



TPA.15

Controlling the cue ball direction in a frozen cue ball shot

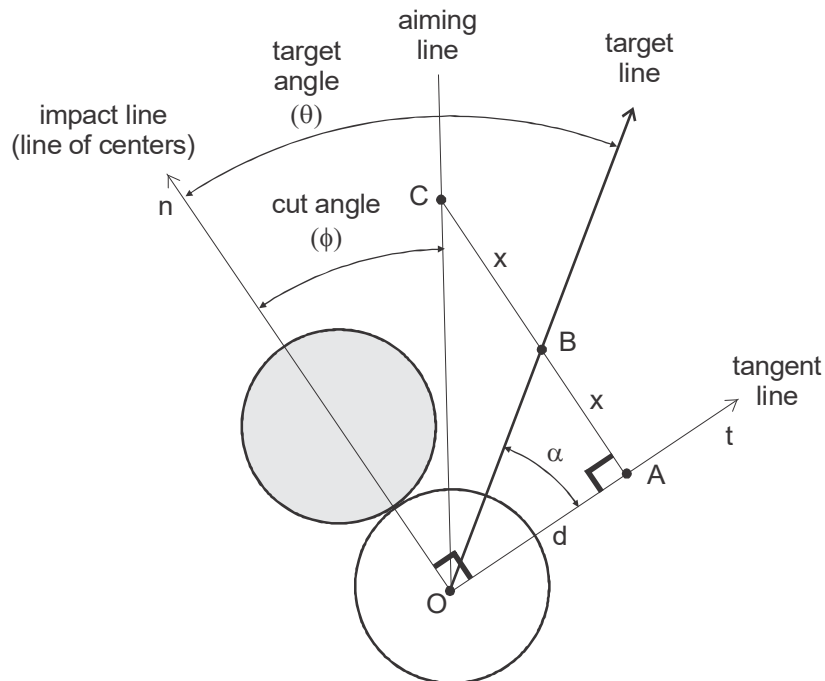
supporting:

“The Illustrated Principles of Pool and Billiards”

<http://billiards.colostate.edu>

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The cue ball is frozen to the object ball, and we want the cue ball to go in the target line direction (e.g., for a carom/billiard shot, break-up shot, or position play). Where do you aim the cue stick to have the cue ball head along the desired target line (i.e., what is the correct aiming line)?

A system proposed by Bob Jewett called the **"twice as full"** or **"two-times fuller"** system is illustrated in the diagram above. To use the method, you extend an imaginary line in the impact line direction from any point along the tangent line (line ABC). The necessary cue ball aiming line should be the same distance (x) away from the target line (segment BC) as the target line is from the tangent line (segment AB). For more information and demonstrations, see:

billiards.colostate.edu/faq/frozen/aiming/

The system is based on the **assumption** that frozen balls initially move as one in the normal direction (along the line of centers), which matches observed ball motion. Here's the **derivation**:

From conservation of momentum in the tangential (t) direction (see TP 3.1), the cue ball tangential speed after impact is related to the initial cue ball speed (v) and cut angle (ϕ) by:

$$v_t = v \cdot \sin(\phi) \quad (1)$$

Assuming the cue ball and object ball move as one in the normal direction, since they are frozen together, conservation of momentum in the normal direction (see TP 3.1) gives the cue ball normal speed after impact:

$$v_n = \frac{1}{2} \cdot v \cdot \cos(\phi) \quad (2)$$

From Equations 1 and 2 and the illustration above, the angle to the target line (α) is given by:

$$\tan(\alpha) = \frac{AB}{d} = \frac{v_n}{v_t} = \frac{1}{2} \cdot \frac{1}{\tan(\phi)} = \frac{1}{2} \cdot \frac{AC}{d}$$

Therefore, length AB is half of length AC, hence the equal x's in the illustration. QED!

To experimentally test the system, I used a high-speed camera to shoot super-slow-motion video clip **HSV A.97** at <http://billiards.colostate.edu>. In the clip, the cut angle is gradually increased to see how the resulting target angle changes. Here are the data from the experiment:

$$M_{\text{exper}} := \begin{pmatrix} 0 & 0 \\ 13 & 8 \\ 34 & 20 \\ 50 & 34 \\ 61 & 46 \\ 70 & 57 \\ 78 & 67 \\ 87 & 77 \\ 90 & 90 \end{pmatrix} \quad \theta_{\text{exper}} := M_{\text{exper}}^{\langle 0 \rangle} \quad \phi_{\text{exper}} := M_{\text{exper}}^{\langle 1 \rangle}$$

Here are the angle relations for the two-times fuller system:

From triangle OAC,

$$\tan(90\text{-deg} - \phi) = \frac{2x}{d} \quad (3)$$

From triangle OAB,

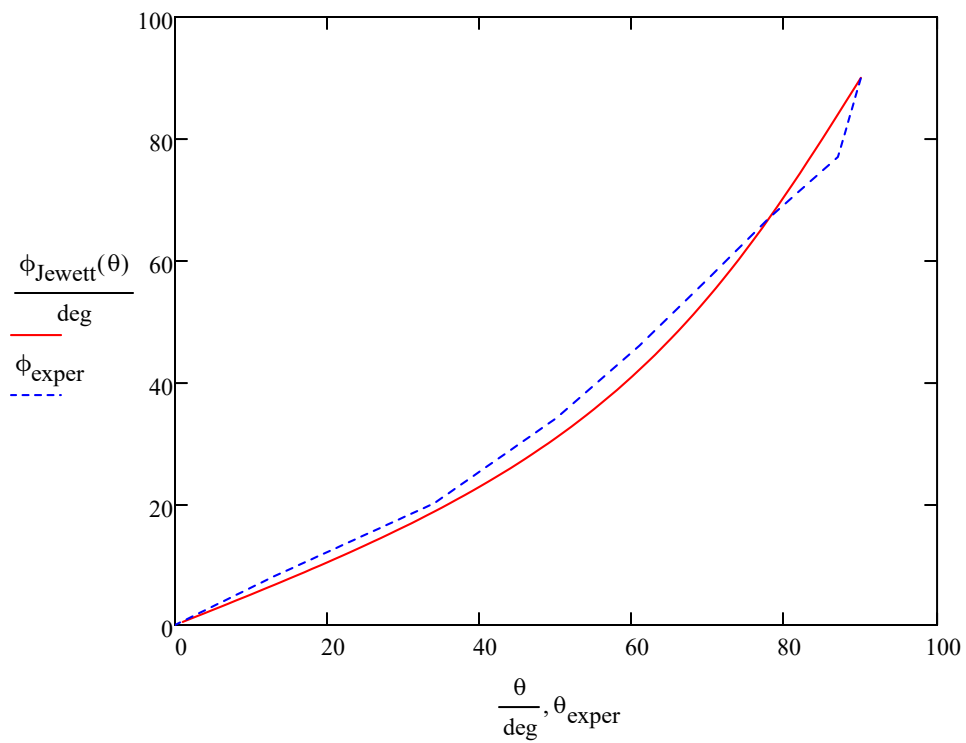
$$\tan(90\text{-deg} - \theta) = \frac{x}{d} \quad (4)$$

Plugging Equation 4 into Equation 3, and solving for ϕ , gives:

$$\phi_{\text{Jewett}}(\theta) := 90\text{-deg} - \text{atan}(2 \cdot \tan(90\text{-deg} - \theta))$$

Here is the comparison of the system predictions to the experimental results:

$$\theta := 1\text{-deg}, 2\text{-deg} .. 90\text{-deg}$$



As you can see, the system matches the experimental results fairly closely over a fairly wide range of target angles.