technical proof technical proof

TP A.28
Throw plots for all types of shots

supporting:
"The Illustrated Principles of Pool and Billiards"
http://billiards.colostate.edu
by David G. Alciatore, PhD, PE ("Dr. Dave")

originally posted: 4/3/2007 last revision: 4/13/2021

Note: These results are based on the analysis in TP A.14.

$$R := \frac{1.125\,in}{m}$$ ball radius converted to meters

Model of friction based on Marlow data (Table 10 on p. 245 in "The Physics of Pocket Billiards," 1995)

$$vd_1 := .1\cdot\sin(45\cdot deg) \qquad \mu d_1 := 0.11$$

$$vd_2 := 1\cdot\sin(45\cdot deg) \qquad \mu d_2 := 0.06$$

$$vd_3 := 10\cdot\sin(45\cdot deg) \qquad \mu d_3 := 0.01$$

We can solve for the coefficients from the set of data above using:

initial guesses:

$$a := \mu d_3 \qquad b := \mu d_1 - \mu d_3 \qquad c := \frac{-1}{vd_2}\cdot\ln\left(\frac{\mu d_2 - a}{b}\right)$$

$$a = 0.01 \qquad b = 0.1 \qquad c = 0.98$$

Given

$$\mu d_1 = a + b\cdot e^{-c\cdot vd_1}$$

$$\mu d_2 = a + b\cdot e^{-c\cdot vd_2}$$

$$\mu d_3 = a + b\cdot e^{-c\cdot vd_3}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} := \mathrm{Find}(a, b, c) \qquad \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 9.951\times 10^{-3} \\ 0.108 \\ 1.088 \end{pmatrix}$$

$$\mu(v) := a + b\cdot e^{-c\cdot v}$$

this is the "model" of friction that will be used

MathCAD formulations of Equations 15 through 17, using the above formulation for friction:

$$v_{rel}(v,\omega_x,\omega_z,\phi) := \sqrt{(v\cdot\sin(\phi) - R\cdot\omega_z)^2 + (R\cdot\omega_x\cdot\cos(\phi))^2}$$

$$\theta_{throw}(v,\omega_x,\omega_z,\phi) := \mathrm{atan}\left[\frac{\min\left(\frac{\mu(v_{rel}(v,\omega_x,\omega_z,\phi))\cdot v\cdot\cos(\phi)}{v_{rel}(v,\omega_x,\omega_z,\phi)}, \frac{1}{7}\right)\cdot(v\cdot\sin(\phi) - R\cdot\omega_z)}{v\cdot\cos(\phi)}\right]$$

cut angle range:

$$\phi := 0\cdot deg, 1\cdot deg .. 75\cdot deg$$

various speeds (slow, medium, fast):

$$v_1 := 0.5\cdot\frac{m}{s} \qquad v_2 := 1.5\cdot\frac{m}{s} \qquad v_3 := 4.5\cdot\frac{m}{s}$$

$$v_1 = 1.118\cdot mph \qquad v_2 = 3.355\cdot mph \qquad v_3 = 10.066\cdot mph$$

$$v1 := \frac{v_1}{\frac{m}{s}} \qquad v2 := \frac{v_2}{\frac{m}{s}} \qquad v3 := \frac{v_3}{\frac{m}{s}}$$

spin rate from percent English (from TP A.25):

$$\omega(v,PE) = SRF\cdot\omega_{roll} = \frac{5}{4}\cdot\frac{v}{R}\cdot PE$$

$$\omega(v,PE) := \frac{5}{4}\cdot\frac{v}{R}\cdot PE$$

throw angle equation variable info

$$\theta_{throw}(v,\omega_x,\omega_z,\phi)$$

ω_x, ω_r: vertical plane spin rate (follow or draw)

ω_z, ω_e: English spin rate

PE_r: % topspin (roll)

PE_e: % English (side spin)

topspin: ω_r, $PE_r > 0$

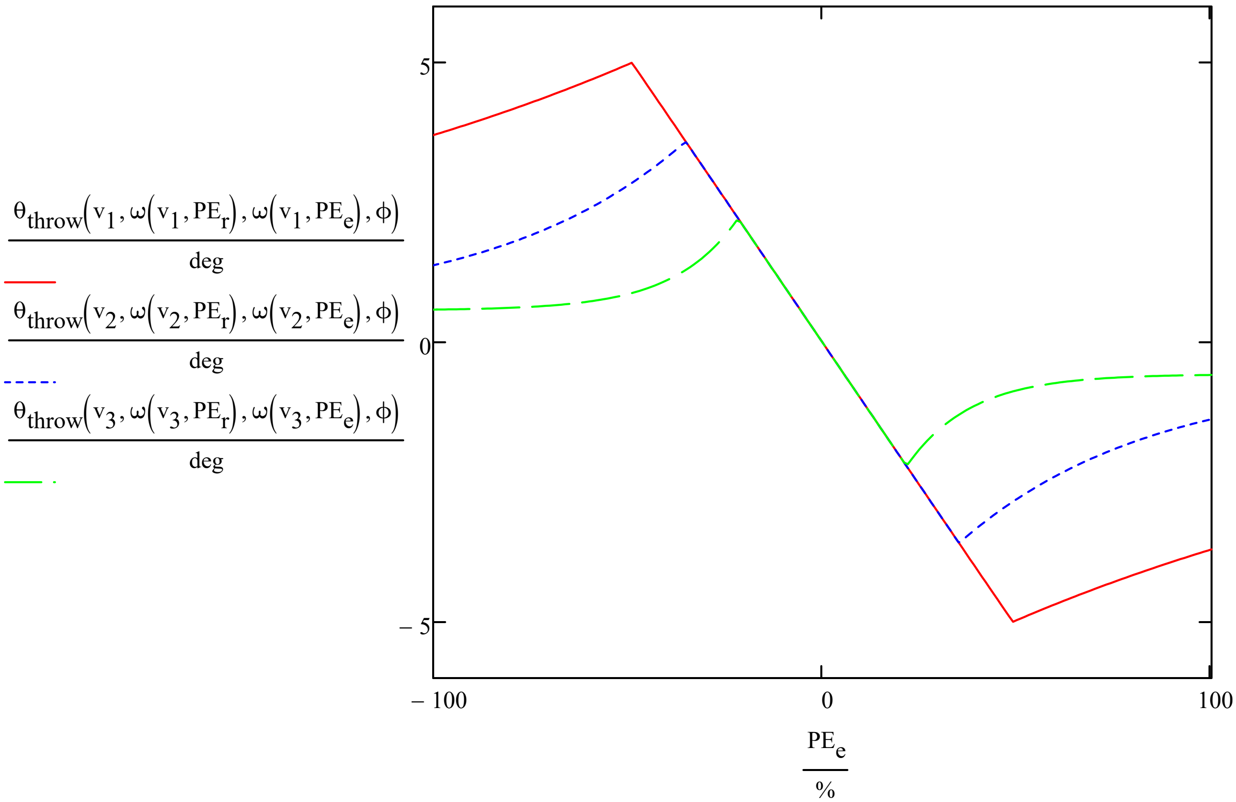
outside English (OE): ω_e, $PE_e > 0$

bottom-spin: ω_r, $PE_r < 0$

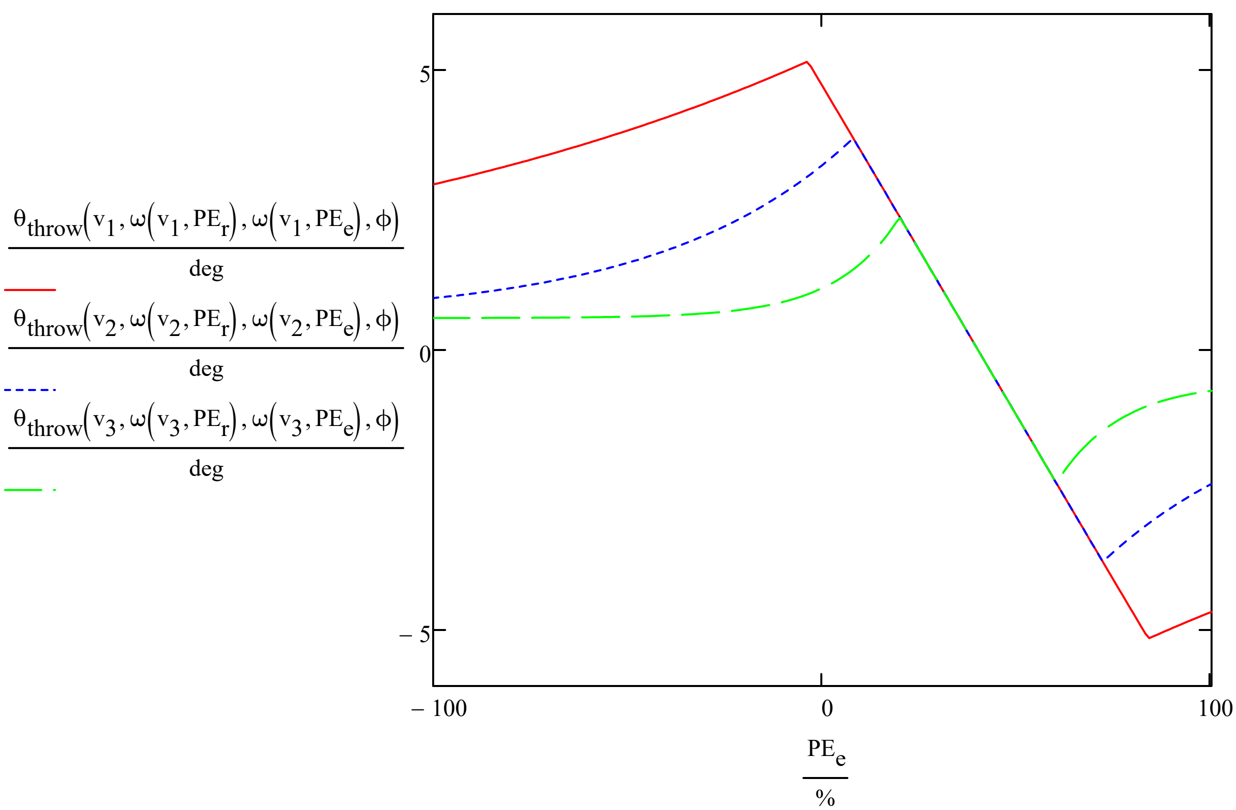
inside English (IE): ω_e, $PE_e < 0$

These plots show how the amount of throw varies with the amount of English for various cut angles, speeds, and amounts of vertical plane spin (draw or follow):

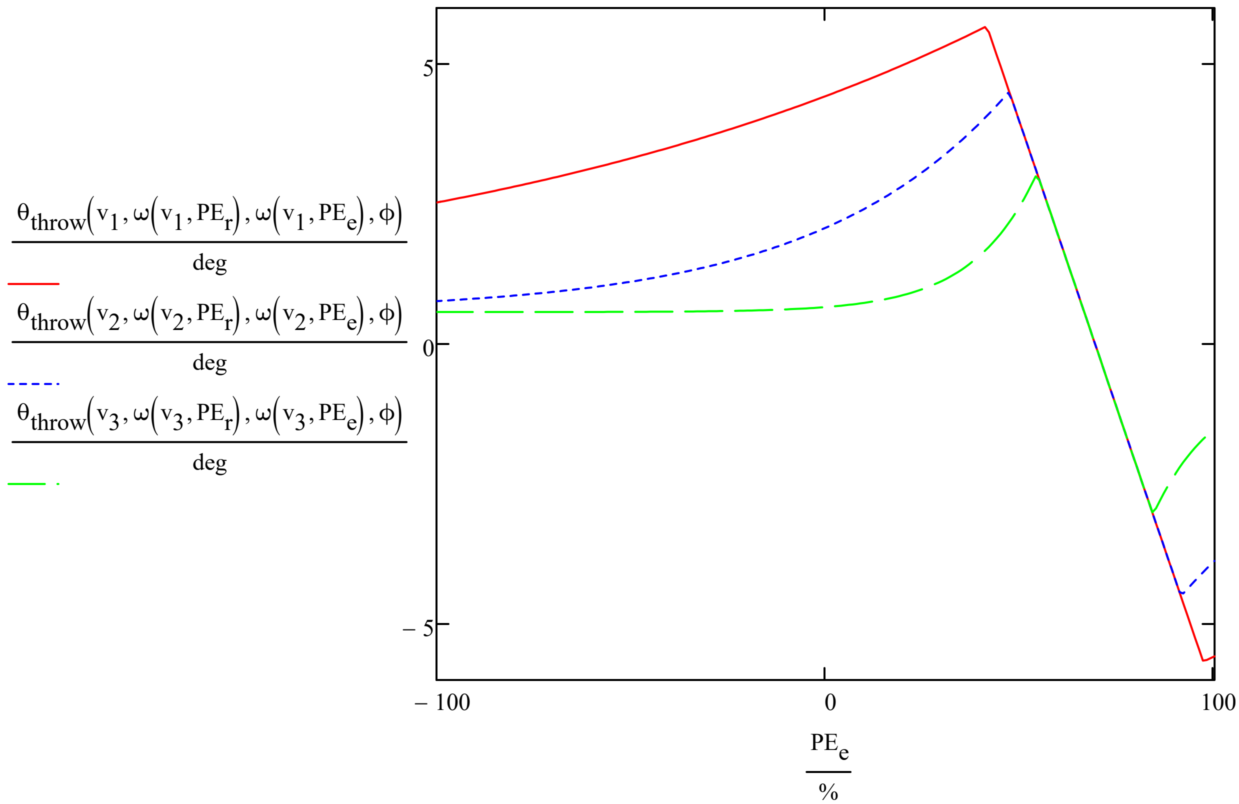
$PE_r := 0\%$ $\quad \phi := 0 \cdot deg$ $\quad PE_e := -100 \cdot \%, -99\% .. 100 \cdot \%$



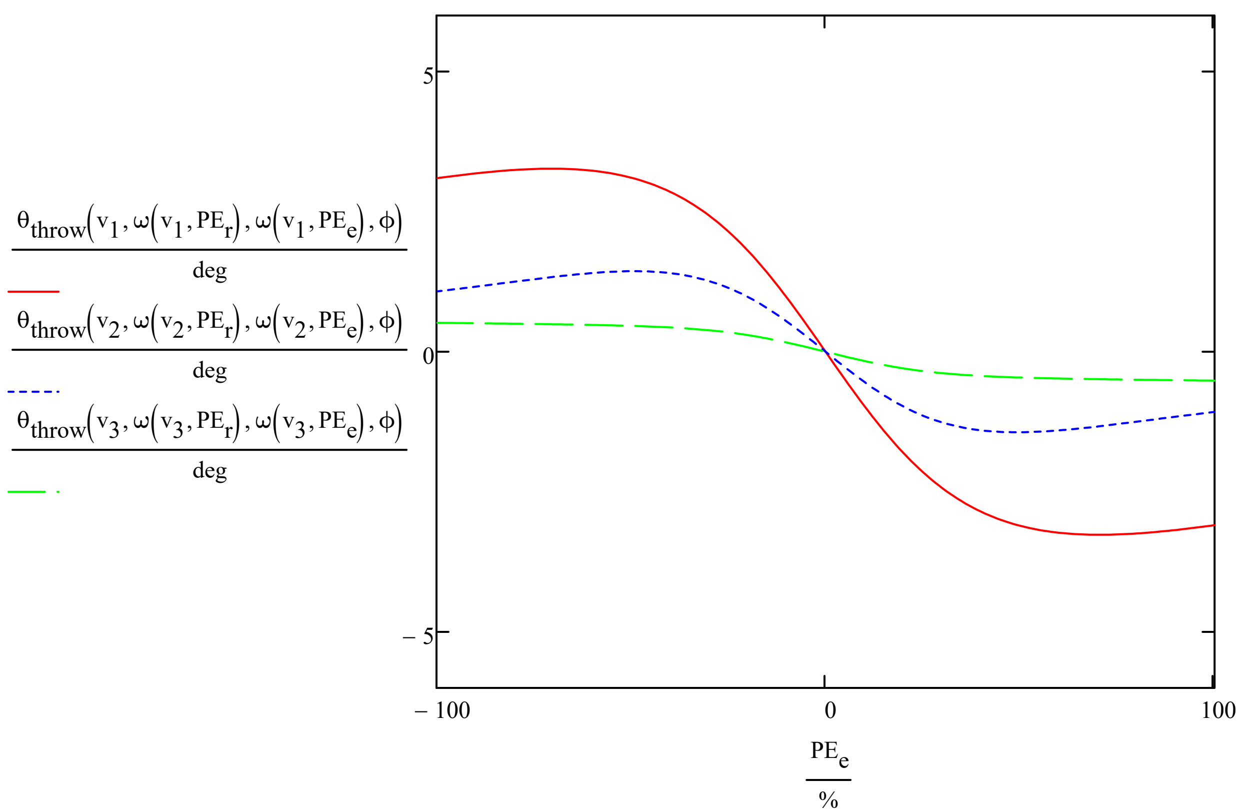
$PE_r := 0\%$ $\quad \phi := 30 \cdot deg$



