technical proof

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TP B.1
Squirt angle, pivot length, and tip shape

supporting:
"The Illustrated Principles of Pool and Billiards"
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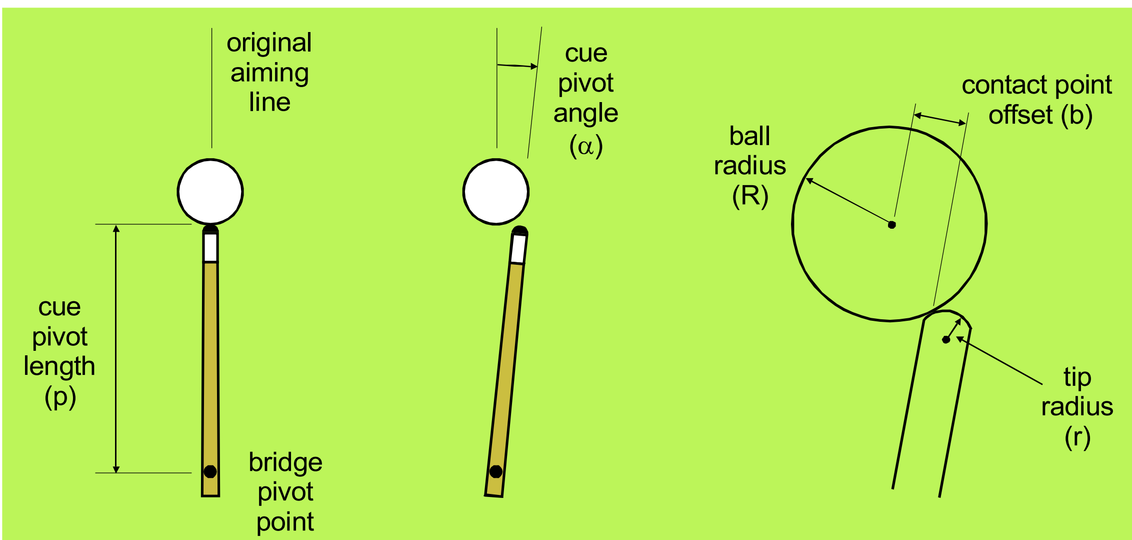
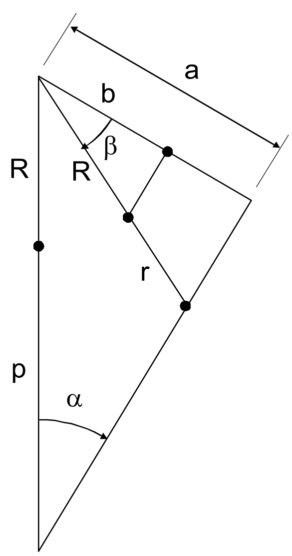




ball radius and mass:

$$R := \frac{2.25 \cdot in}{2} \qquad m_b := 6 \cdot oz$$

tip radii:

$$r_{dime} := \frac{0.705 \cdot in}{2}$$

$$r_{nickel} := \frac{0.835 \cdot in}{2}$$

$$r_{break} := 0.5 \cdot in$$

From the drawings above,

$$b = R\cos(\beta) \qquad \cos(\beta) = \left(\frac{a}{R + r}\right) \qquad a = (p + R)\cdot\sin(\alpha) \tag{1}$$

so the contact point tip offset distance "b" is related to the cue pivot angle (α), bridge length (p), and tip radius (r) according to:

$$b(\alpha, p, r) := \frac{R}{R + r}\cdot(p + R)\cdot\sin(\alpha) \tag{2}$$

This equation can be rearranged to solve for the natural pivot length for the cue for a given squirt angle (α), tip offset (b), and tip radius (r):

$$p(\alpha, b, r) := \frac{(R + r)\cdot b}{R\cdot\sin(\alpha)} - R \tag{3}$$

The assumption here is the cue pivot angle (α) exactly cancels the cue squirt angle (α), resulting in a straight shot.

