\Delta y)\) over a given distance \((\Delta x)\) around the tip contact point:

\[ \Delta x := 4 \cdot \text{in} \]
\[ x_{\text{start}} := b - \frac{\Delta x}{2} = 10 \text{ in} \]
\[ x_{\text{end}} := b + \frac{\Delta x}{2} = 14 \text{ in} \]

\[ \theta := 0 \]
\[ \theta_{\text{start}} := \text{root} \left( x(\theta) - x_{\text{start}}, \theta \right) = -8.212 \ \text{deg} \]
\[ x(\theta_{\text{start}}) = 10 \text{ in} \]

\[ \theta := 0 \]
\[ \theta_{\text{end}} := -\theta_{\text{start}} \]
\[ x(\theta_{\text{end}}) = 14 \text{ in} \]

\[ \Delta y := y(\theta_{\text{start}}) - y(\theta_{\text{end}}) = 0.013 \text{ in} \]
\[ \Delta y = 0.325 \text{ mm} \]

Obviously, from the plots and numbers above, a pure pendulum stroke results in the tip moving very straight into and through the CB position, with an accurate tip contact point.