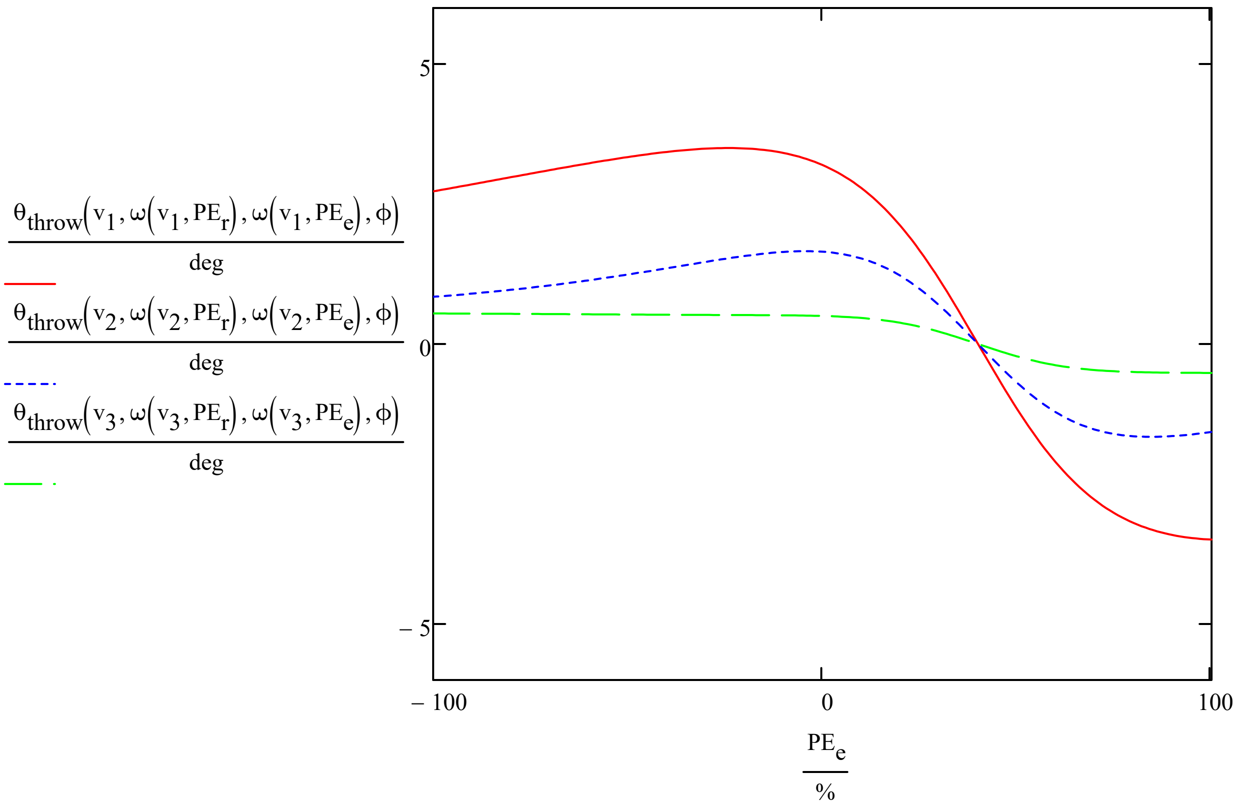
$PE_r := 0\%$ $\phi := 60 \cdot deg$



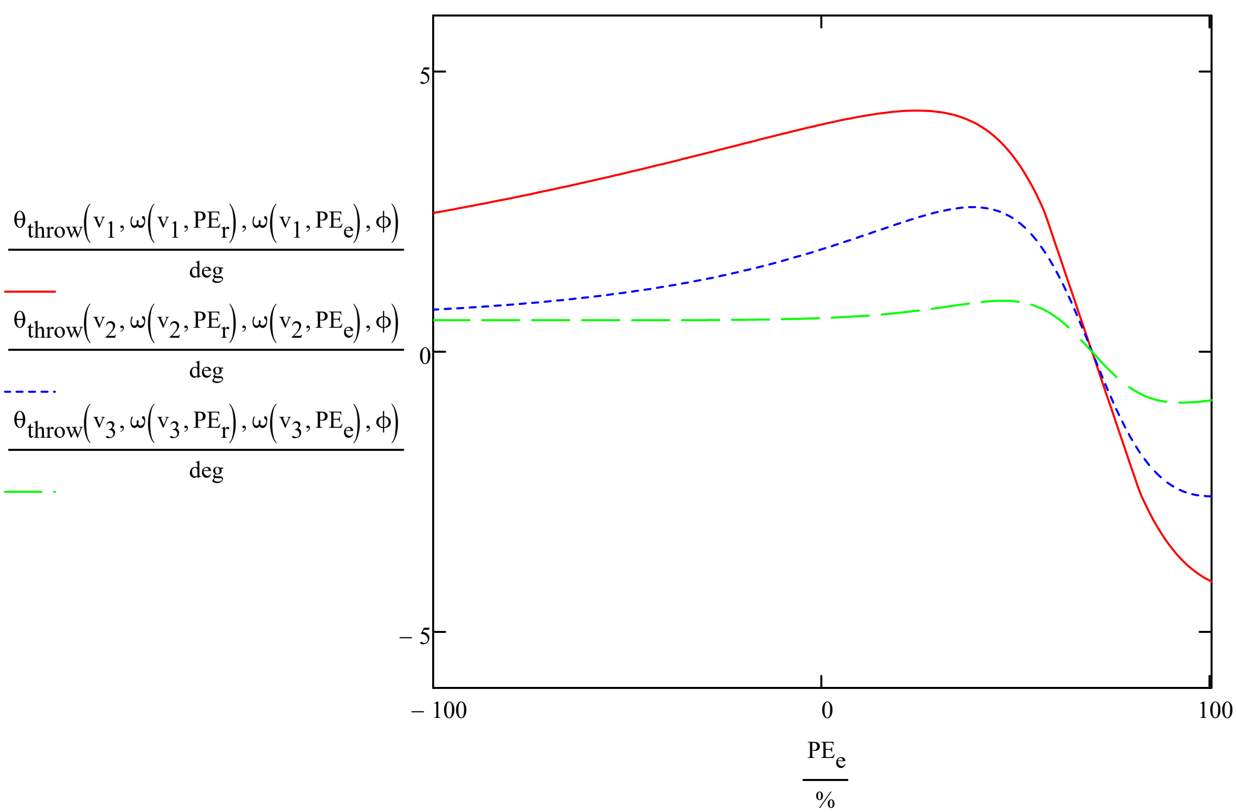
$PE_r := 50\%$ $\phi := 0 \cdot deg$



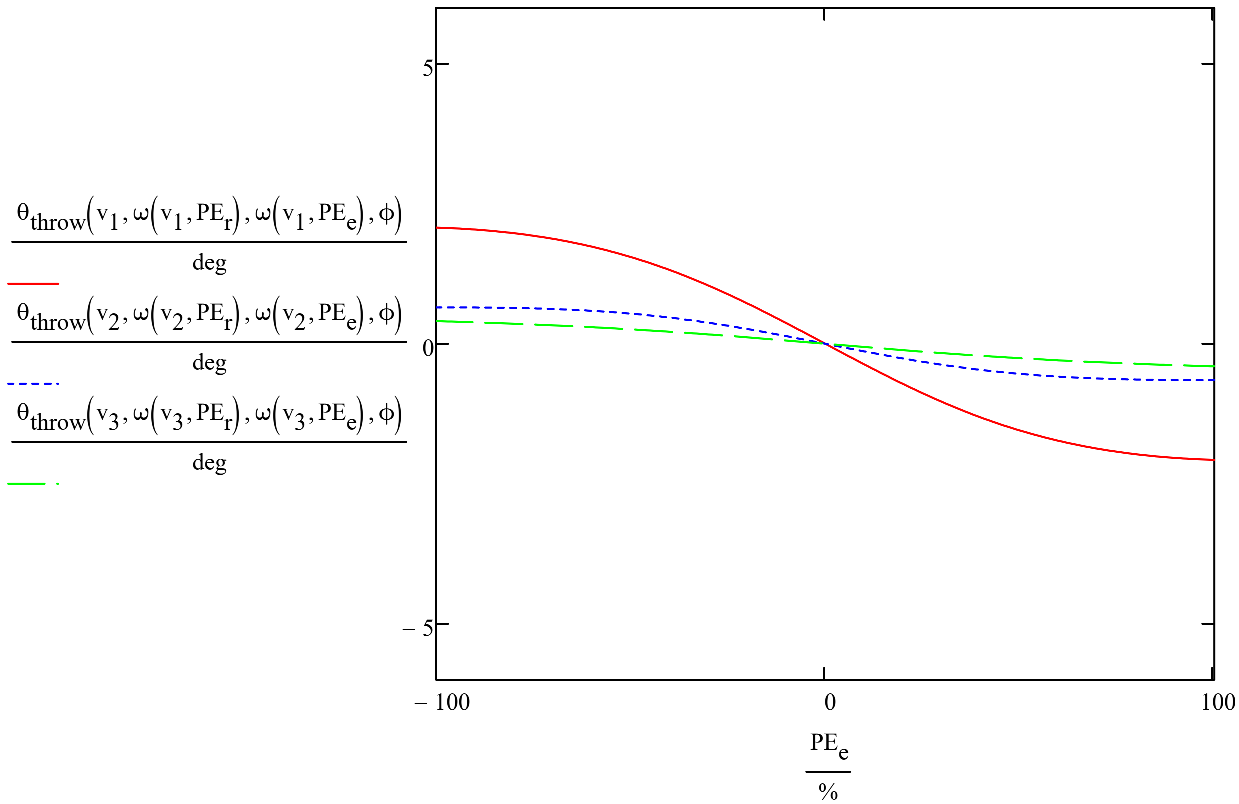
$PE_r := 50\%$ $\qquad \phi := 30 \cdot deg$



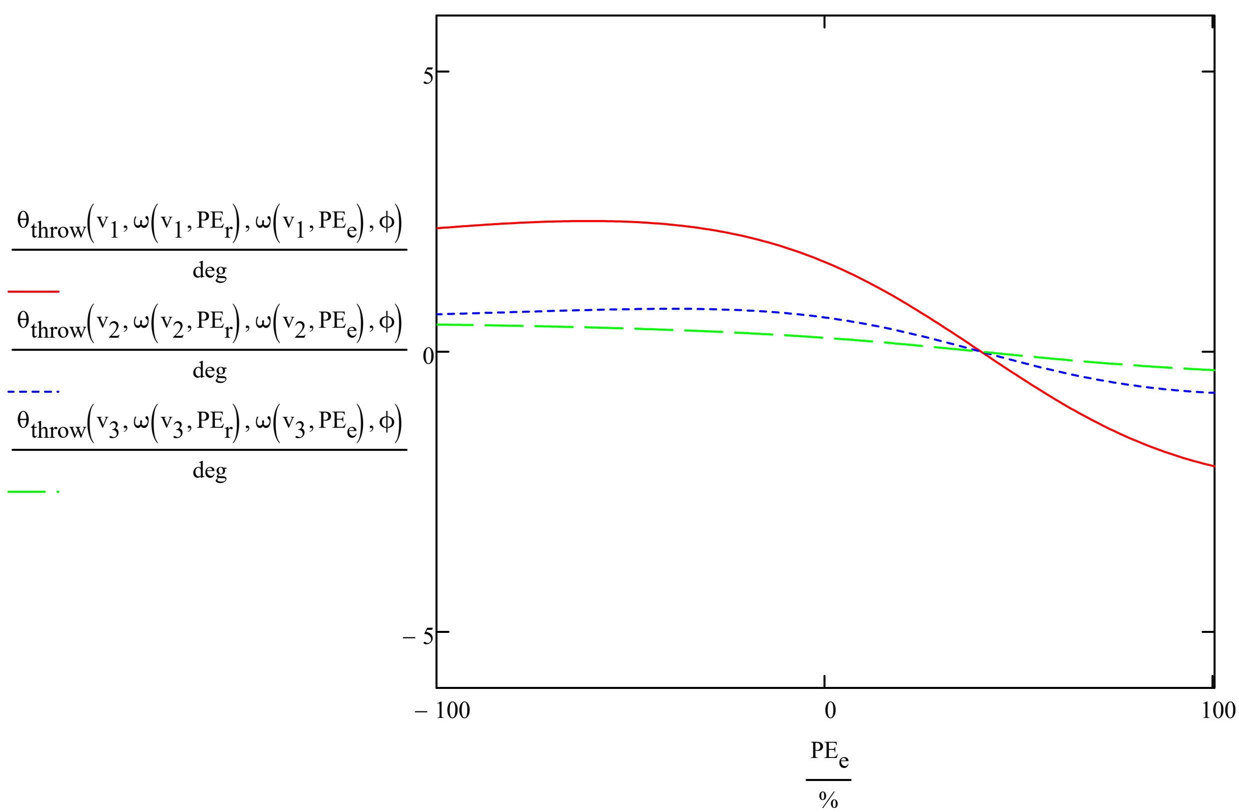
$PE_r := 50\%$ $\qquad \phi := 60 \cdot deg$