From TP A.31 (Equation 13), the squirt angle is a function of tip offset (b) and the endmass properties of the cue:

$$\alpha\left(b, m_r\right) := \operatorname{atan}\left[\frac{\frac{5}{2}\cdot\frac{b}{R}\cdot\sqrt{1 - \left(\frac{b}{R}\right)^2}}{1 + m_r + \frac{5}{2}\cdot\left[1 - \left(\frac{b}{R}\right)^2\right]}\right] \tag{4}$$

where m_r is the **endmass ratio** for the cue defined by:

$$m_r = \frac{m_b}{m_e} \tag{5}$$

where m_b is the mass of the ball and m_e is the effective endmass of the cue. The mass ratio m_r can be determined experimentally if a squirt value (α_{exp}) is measured for a known offset (b_{exp}). From Equation 4,

$$m_r(b, \alpha) := \frac{5}{2}\cdot\left[\frac{\frac{b}{R}\cdot\sqrt{1 - \left(\frac{b}{R}\right)^2}}{\tan(\alpha)} - \left[1 - \left(\frac{b}{R}\right)^2\right]\right] - 1 \tag{6}$$

Now let's look at example data and plots for some real cues. From my September '07 BD article, here is some data for several example cues:

Players ("regular" cue):

$b_{exp} := 0.51 \cdot in \qquad \alpha_{exp} := 2.5 \cdot deg$

$mr_{exp} := m_r(b_{exp}, \alpha_{exp}) \qquad mr_{exp} = 20.151$

$p_{exp} := p(\alpha_{exp}, b_{exp}, r_{dime}) \qquad p_{exp} = 14.231\ in$

Predator Z ("low-squirt" cue):

$b_{exp} := 0.51 \cdot in \qquad \alpha_{exp} := 1.8 \cdot deg$

$m_r(b_{exp}, \alpha_{exp}) = 29.158$

$p(\alpha_{exp}, b_{exp}, r_{dime}) = 20.199\ in$

Stinger (break/jump cue):

$b_{exp} := 0.3 \cdot in \qquad \alpha_{exp} := 2.4 \cdot deg$

$m_r(b_{exp}, \alpha_{exp}) = 12.008$

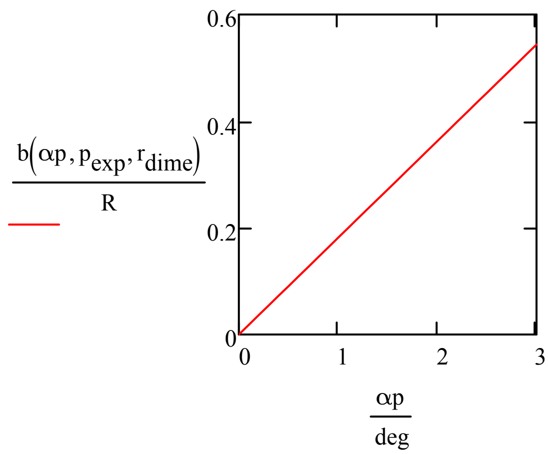
$p(\alpha_{exp}, b_{exp}, r_{break}) = 9.223\ in$

So, as expected, the natural pivot length (p) for a "low-squirt" cue is longer than for a "regular" cue, and the pivot length for a break cue is much shorter. These calculated numbers are consistent with experimentally-determined pivot lengths (see my November '07 article for more information). Note that the range of endmass ratios reported here (12 to 21) is much smaller and lower than the expected range reported in Ron Shepard's 2001 "Everything you always wanted to know about cue ball squirt, but were afraid to ask" paper (20 to 100) Also, the pivot lengths are much longer than the values reported by Platinum Billiards (www.platinumbilliards.com/rating_deflect.php).

The Players cue numbers (mr_{exp} and p_{exp}) are used in the subsequent analysis as an example.

