TP B.19
Comparison of cue ball deflection (squirt) "endmass" and stiffness effects

supporting:
“The Illustrated Principles of Pool and Billiards”
http://billiards.colostate.edu
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From an actual tip-deflection measurement due to a weight applied to the end of a Predator Z-2 shaft:

\[ F_{applied} = 3.1 \text{ lbf} \]
\[ \Delta x_{measured} = 1.4 \text{ \( \cdot \) in} \]

Therefore, the transverse stiffness of the shaft is:

\[ k_{shaft} = \frac{F_{applied}}{\Delta x_{measured}} = 2.214 \frac{\text{lbf}}{\text{in}} \]

For a medium-fast shot, the impulse pushing the CB forward with momentum \( mV \) is about:

\[ m = 6 \text{ oz} \quad v = 5 \text{ mph} \]
\[ F_{imp} = m \cdot v = 0.085 \text{ lbf} \cdot \text{s} \]

Over a typical cue-tip contact time, the peak forward force generated, assuming a triangular impulse curve, is:

\[ \Delta t = 1 \text{ ms} = 0.001 \text{ s} \]
\[ F_{fwd} = \frac{2}{\Delta t} \cdot F_{imp} = 170.945 \text{ lbf} \]

For a near-maximum sidespin shot, the peak angular speed as the tip releases from the ball is about:

\[ R = 2.25 \text{ in} \quad \omega = \frac{v}{R} = 373.484 \text{ rpm} \]

The amount the CB rotates during tip contact, using an average CB speed and spin during contact is about:

\[ \Delta \theta = \frac{\omega}{2} \cdot \Delta t = 1.12 \text{ deg} \]

Therefore, for a near-maximum spin shot (where the tip contact point is about 30 degrees from the center), the amount the tip moves sideways during tip contact is about:

\[ \Delta x = R \cdot \Delta \theta \cdot \cos (30 \text{ } ^\circ) = 0.038 \text{ in} \]
This corresponds to a peak sideways force, due to stiffness $k$, of about:

$$F_k := k_{\text{shaft}} \cdot \Delta x = 0.084 \text{ lb}$$

From my September '07 article, for a typical shot with sidespin, the squirt angle for the Z-2 is about:

$$\alpha := 1.8^\circ$$

Therefore, the total peak sideways force delivered to the CB (which acts equal and opposite on the tip) is about:

$$F_{\text{side}} := \tan(\alpha) \cdot F_{\text{fwd}} = 5.372 \text{ lb}$$

The force resulting from transverse stiffness is a small fraction of this total sideways force:

$$\frac{F_k}{F_{\text{side}}} = 1.571\%$$

Since only a portion of the shaft flexes (5-8 inches) during tip contact, I took another set of static stiffness measurements, with only 8 inches of the end of the shaft allowed to flex. Here is how this changes the results:

$$\Delta x_{\text{measured}} := 0.156 \text{ in}$$

$$F_k := k_{\text{shaft}} \cdot \Delta x = 0.757 \text{ lb}$$

$$\frac{F_k}{F_{\text{side}}} = 14.095\%$$

Therefore, as has been shown with experiments dealing with adding and removing mass from the end of the shaft, **endmass is much more important than shaft flex concerning how much CB deflection (squirt) a shaft creates** (although, as pointed out on the following resource page, lateral or transverse stiffness does affect "effective endmass").

For more information, see:
http://billiards.colostate.edu/threads/squirt.html#endmass

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