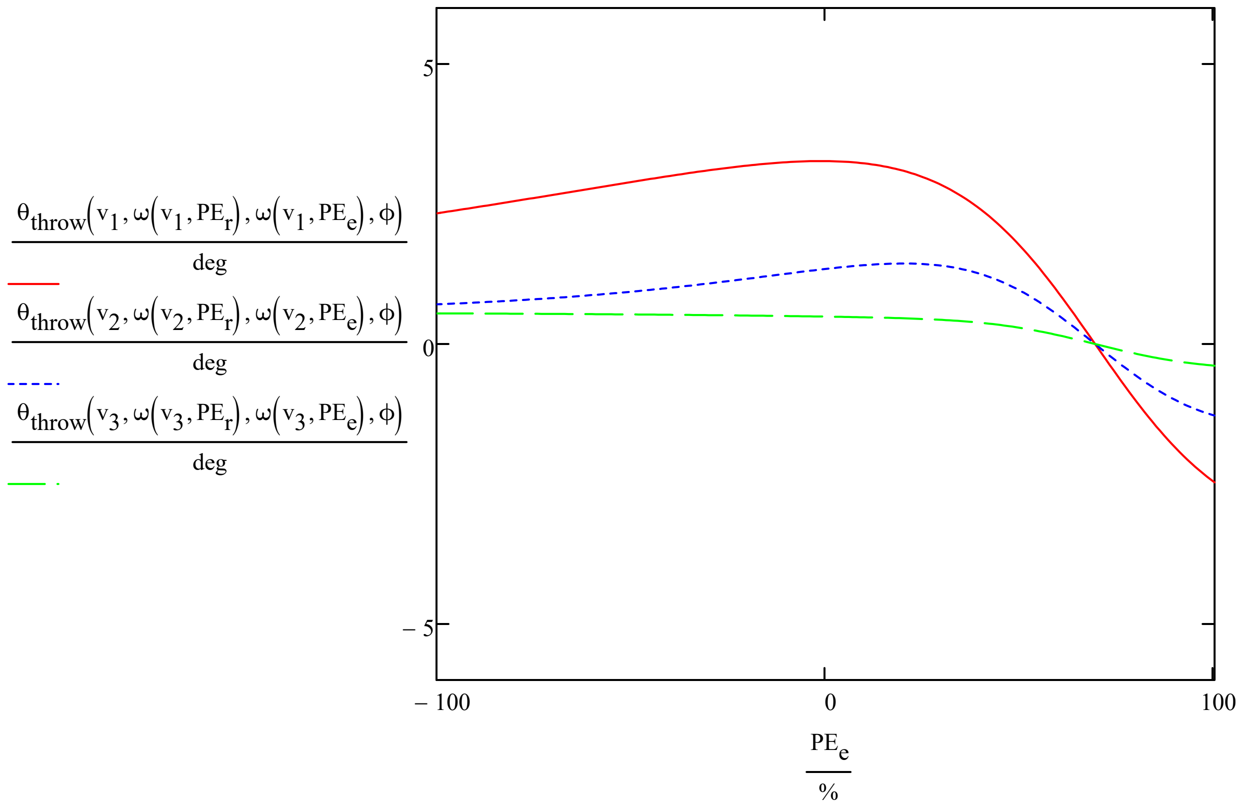
$PE_r := 100\%$ $\phi := 0 \cdot deg$



$PE_r := 100\%$ $\phi := 30 \cdot deg$

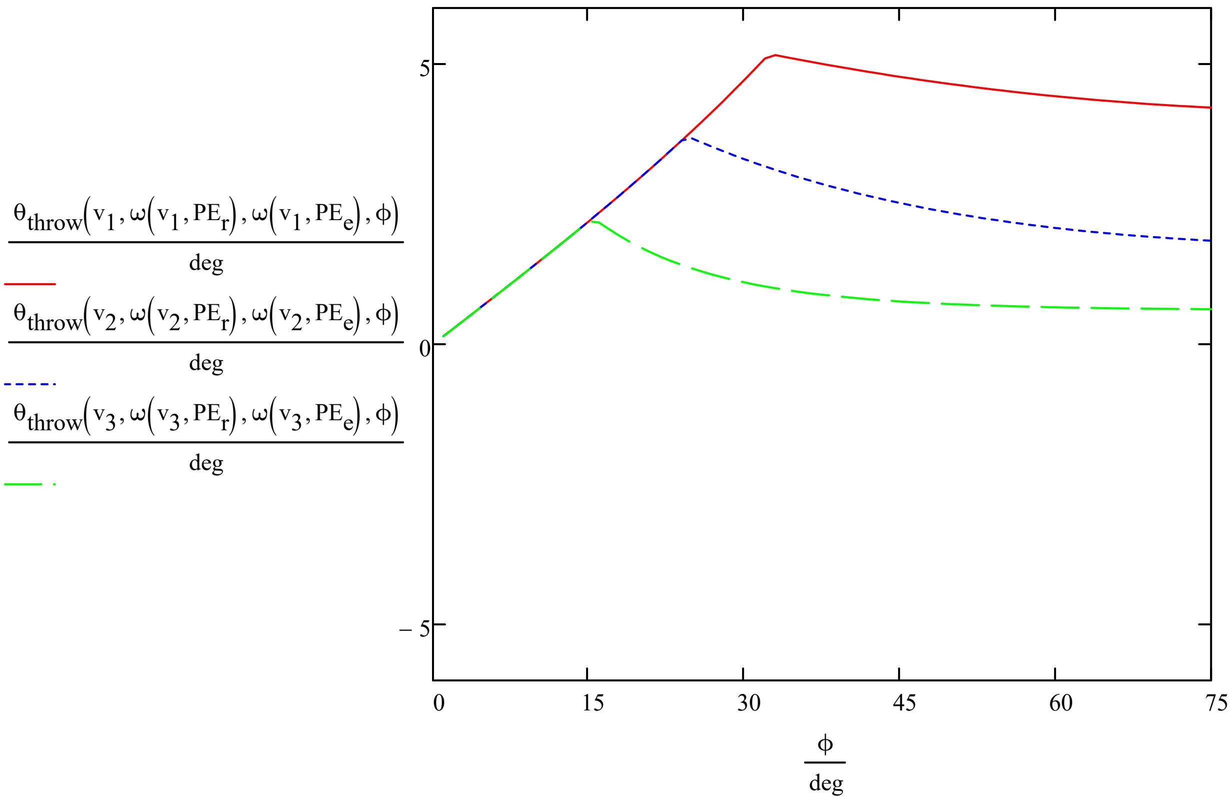


$PE_r := 100\%$ $\phi := 60\cdot deg$

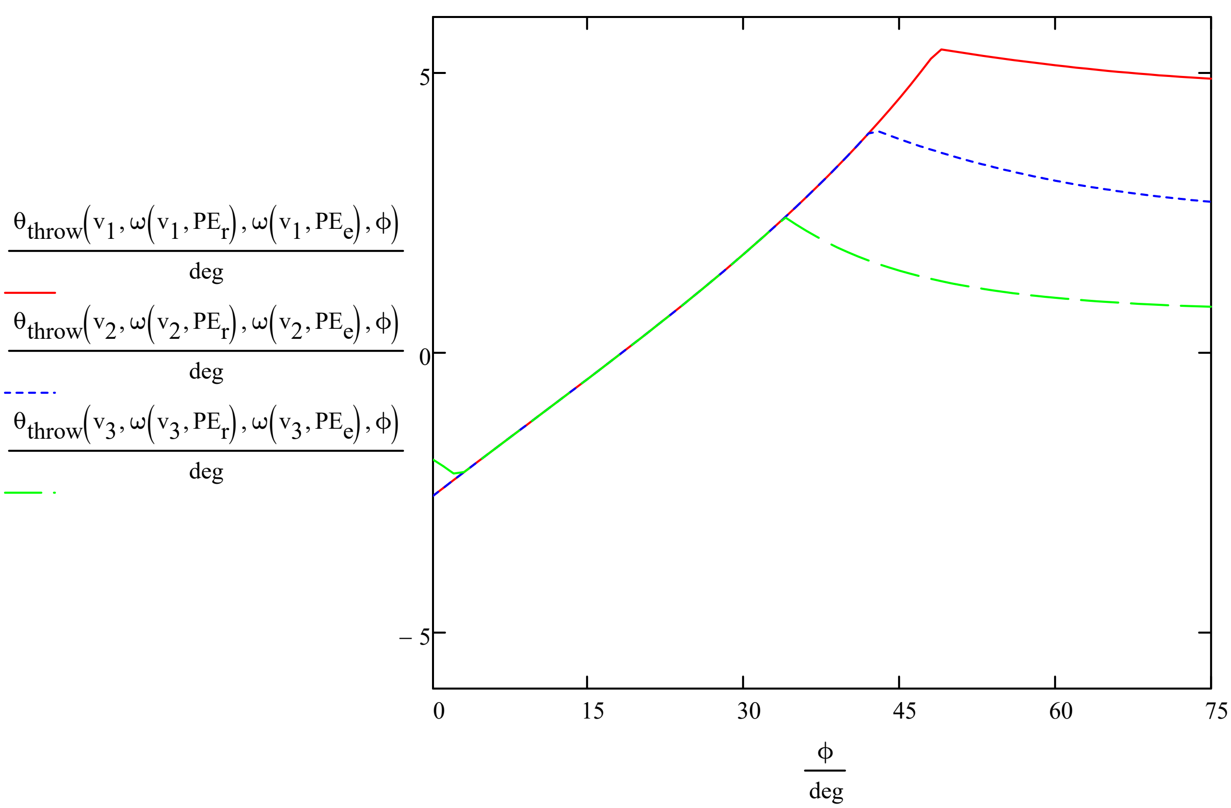


The remaining plots show how throw varies with cut angle for various speed shots with various amounts of English and vertical plane spin (follow or draw):

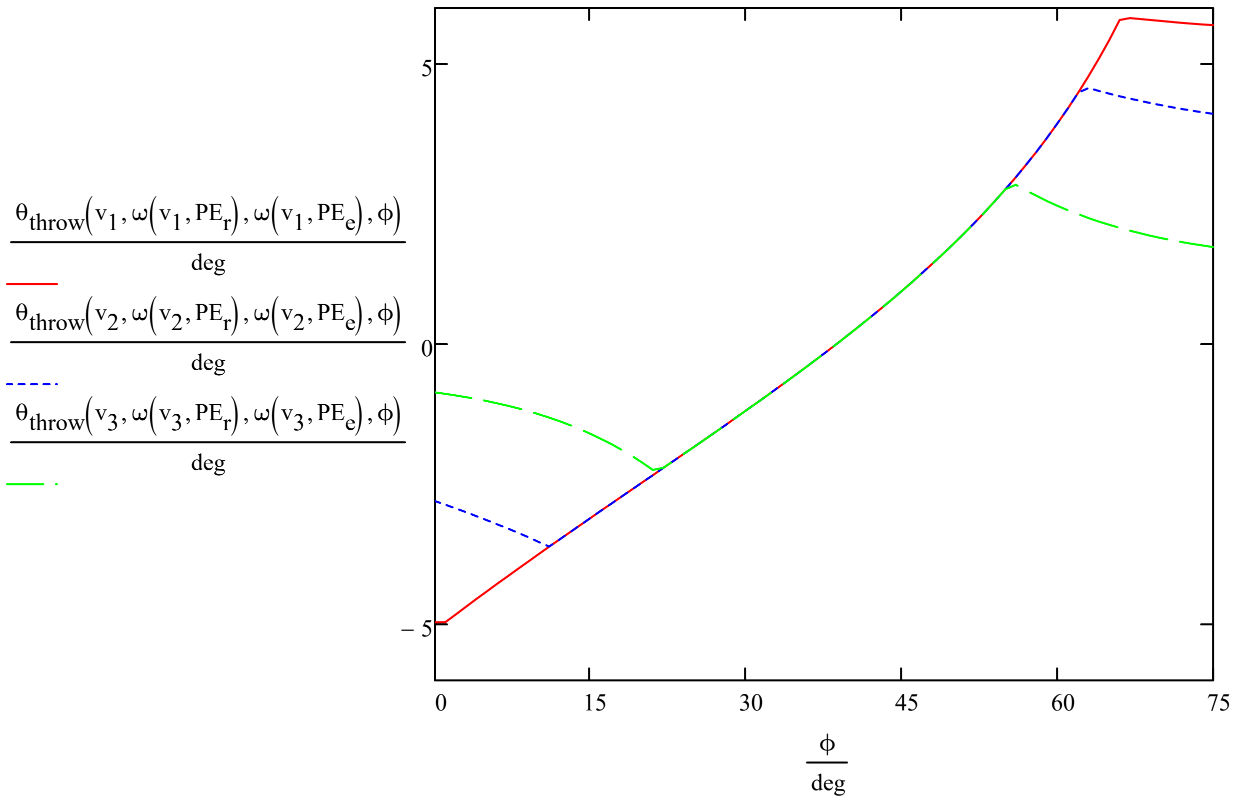
$PE_r := 0\%$ $\quad PE_e := 0\%$ $\qquad \phi := 0 \cdot deg, 1 \cdot deg .. 75 \cdot deg$