So for a fixed bridge length, how does tip offset vary with the cue pivot angle:

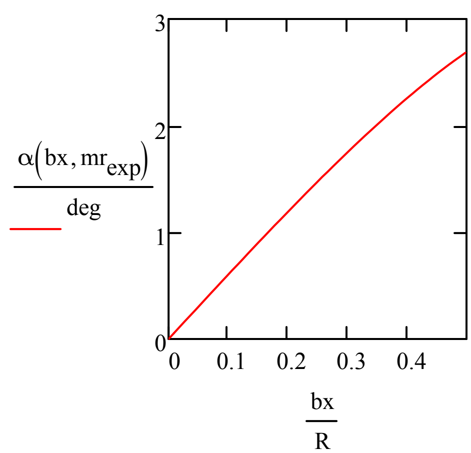
$$\alpha p := 0 \cdot deg, 0.1 \cdot deg .. 3 \cdot deg$$



As you can see, **the offset varies nearly linearly over typical cue pivot angles**.

So how does a cue's squirt angle vary with tip offset:

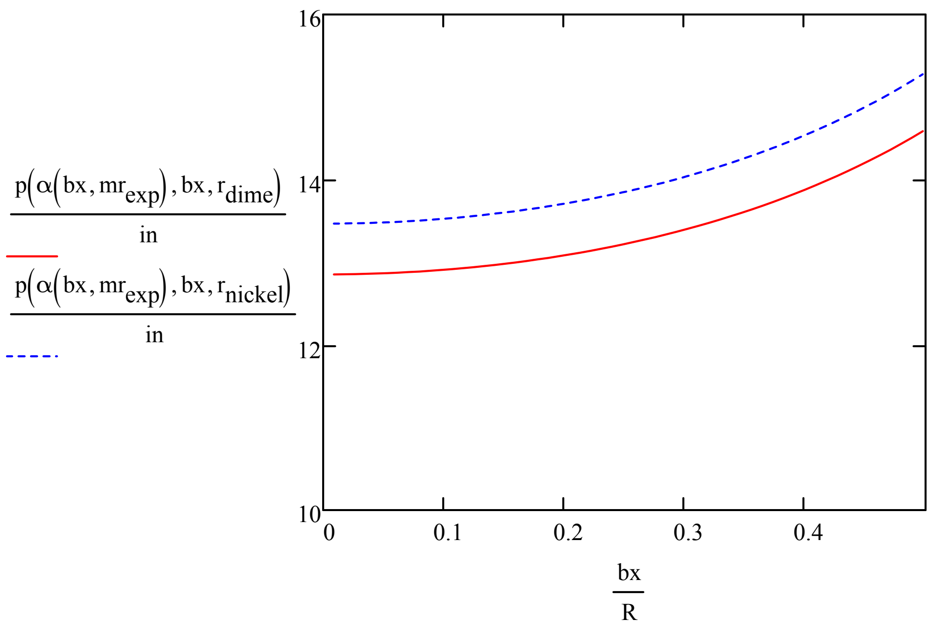
$$bx := 0 \cdot in, 0.01 \cdot in .. 0.5 \cdot R$$



As you can see, **the squirt also varies nearly linearly over the range of possible tip offsets**. This is why the BHE and FHE aim compensation methods are effective (see my November '07 article).

For the cue pivot angle to exactly cancel the squirt angle, the pivot length needs to be a certain value. Does this value change with tip offset and the size of the tip:

$$bx := 0.01 \cdot in, 0.02 \cdot in .. 0.5 \cdot R$$



The answer is yes ... **the required pivot length does vary a little with both tip offset and tip radius**. So when comparing different cues with experimental measurements, it is important to use the same tip shape and the same amount of tip offset (English) for each cue.

