TP B.19
Comparison of cue ball deflection (squirt) "endmass" and stiffness effects

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
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From an actual tip-deflection measurement due to a weight applied to the end of a Predator Z-2 shaft:

$$F_{applied} := 3.1\ lbf$$

$$\Delta x_{measured} := 1.4 \cdot in$$

Therefore, the transverse stiffness of the shaft is:

$$k_{shaft} := \frac{F_{applied}}{\Delta x_{measured}} = 2.214\ \frac{lbf}{in}$$

For a medium-fast shot, the impulse pushing the CB forward with momentum *mv* is about:

$$m := 6 \cdot oz \qquad v := 5 \cdot mph$$

$$F_{imp} := m \cdot v = 0.085\ lbf \cdot s$$

Over a typical cue-tip contact time, the peak forward force generated, assuming a triangular impulse curve, is:

$$\Delta t := 1 \cdot ms = 0.001\ s$$

$$F_{fwd} := \frac{2}{\Delta t} \cdot F_{imp} = 170.945\ lbf$$

For a near-maximum sidespin shot, the peak angular speed as the tip releases from the ball is about:

$$R := 2.25 \cdot in \qquad \omega := \frac{v}{R} = 373.484\ rpm$$

The amount the CB rotates during tip contact, using an average CB speed and spin during contact is about:

$$\Delta\theta := \frac{\omega}{2} \cdot \Delta t = 1.12\ deg$$

Therefore, for a near-maximum spin shot (where the tip contact point is about 30 degrees from the center), the amount the tip moves sideways during tip contact is about:

$$\Delta x := R \cdot \Delta\theta \cdot \cos(30\ °) = 0.038\ in$$

This corresponds to a peak sideways force, due to stiffness k, of about:

$$F_k := k_{shaft} \cdot \Delta x = 0.084 \; \mathbf{lbf}$$

From my September '07 article, for a typical shot with sidespin, the squirt angle for the Z-2 is about:

$$\alpha := 1.8 \; ^\circ$$

Therefore, the total peak sideways force delivered to the CB (which acts equal and opposite on the tip) is about:

$$F_{side} := \tan(\alpha) \cdot F_{fwd} = 5.372 \; \mathbf{lbf}$$

The force resulting from transverse stiffness is a small fraction of this total sideways force:

$$\frac{F_k}{F_{side}} = 1.571\%$$

Since only a portion of the shaft flexes (5-8 inches) during tip contact, I took another set of staic stiffness measurements, with only 8 inches of the end of the shaft allowed to flex. Here is how this changes the results:

$$\Delta x_{measured} := 0.156 \cdot \mathbf{in} \qquad k_{shaft} := \frac{F_{applied}}{\Delta x_{measured}} = 19.872 \; \frac{\mathbf{lbf}}{\mathbf{in}}$$

$$F_k := k_{shaft} \cdot \Delta x = 0.757 \; \mathbf{lbf} \qquad \frac{F_k}{F_{side}} = 14.095\%$$

Therefore, as has been shown with experiments dealing with adding and removing mass from the end of the shaft, **endmass is much more important than shaft flex concerning how much CB deflection (squirt) a shaft creates** (although, as pointed out on the following resource page, lateral or transverse stiffness does affect "effective endmass").

For more information, see:
http://billiards.colostate.edu/threads/squirt.html#endmass

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