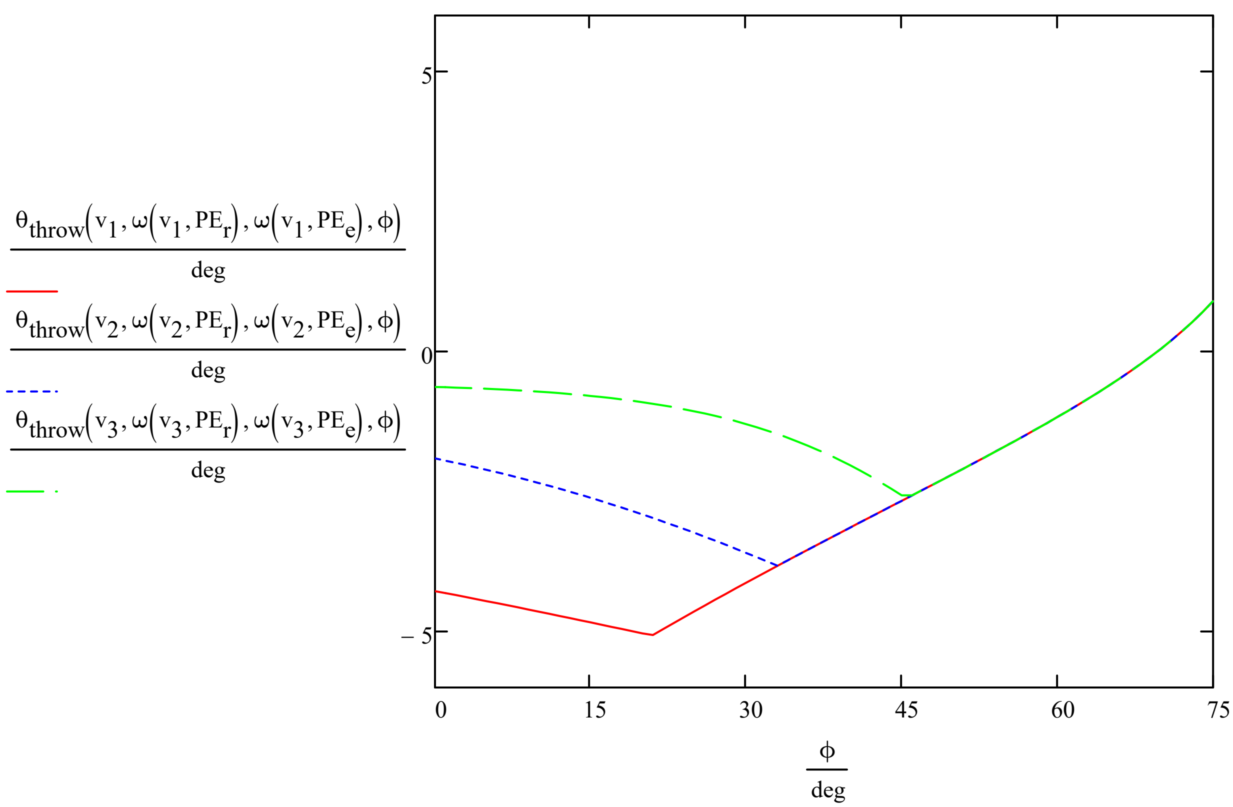
$PE_r := 0\%$ $\quad PE_e := 25\%$



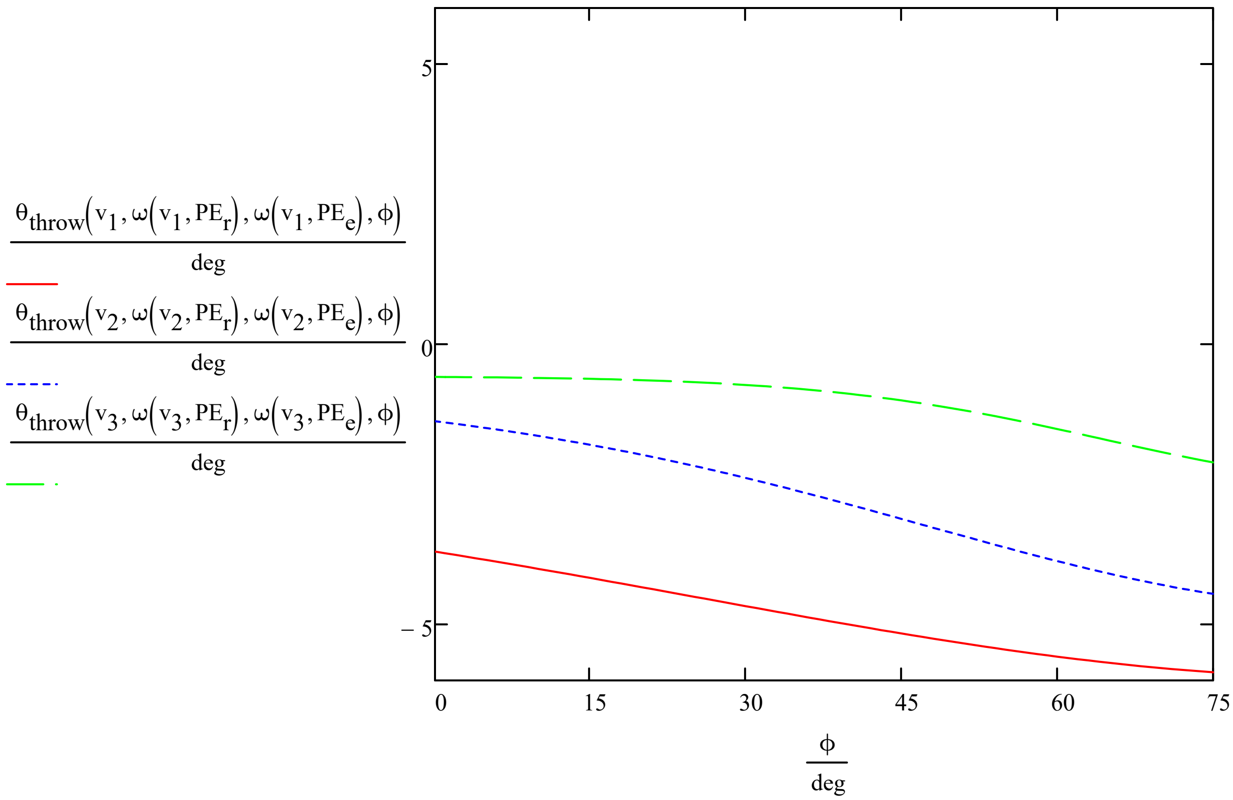
$PE_r := 0\%$ $PE_e := 50\%$



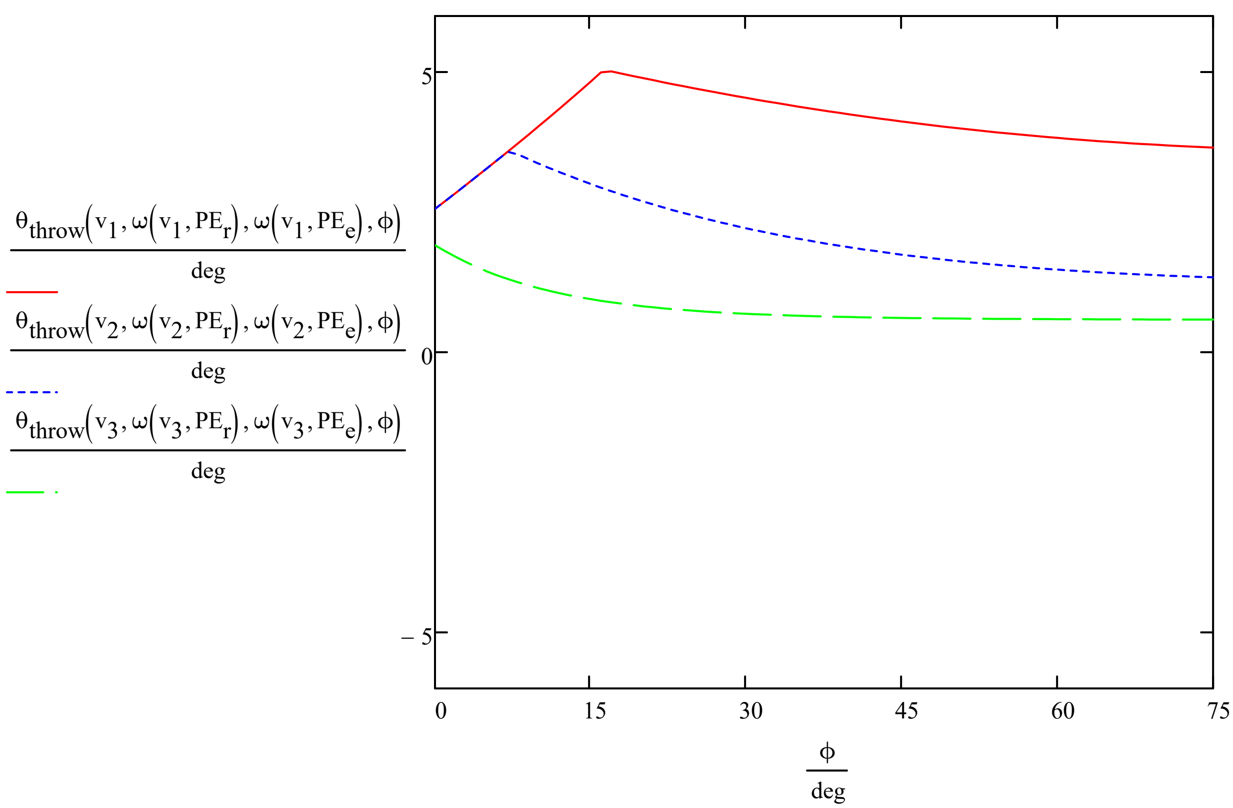
$PE_r := 0\%$ $PE_e := 75\%$



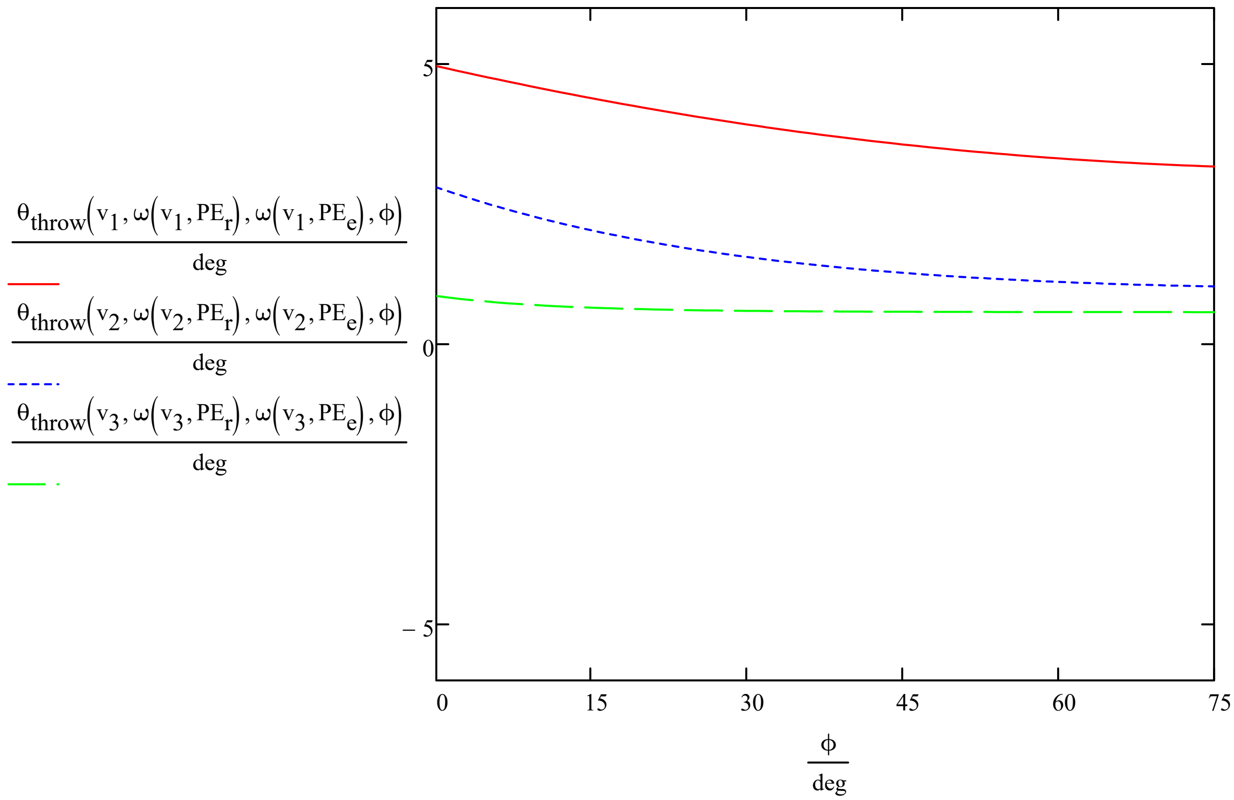
$PE_r := 0\% \qquad PE_e := 100\%$



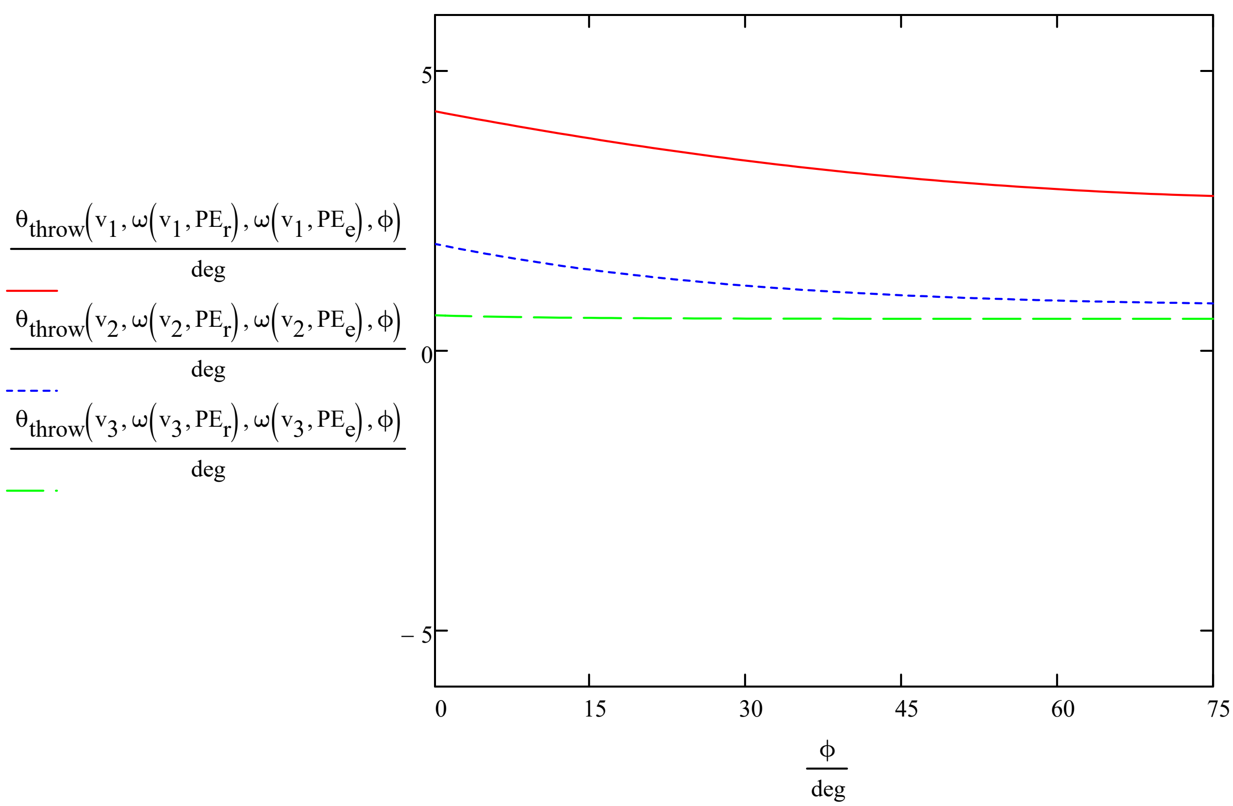
$PE_r := 0\% \qquad PE_e := -25\%$



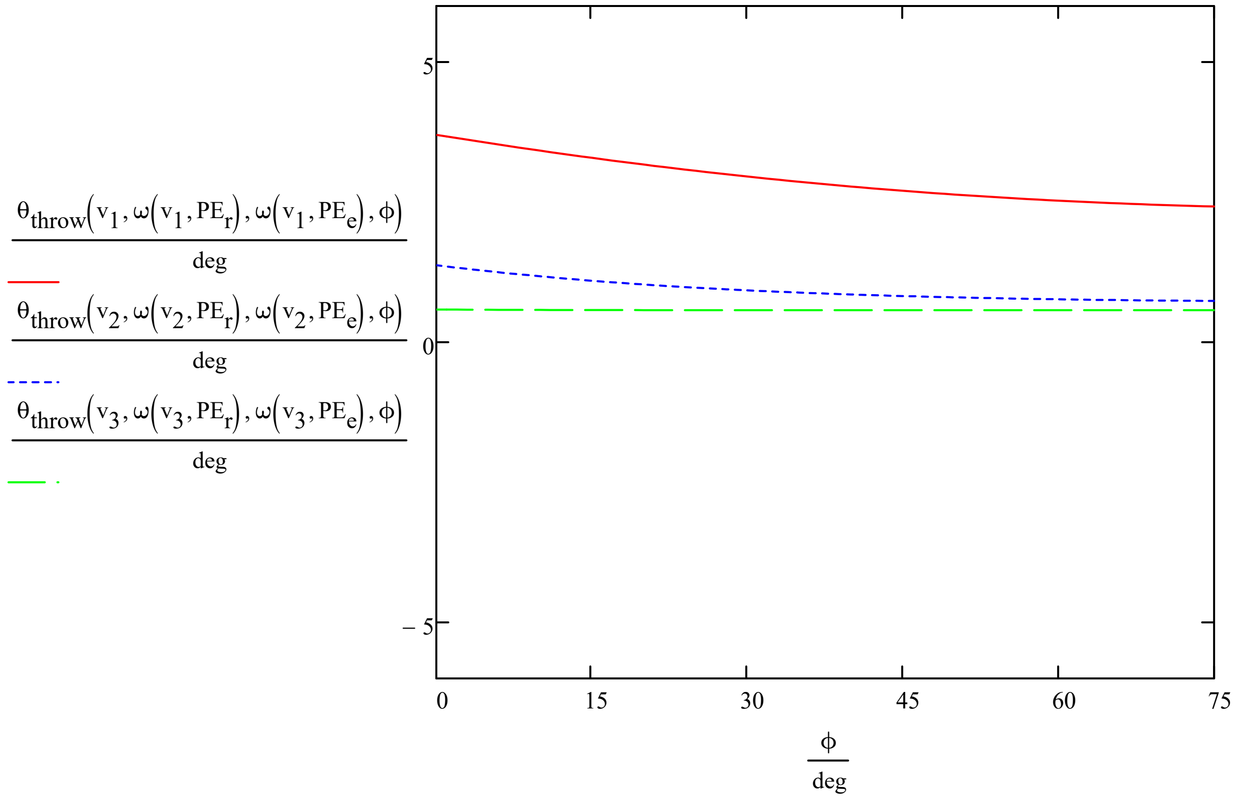