For 50% English,

$$bx := 0.25 \cdot R$$

$$\frac{p(\alpha(bx, mr_{exp}), bx, r_{nickel})}{p(\alpha(bx, mr_{exp}), bx, r_{dime})} - 1 = 4.774\,\%$$

So a cue with a tip of nickel radius has a natural pivot length about 5% longer than the same cue with a tip of dime radius.

Now, what is the effect of tip size on the accuracy of a near center-ball hit? This is an extension of the analysis in TP A.10. We will compare two different bridge lengths:

$$p_{short} := 6 \cdot in \qquad\qquad p_{long} := 18 \cdot in$$

Let's assume a near cent-ball hit, where the shooter has a cue pivot-angle error of up to 1/2 a degree:

$$\Delta\alpha := 0.5 \cdot deg$$

If the bridge distance happens to match the natural pivot length for the cue, the cue pivot angle will cancel the squirt angle, and the cue ball will still head in the aiming-line direction. However, the cue pivot angle will create tip offset and English. Here's how the percent English changes with pivot angle error for various combinations of bridge lengths and tip shapes:

$$\frac{b\left(\Delta\alpha, p_{short}, r_{dime}\right)}{0.5R} = 8.416\,\% \qquad \frac{b\left(\Delta\alpha, p_{long}, r_{dime}\right)}{0.5R} = 22.592\,\%$$

$$\frac{b\left(\Delta\alpha, p_{short}, r_{nickel}\right)}{0.5R} = 8.062\,\% \qquad \frac{b\left(\Delta\alpha, p_{long}, r_{nickel}\right)}{0.5R} = 21.64\,\%$$

$$\frac{b\left(\Delta\alpha, p_{short}, r_{break}\right)}{0.5R} = 7.653\,\% \qquad \frac{b\left(\Delta\alpha, p_{long}, r_{break}\right)}{0.5R} = 20.541\,\%$$

So a shorter bridge is dramatically better if you want to minimize English effects with a non-perfect center-ball hit. A flatter tip also reduces the impact of error with a center-ball hit. However, a shorter bridge can cause stroke problems (e.g., not enough length for smooth acceleration) and it fails to take advantage of the squirt-canceling natural-pivot-length effect of the cue (see my November '07 article). And a flatter tip is not suitable for applying English on non-center-ball hits. See my January '08 article for more information.

So what affect does ball mass have on the natural pivot length for a cue and cue ball combination?

Here is the data for the Players cue with a regulation-weight cue ball:

$$b_{exp} := 0.51 \cdot in \qquad \alpha_{exp} := 2.5 \cdot deg$$

$$mr_{exp} := m_r(b_{exp}, \alpha_{exp}) \qquad mr_{exp} = 20.151$$

$$p_{exp} := p(\alpha_{exp}, b_{exp}, r_{dime}) \qquad p_{exp} = 14.231\ in$$

For a 10% heavier cue ball, the mass ratio (mb/me) would be slightly larger, creating a smaller squirt angle and a longer pivot length:

$$mr_{heavy} := 1.1 \cdot mr_{exp}$$

$$\alpha_{heavy} := \alpha(b_{exp}, mr_{heavy}) \qquad \alpha_{heavy} = 2.3\ deg$$

$$p(\alpha_{heavy}, b_{exp}, r_{dime}) = 15.565\ in$$

$$\frac{p(\alpha_{heavy}, b_{exp}, r_{dime})}{p_{exp}} - 1 = 9.381\ \%$$

9% longer pivot length

For a 10% lighter cue ball, the mass ratio (mb/me) would be slightly smaller, creating a larger squirt angle and a shorter pivot length:

$$mr_{light} := 0.9 \cdot mr_{exp}$$

$$\alpha_{light} := \alpha(b_{exp}, mr_{light}) \qquad \alpha_{light} = 2.738\ deg$$

$$p(\alpha_{light}, b_{exp}, r_{dime}) = 12.896\ in$$

$$\frac{p(\alpha_{light}, b_{exp}, r_{dime})}{p_{exp}} - 1 = -9.379\ \%$$

9% shorter pivot length