TP B.22  

How peak tip contact force and contact patch size vary with shot speed, and drop tests

supporting:
“The Illustrated Principles of Pool and Billiards”  
http://billiards.colostate.edu  
by Dr. Dave Alciatore, PhD, PE ("Dr. Dave")

originally posted: 5/19/2018     last revision: 5/19/2018

Mass of a pool ball and typical cue stick:

\[ m_b := 6 \text{ oz} \quad m_s := 19 \text{ oz} \]

Typical tip-ball contact times for phenolic and leather tips with fast-speed shots, from the DBKcue link here:  
http://billiards.colostate.edu/threads/cue_tip.html#contact

\[ \Delta t_{\text{phenolic}} := 0.0008 \cdot s \quad \Delta t_{\text{leather}} := 0.0012 \cdot s \]

Typical coefficients of restitution (CORs) for a phenolic tip on a break cue and a typical leather tip on playing cue, from:  
http://billiards.colostate.edu/threads/cue_tip.html#efficiency

\[ e_{\text{phenolic}} := 0.85 \quad e_{\text{leather}} := 0.73 \]

Typical contact patch sizes for a fast-speed shot with phenolic and leather tips:

\[ v_{\text{fast}} := 10 \cdot \text{mph} \quad cps_{\text{phenolic}} := 3 \cdot \text{mm} \quad cps_{\text{leather}} := 4 \cdot \text{mm} \]

From TP B.20, the peak force between the cue tip and CB during impact, for a given CB speed \( v_b \) and tip contact time \( \Delta t \) is:

\[ F_{\text{peak}}(v_b, \Delta t) := \frac{2 \cdot m_b \cdot v_b}{\Delta t} \]

Hertz elastic contact-stress equations (e.g., from "Impact Mechanics" by Strong, pp.117-118, 2004) can be used to approximate how contact patch size (cps) varies with peak force (\( F \)) according to:

\[ cps = \left( \frac{3 \cdot F \cdot E}{R} \right)^{\frac{1}{3}} = c \cdot F^{\frac{1}{3}} \]

where \( E \) depends on tip and CB material properties, \( R \) depends on the radii of curvature of the tip and CB, and \( c \) is the resulting constant.

Therefore, the approximate contact patch size can be related to CB speed and tip contact time according to:

\[ cps(v_b, \Delta t, c) := c \left( \frac{2 \cdot m_b \cdot v_b}{\Delta t} \right)^{\frac{1}{3}} \]
And the Hertz constant $c$ can be related to contact patch size according to:

$$c(v_b, \Delta t, \text{cps}) := \text{cps} \cdot \left(\frac{\Delta t}{2 \cdot m_b \cdot v_b}\right)^{\frac{1}{3}}$$

We can approximate the Hertz equation constant $c$ for both phenolic and leather tips using the data above:

$$c_{\text{phenolic}} := c(v_{\text{fast}}, \Delta t_{\text{phenolic}}, \text{cps}_{\text{phenolic}}) = 0.242 \frac{\text{mm}}{N^{\frac{1}{3}}}$$  

$$c_{\text{leather}} := c(v_{\text{fast}}, \Delta t_{\text{leather}}, \text{cps}_{\text{leather}}) = 0.37 \frac{\text{mm}}{N^{\frac{1}{3}}}$$

As a check to make sure these values are correct, we can see if the cps equation predicts the correct contact patch sizes:

$$\text{cps}(v_{\text{fast}}, \Delta t_{\text{phenolic}}, c_{\text{phenolic}}) = 3 \text{ mm}$$

$$\text{cps}(v_{\text{fast}}, \Delta t_{\text{leather}}, c_{\text{leather}}) = 4 \text{ mm}$$

Now we can look at how both peak contact force (in pounds) and contact patch size (in mm) vary with shot speed for both phenolic and leather tips:

$$v_b := 1 \cdot \text{mph}, 2 \cdot \text{mph} \ldots 30 \text{ mph}$$

As expected, the peak contact force increases with CB speed, and is greater for a phenolic tip as compared to a leather tip. With a powerful break (25 mph), the peak forces on both phenolic and leather tips are:

$$F_{\text{peak}}(25 \cdot \text{mph}, \Delta t_{\text{phenolic}}) = 1068 \text{ lbf}$$

$$F_{\text{peak}}(25 \cdot \text{mph}, \Delta t_{\text{leather}}) = 712 \text{ lbf}$$

$$F_{\text{peak}}(25 \cdot \text{mph}, \Delta t_{\text{phenolic}}) = 4753 \text{ N}$$

$$F_{\text{peak}}(25 \cdot \text{mph}, \Delta t_{\text{leather}}) = 3168 \text{ N}$$
As expected, the contact patch size increases with CB speed, and is larger for a leather tip as compared to a phenolic tip. With a powerful break (25 mph), the contact patch sizes for phenolic and leather tips are approximated to be:

\[
cps(v_b, \Delta t_{\text{phenolic}}, c_{\text{phenolic}}) = 4.1 \text{ mm}
\]

\[
cps(v_b, \Delta t_{\text{leather}}, c_{\text{leather}}) = 5.4 \text{ mm}
\]
One way to simulate cue-tip-CB impact is to drop a cue from different heights onto a heavy/solid/hard/flat/smooth surface (e.g., a big steel block). From conservation of energy, the cue speed \( v \) after falling height \( h \) is:

\[
v = \sqrt{2 \cdot g \cdot h}
\]

From impulse-momentum principles, if we want the impulse (and peak force) with a drop test to match the impulse (and peak force) of a CB hit, we can relate drop height \( h \) to CB speed \( v_b \) and drop rebound COR \( e \) with:

\[
m_b \cdot v_b = m_s \cdot (v + e \cdot v) = m_s \cdot \sqrt{2 \cdot g \cdot h \cdot (1 + e)}
\]

Solving for \( h \) gives us the required drop height to simulate different CB speeds:

\[
h(v_b, e) := \frac{1}{2} \frac{m_b \cdot v_b}{m_s \cdot (1 + e)}^2
\]

Here's a plot of how required drop height varies with simulated CB speed for both phenolic and leather tips:

As expected, a larger drop height is required to simulate faster CB speeds, and the drop height for a leather tip needs to be a little higher compared to a phenolic tip. With a powerful break (25 mph), the required drop heights for both phenolic and leather tips are approximately:

\[
h(25 \cdot mph, e_{\text{phenolic}}) = 0.61 \ ft \quad h(25 \cdot mph, e_{\text{phenolic}}) = 18.6 \ cm
\]

\[
h(25 \cdot mph, e_{\text{leather}}) = 0.7 \ ft \quad h(25 \cdot mph, e_{\text{leather}}) = 21.2 \ cm
\]