$PE_r := 0\%$ $\quad PE_e := -50\%$



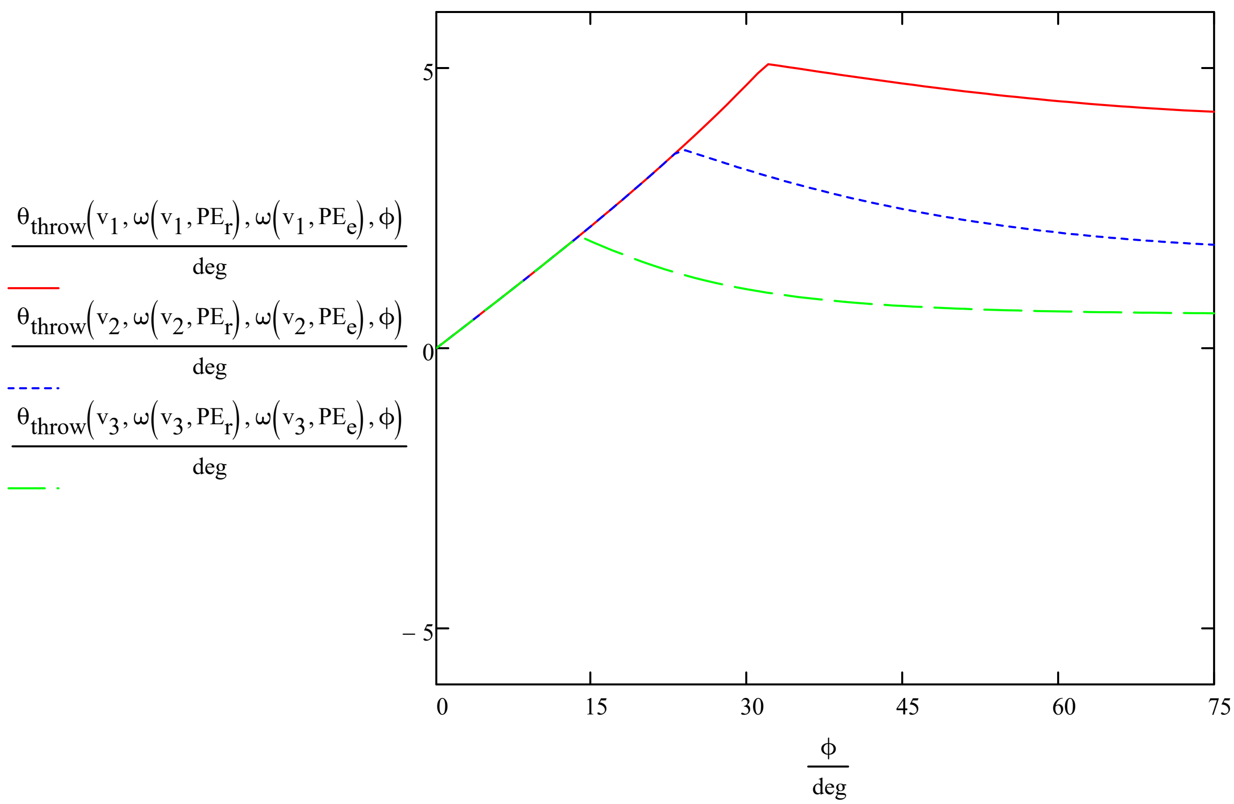
$PE_r := 0\%$ $\quad PE_e := -75\%$



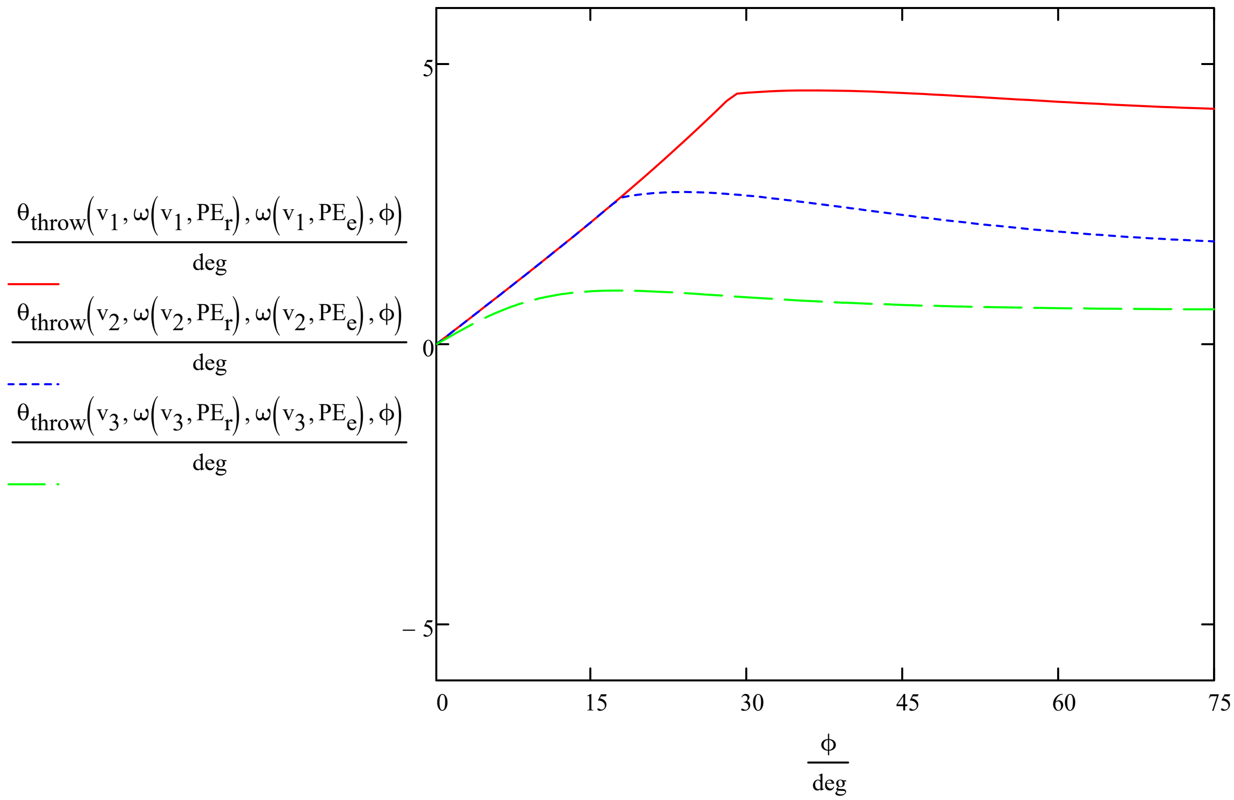
$PE_r := 0\%$ $\quad PE_e := -100\%$



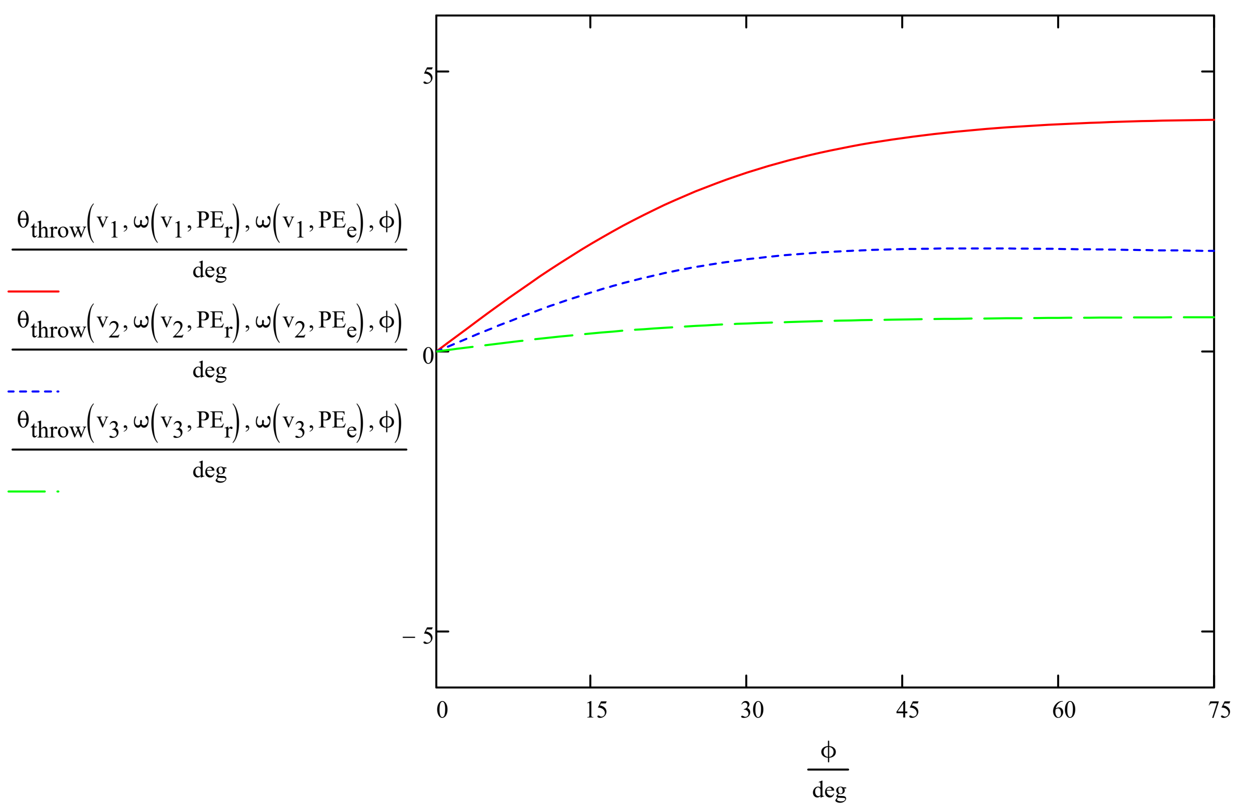
$PE_r := 10\%$ $\quad PE_e := 0\%$



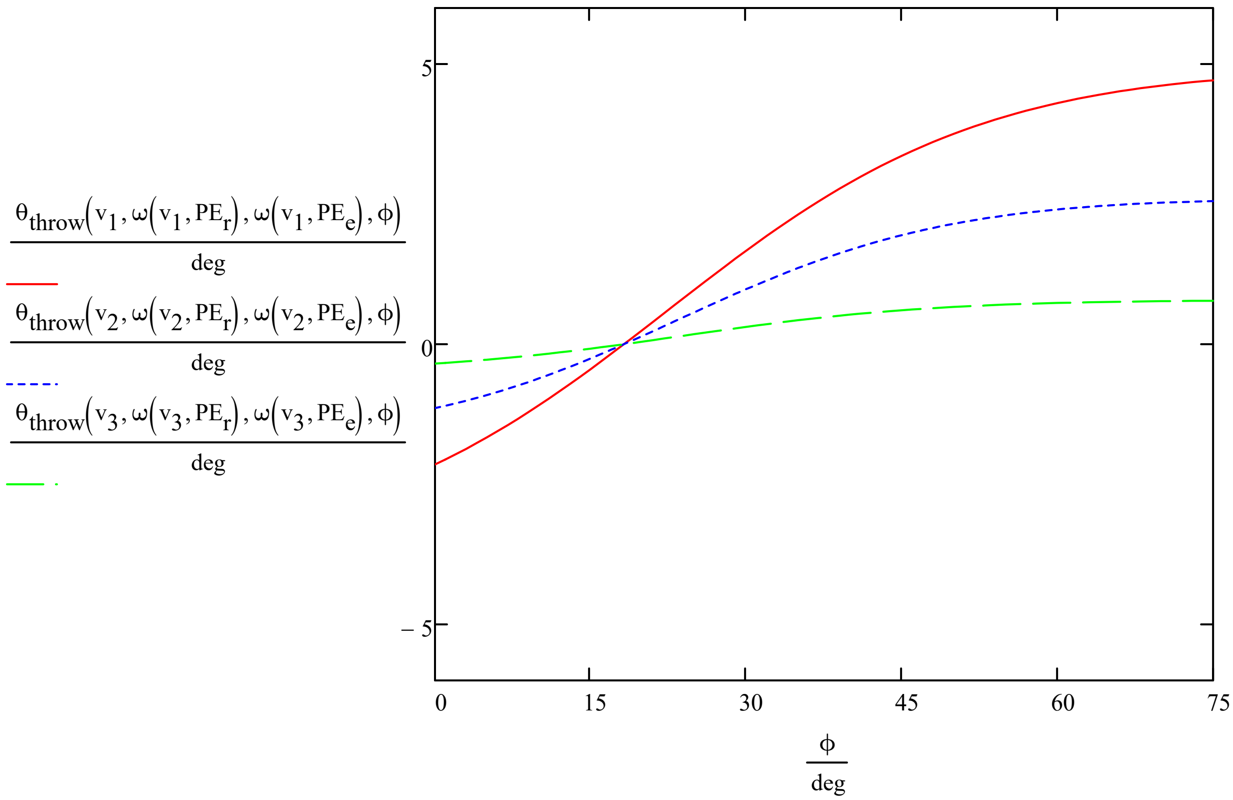
$PE_r := 25\%$ $PE_e := 0\%$



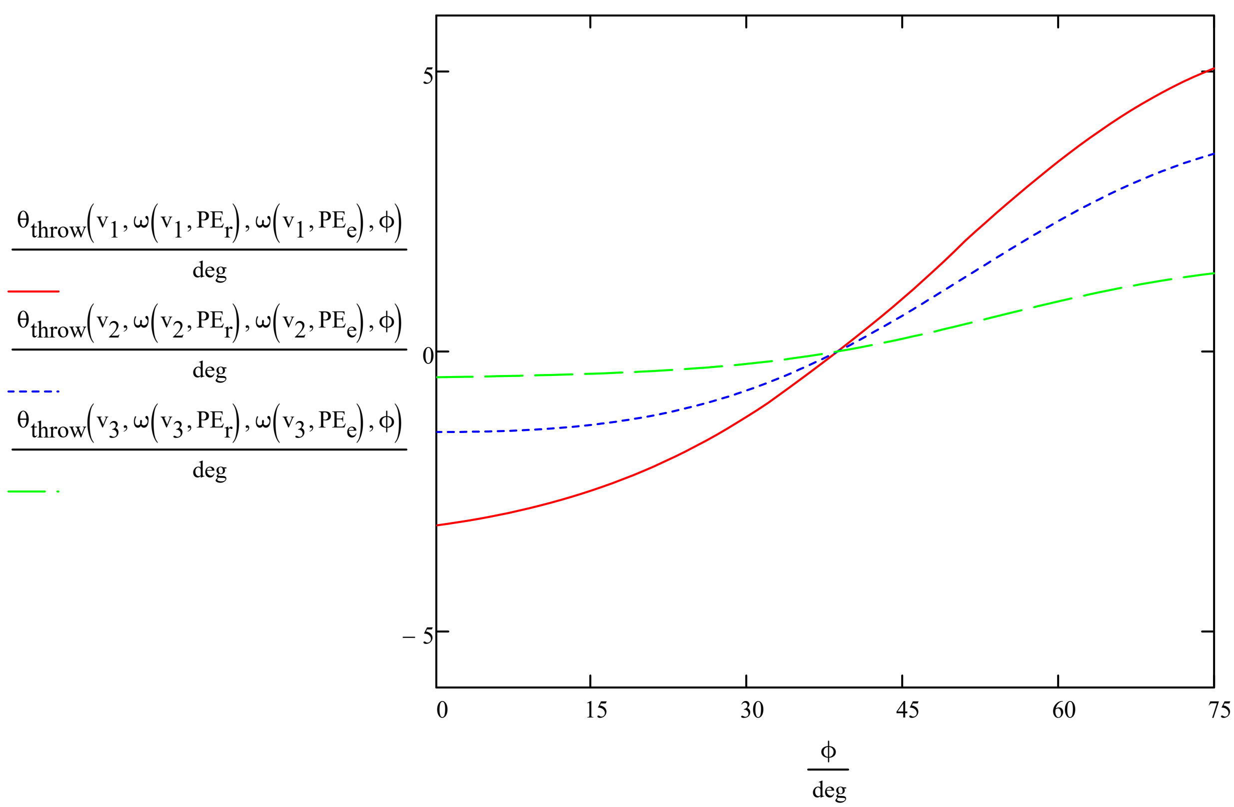
$PE_r := 50\%$ $PE_e := 0\%$



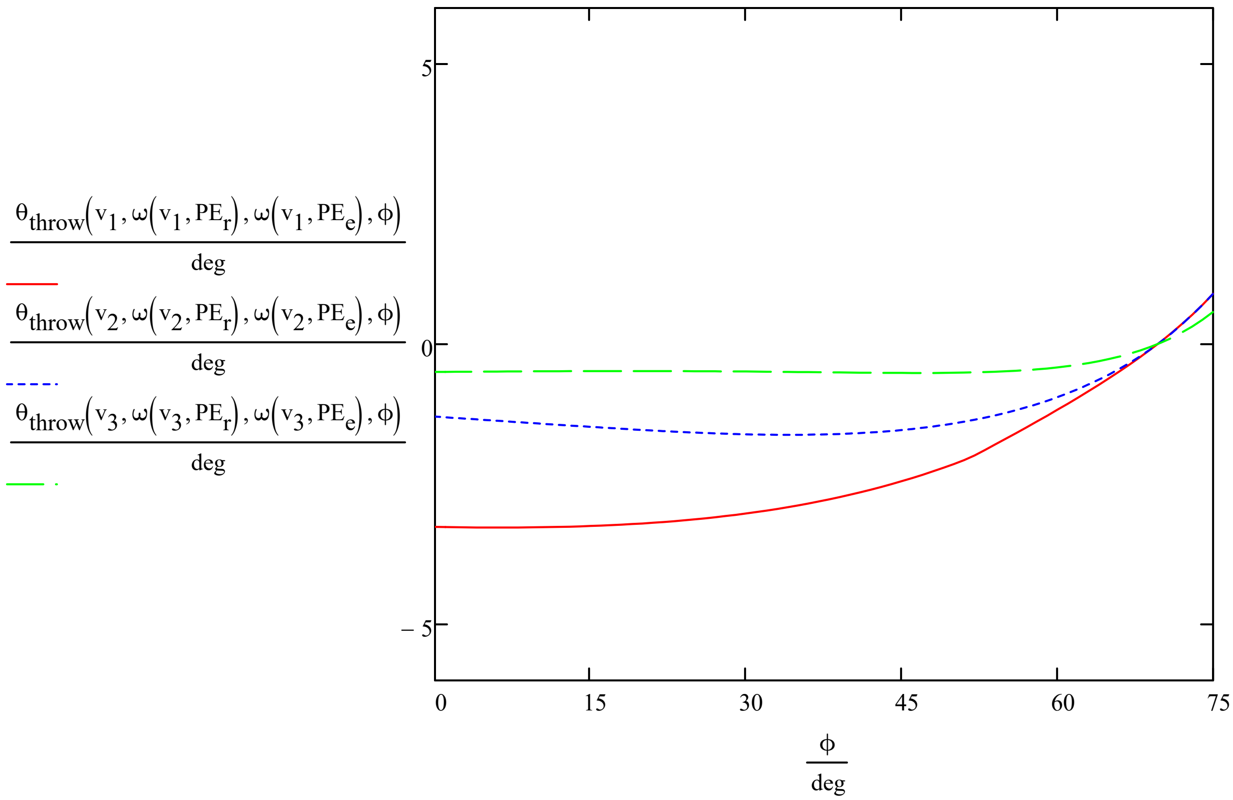
$PE_r := 50\%$ $PE_e := 25\%$



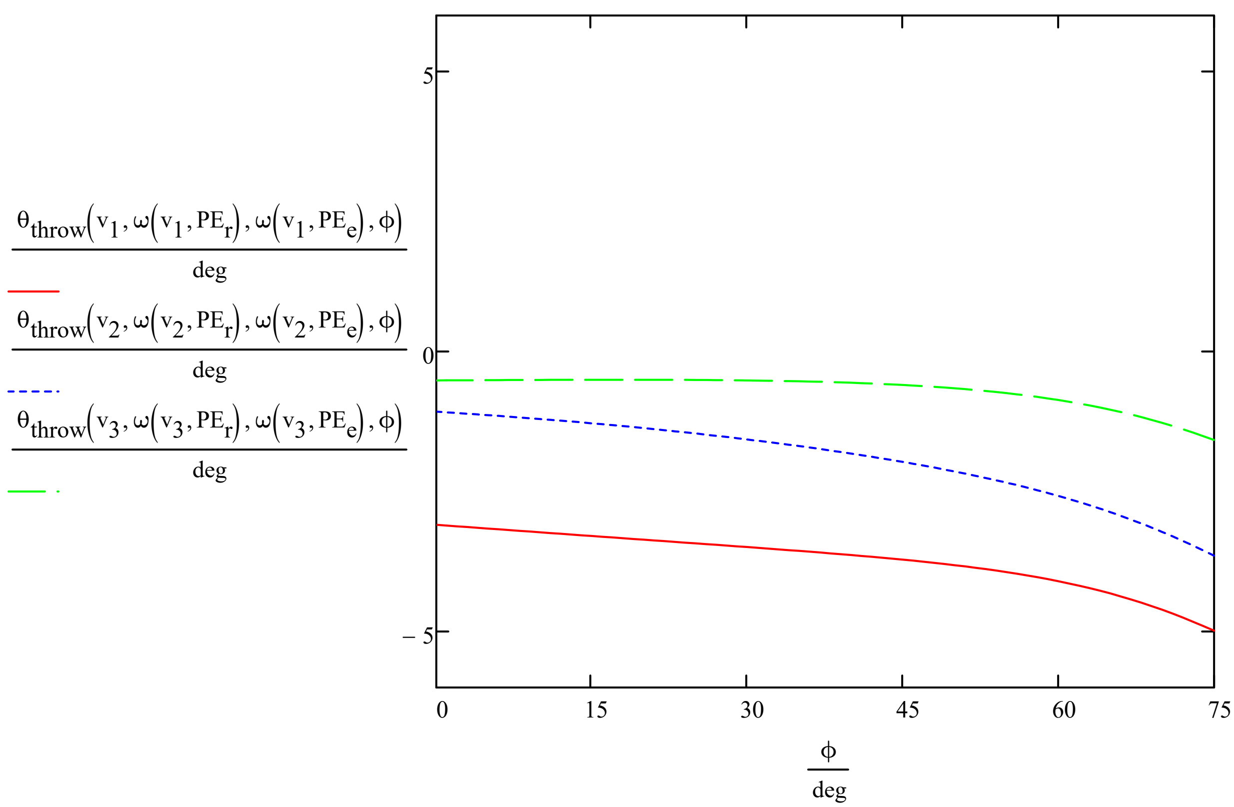
$PE_r := 50\%$ $PE_e := 50\%$



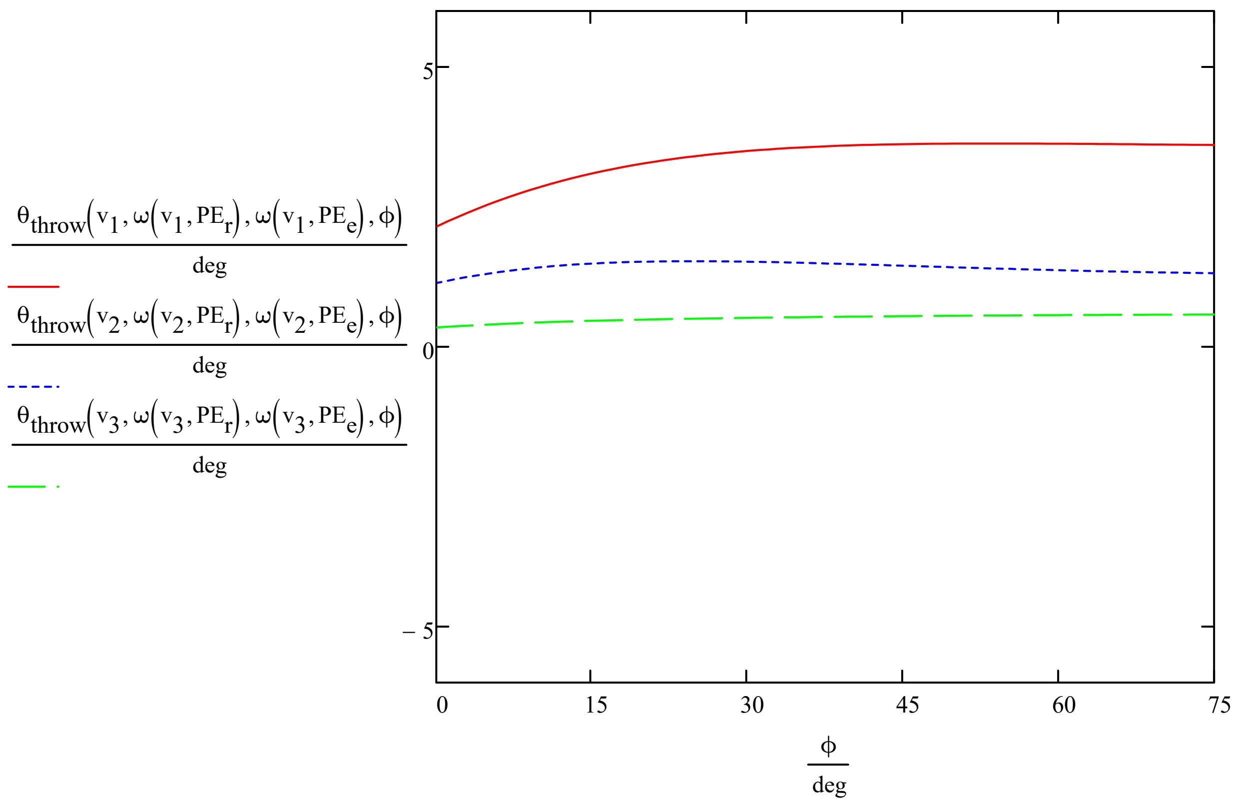
$PE_r := 50\%$ $\quad$ $PE_e := 75\%$



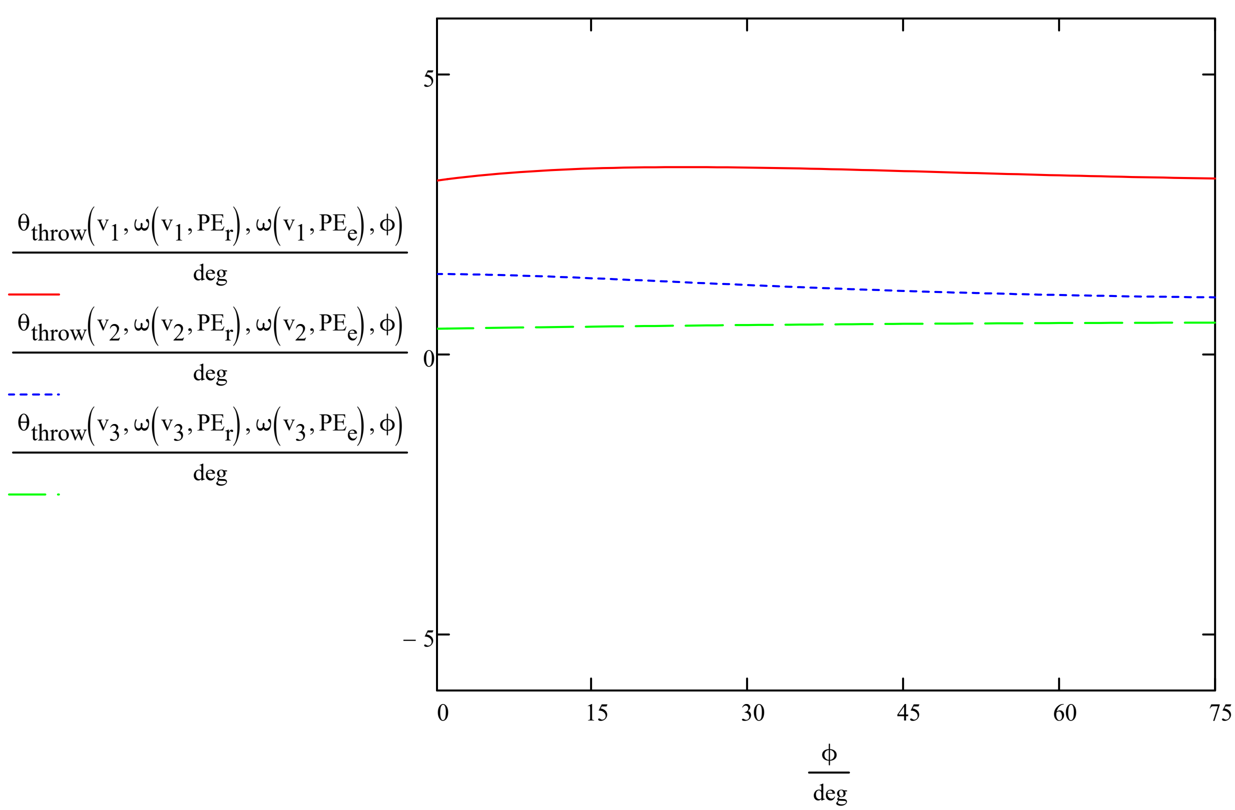
$PE_r := 50\%$ $\quad$ $PE_e := 100\%$



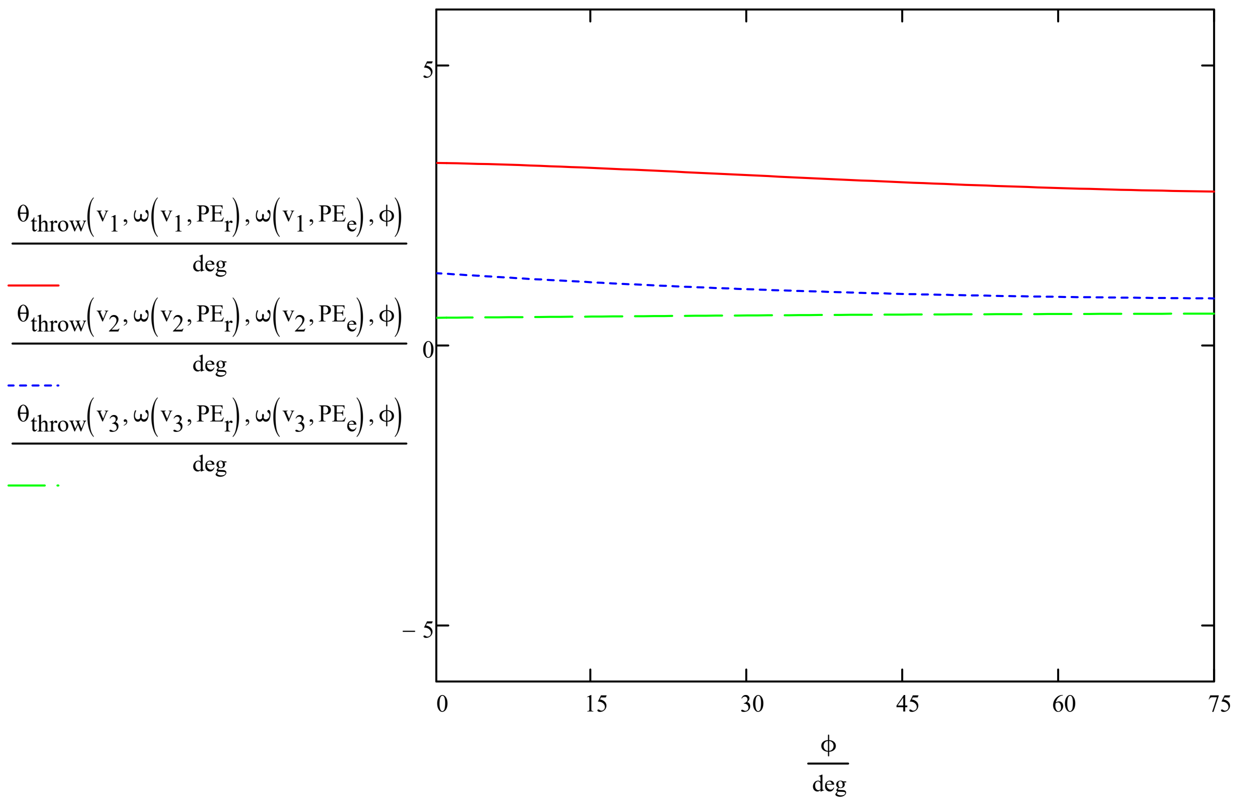
$PE_r := 50\%$ $PE_e := -25\%$



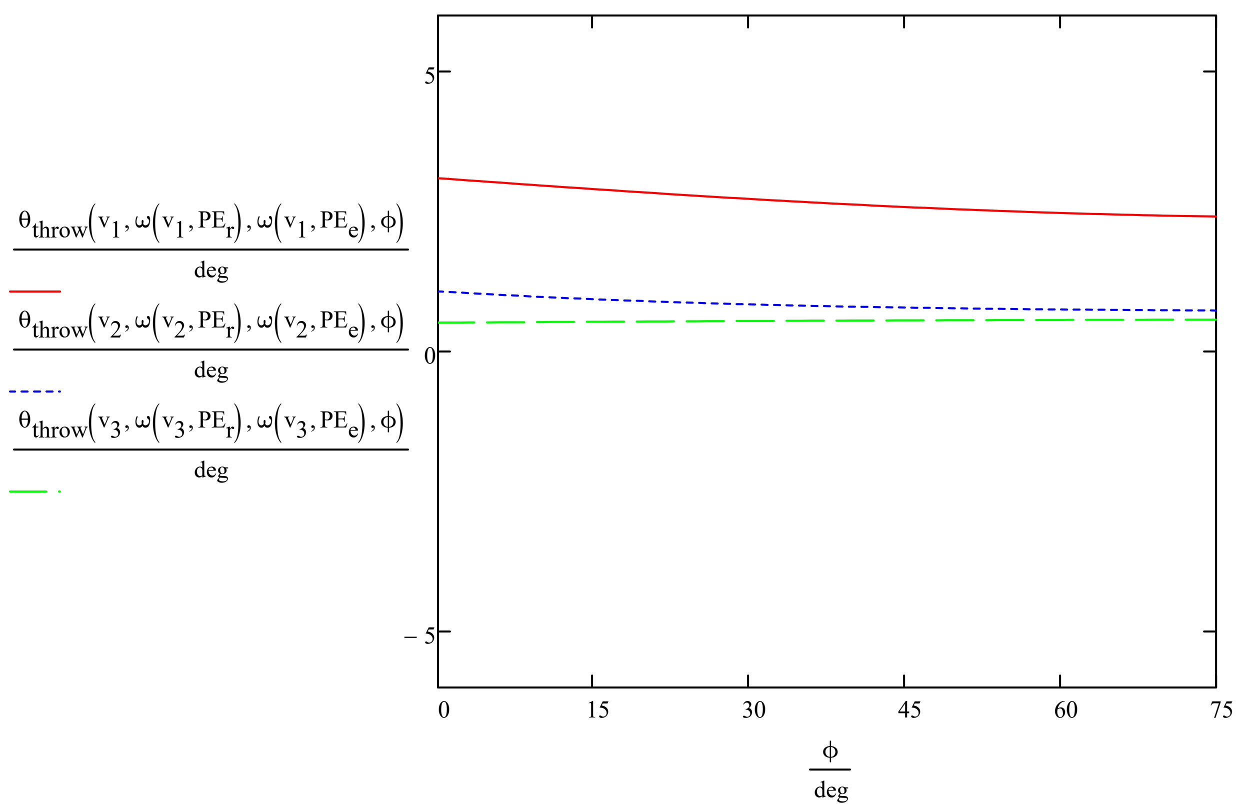
$PE_r := 50\%$ $PE_e := -50\%$



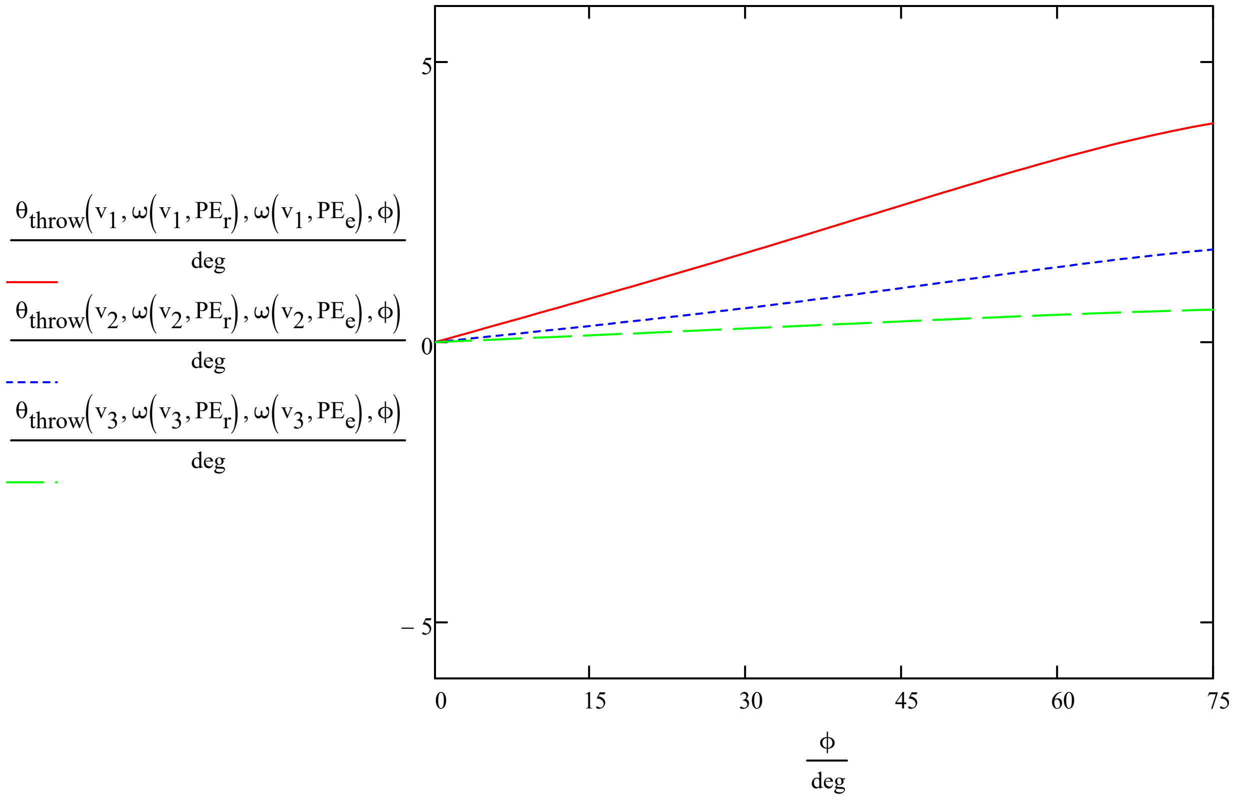
$PE_r := 50\%$ $\quad$ $PE_e := -75\%$



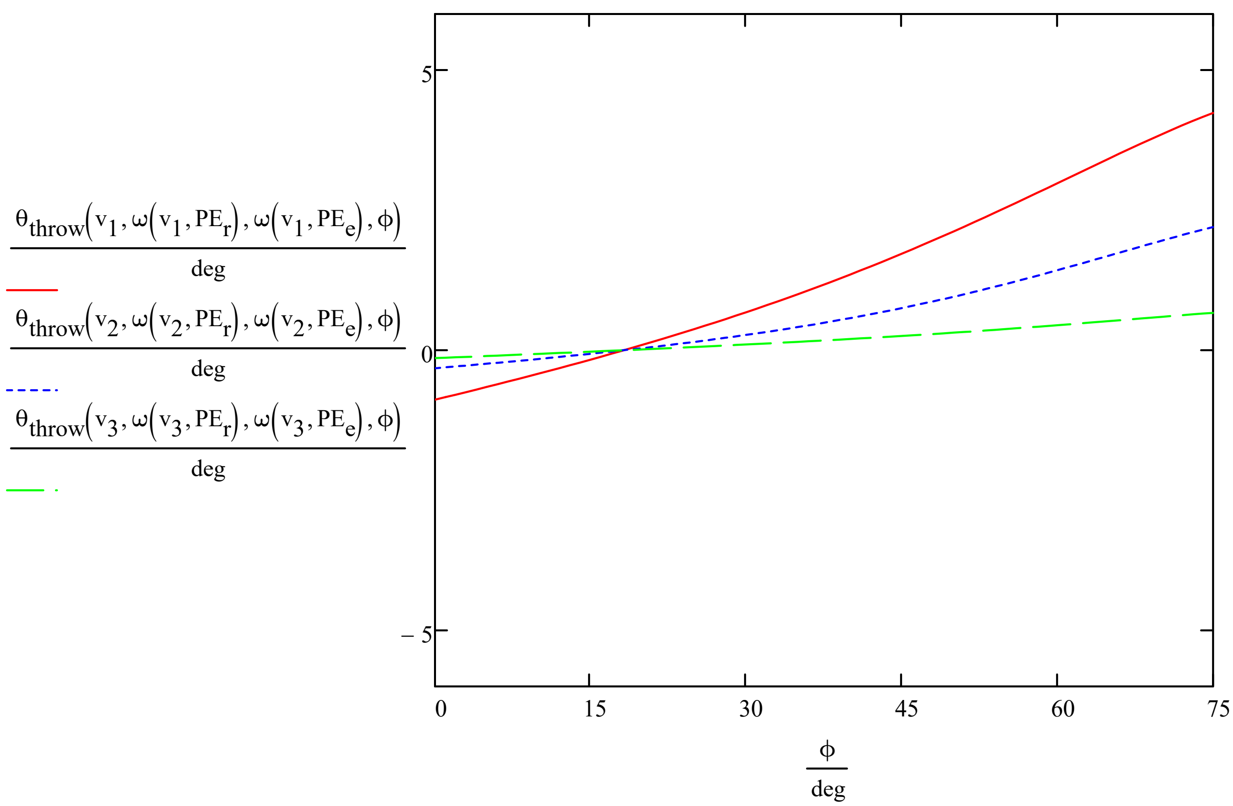
$PE_r := 50\%$ $\quad$ $PE_e := -100\%$



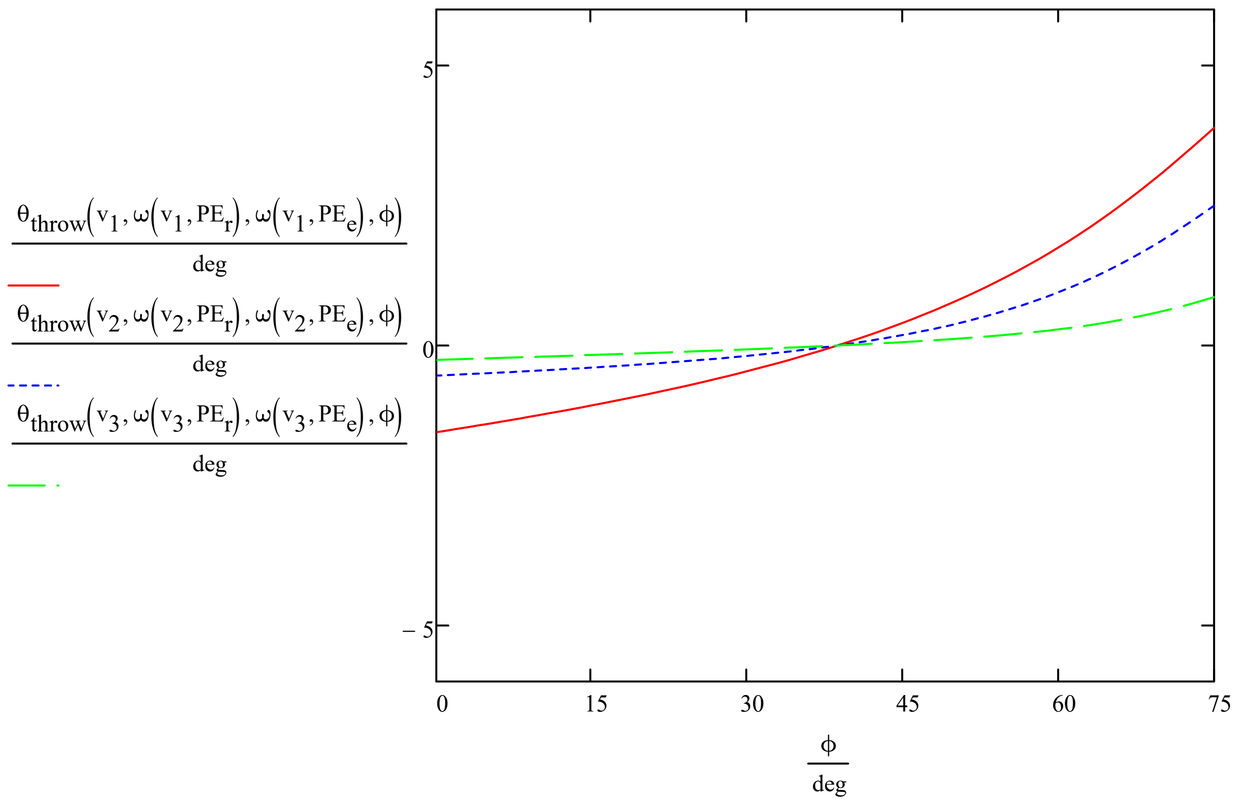
$PE_r := 100\%$ $PE_e := 0\%$



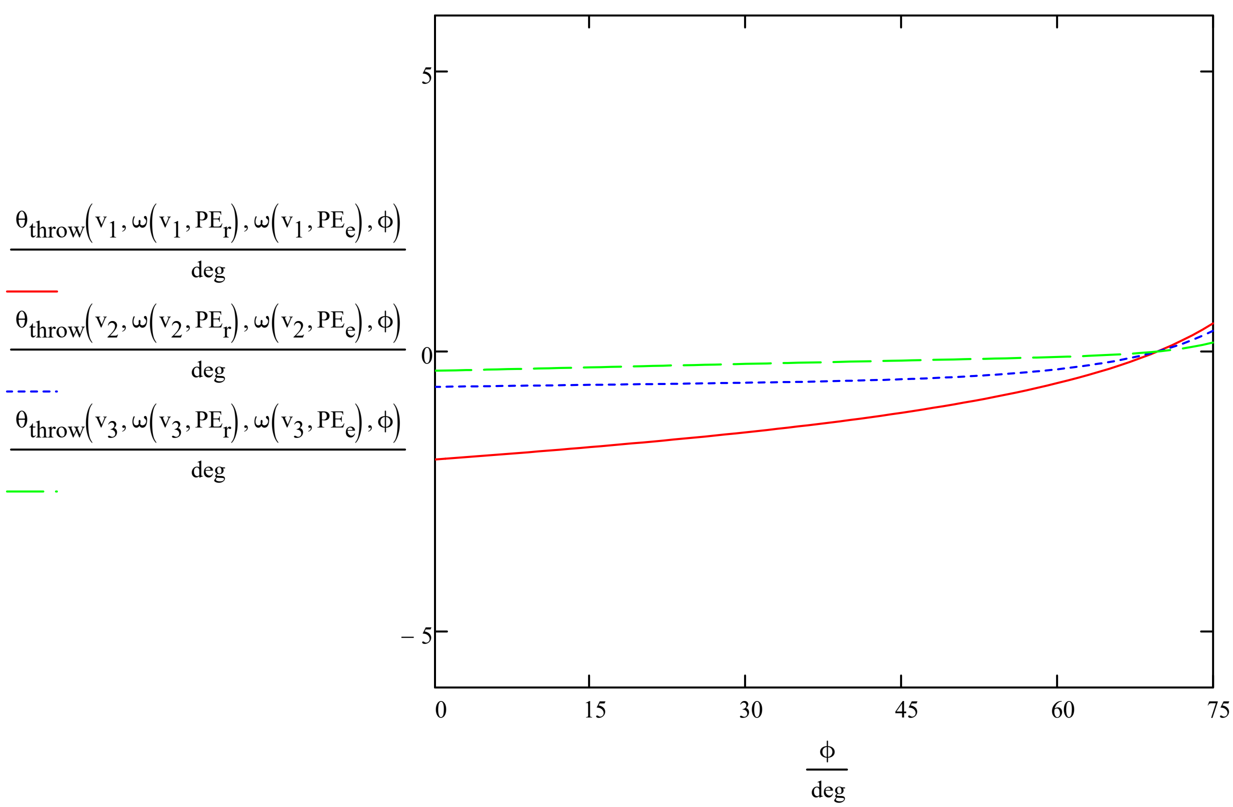
$PE_r := 100\%$ $PE_e := 25\%$



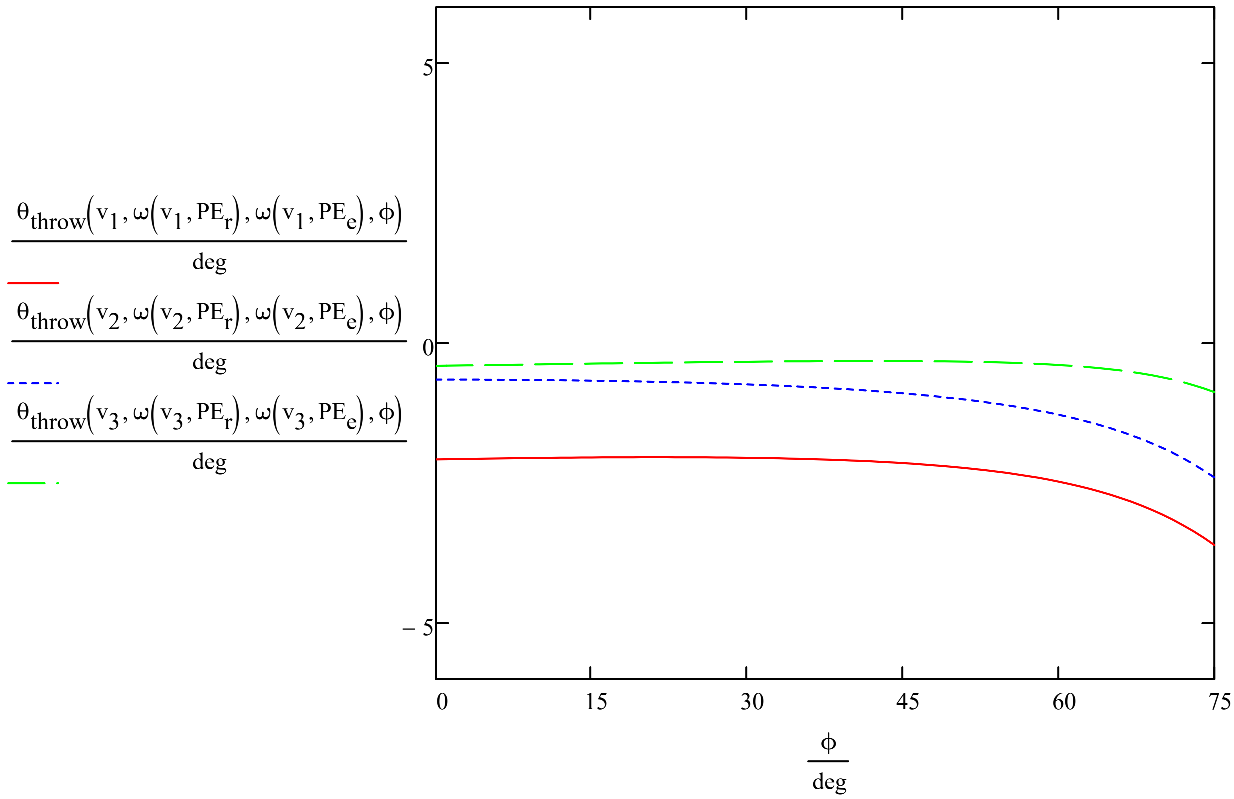
$PE_r := 100\%$ $PE_e := 50\%$



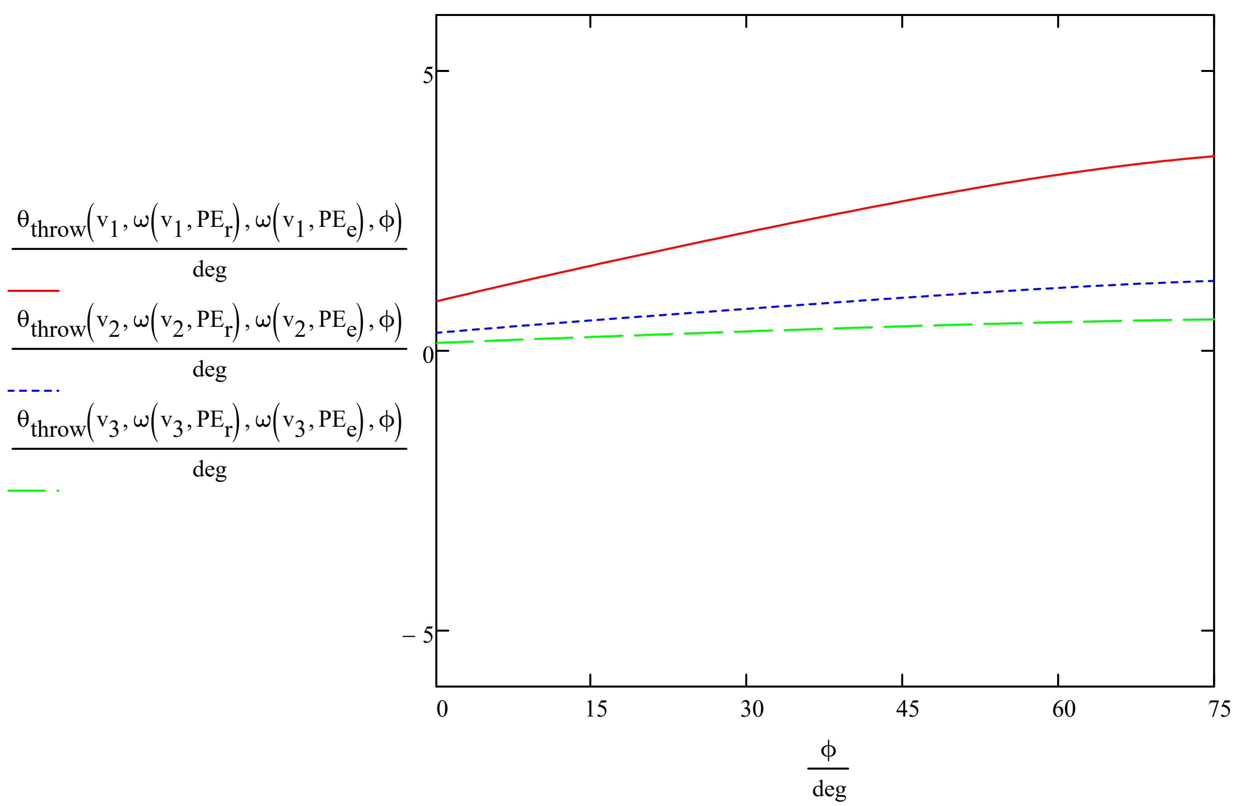
$PE_r := 100\%$ $PE_e := 75\%$



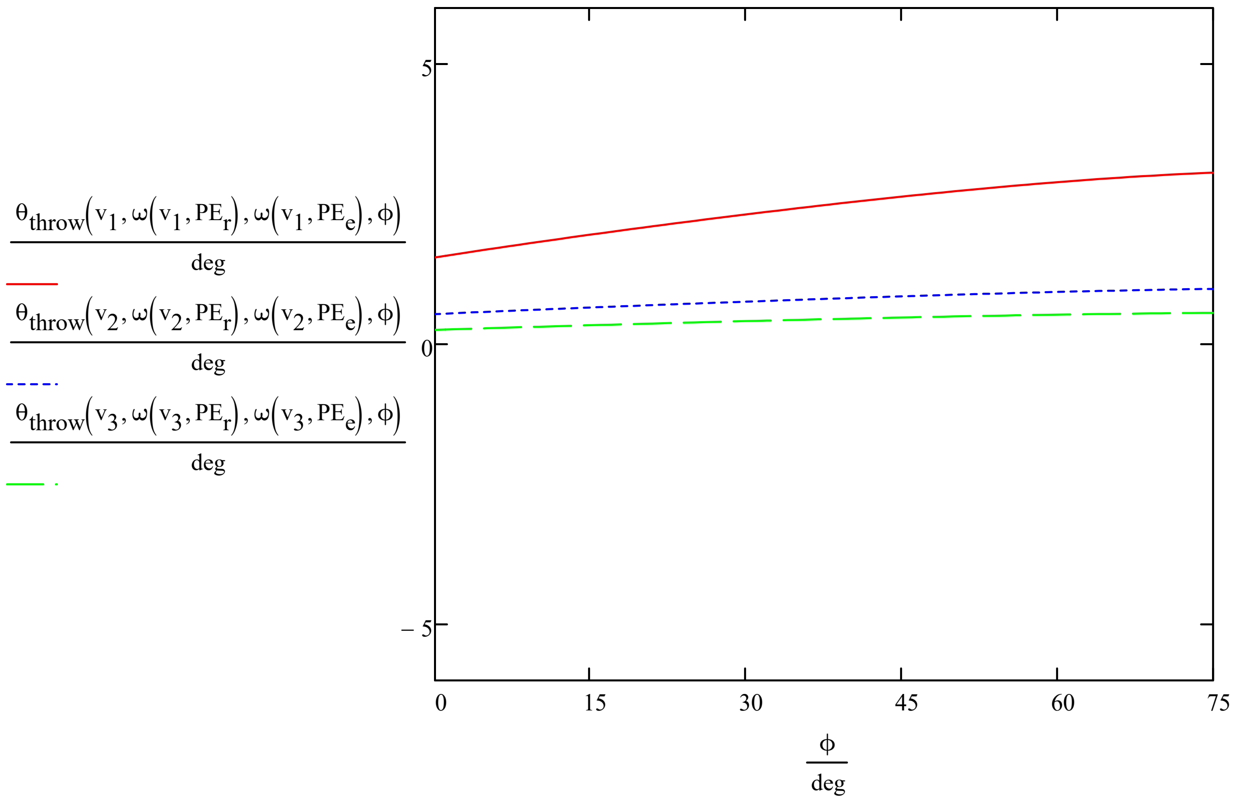
$PE_r := 100\%$ $PE_e := 100\%$



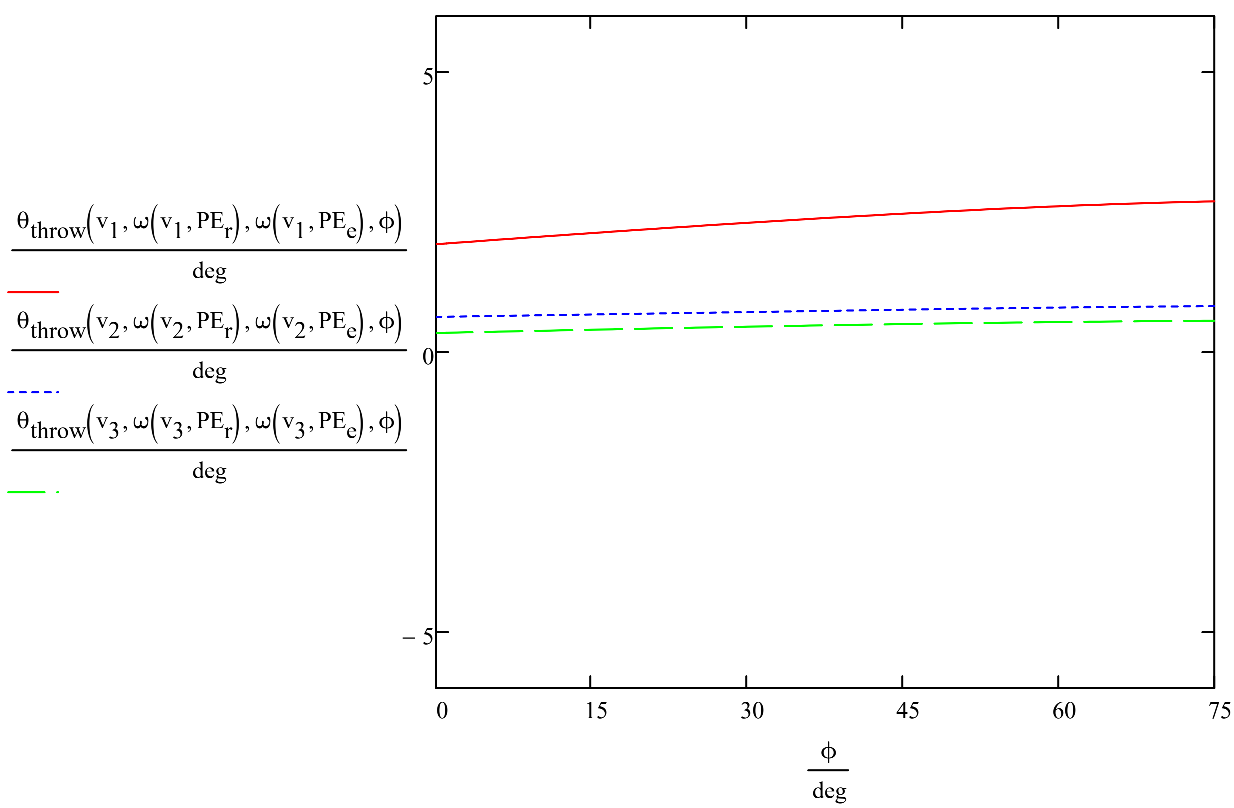
$PE_r := 100\%$ $PE_e := -25\%$



$PE_r := 100\%$ $PE_e := -50\%$



$PE_r := 100\%$ $PE_e := -75\%$



$PE_r := 100\%$ $\quad PE_e := -100\%$